Teaching Derivatives Concepts with Computational Techniques

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Abstract

This work presents a qualitative analysis about teaching Calculus I using computational software in a classroom at the interdisciplinary Science and Technology Bachelor Course from the Federal University of Jequitinhonha and Mucuri Valleys. This analysis presents a study about the work that we are developing in UFVJM teaching Calculus and Physics with computational modeling techniques. The main purpose is to discuss and evaluate the challenge of teaching and learning derivatives concepts using GeoGebra.

1. Introduction

The Federal University of Jequitinhonha and Mucuri Valleys (UFVJM) mission is to produce and spread knowledge and innovation by integrating teaching, research and extension services as propellers of regional and national development. Therefore it is a great challenge to teach Mathematics in the Interdisciplinary Science and Technology Bachelor (STB) undergraduate course for students with so many social and structural problems [1]. After three years of basic science study, UFVJM provides an undergraduate degree on STB and gives to these students the possibility of transitioning into one of the engineering courses offered by the university. Then, after two more years studying specific engineering disciplines, the student can have another undergraduate degree as an Engineer [1]. The freshman students of the STB course have a great lack of mathematical and physical concepts brought from High School and is very important that these students are able to describe and understand natural phenomena. These are the fundamental reasons that motivate us to teach carefully the fundamental Calculus concepts for STB's students [1,2]. This work presents a teaching strategy that uses technology as an important modeling tool using the GeoGebra software to improve the students' learning process and modeling thinking. We believe that technology can provide a better understanding of some mathematical concepts and ideas that are still abstract to the students. As *D'*Ambrosio said, the transfer of knowledge, particularly of technology, is a crucial issue in the analysis of the development process [1]. *D'*Ambrosio also said that the learning process should not be just to stimulate the reproduction of concepts, neither repetition nor memorization [3]. Researchers who follow the didactics of mathematics line search to adapt teaching methodologies through a sequential structure for the learning process, focusing on the classroom and challenging activities as a stimulus in the production and construction of mathematical knowledge. The Didactic Engineering, proposed by researchers in France and presented by Artigue in [4] for example, follow this thinking. The work proposed here is part of a step sequence presented by the Didactic Engineering and uses technology to encourage and propose challenges to students in the classroom, enhancing the work of the students and the teaching activity. By analyzing the characteristics of mathematical teaching and learning cases, this work presents two points of view on teaching this course, namely what should the instructor teach (i.e., the teaching goal) and what should be learnt by the students (i.e., the orientation of the learning).

2. The Classroom Activities

The activities with GeoGebra were given to two Calculus I classes: STB-A, with 59 students, and STB-B, with 58 students. After twenty five hours of teaching training using theoretical concepts and exploring definitions, theorems and differentiability techniques by the teacher, a research group called Group of Studies in Free Software (GESE) proposed a two-hour workshop on the usage of GeoGebra to explore the derivatives concepts that the teacher has taught in the classroom. Along this small workshop, the GESE also helped the students to finish and understand some exercises proposed by the teacher. To explore the discussion about the teaching and learning process we focus on two

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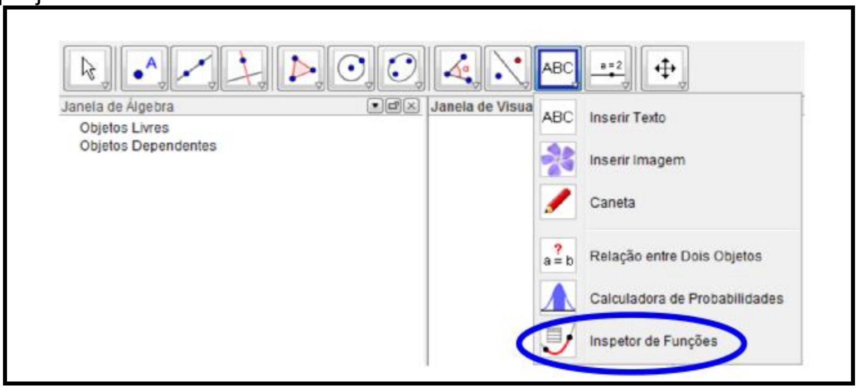
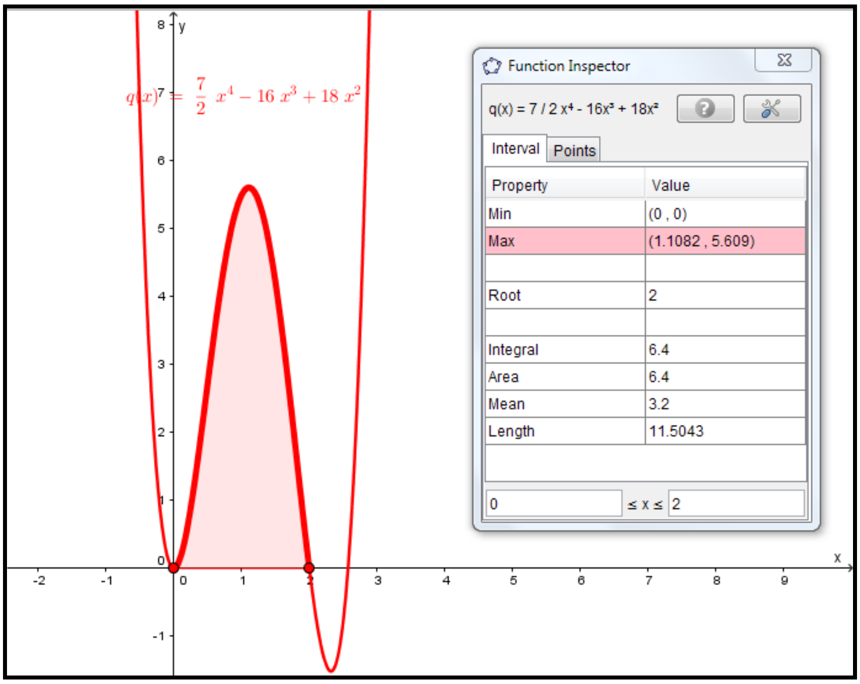
examples that follows and that were addressed in classroom by the teacher and later by GESE during the small workshop in a co-operative learning environment. The first example is presented in Figure 1 and shows the behavior of a forth degree polynomial function along the domain exploring their maximum and minimums points. The second example is presented in Figure 2 and presents the launching of a projectile in three different situations.



(a) Functions Inspector

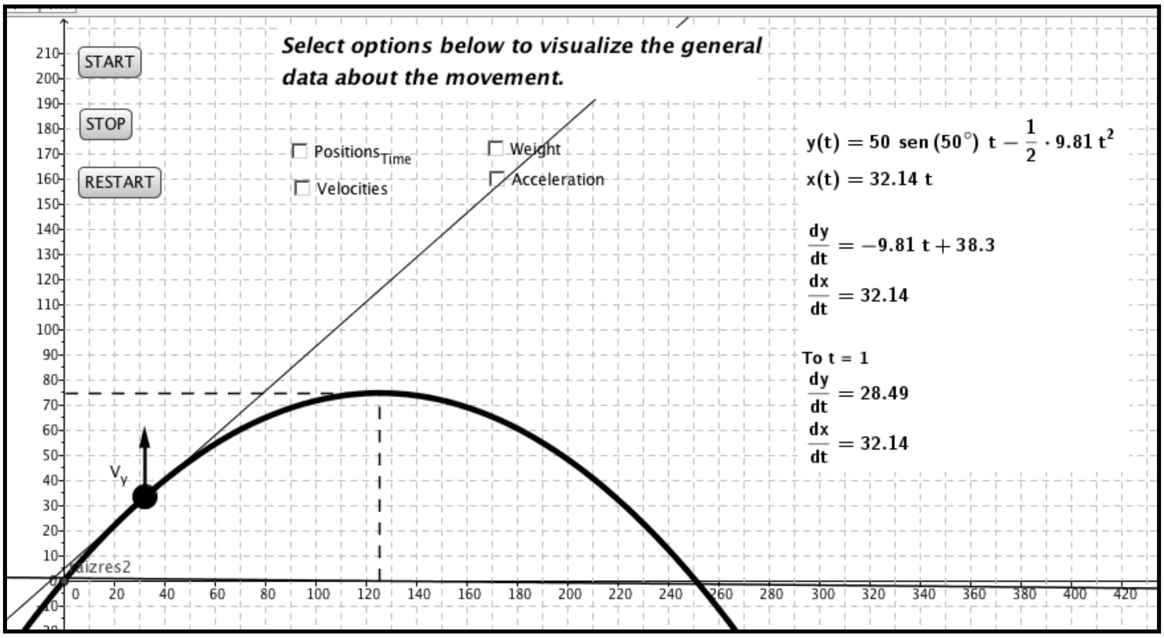


(b) Extreme Value Theorem

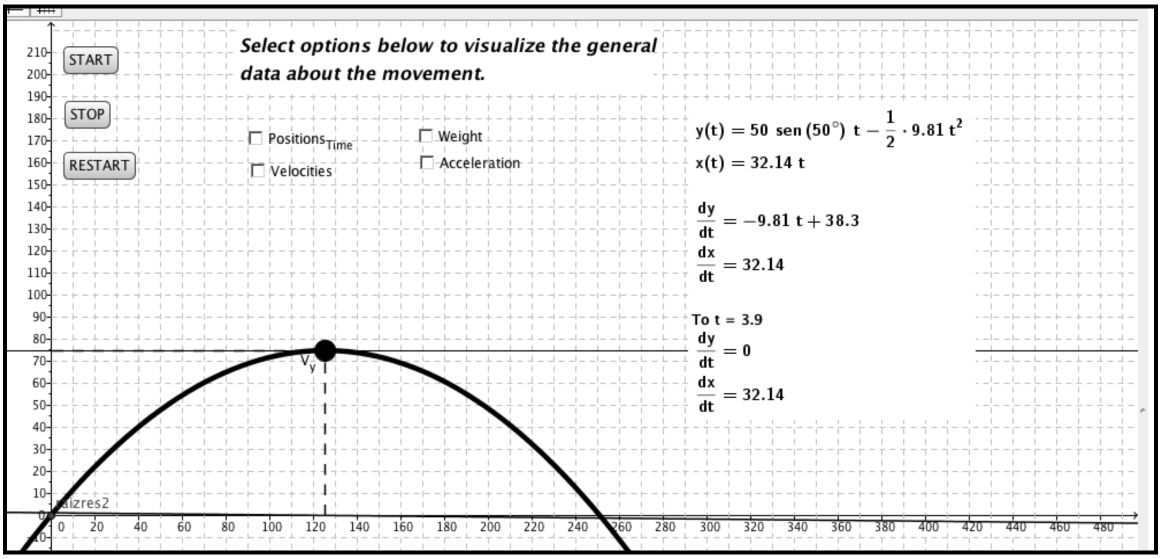
Figure 1: Maximum and Minimum Values.

The activity presented at Figure 1 was proposed with the goal of showing the students the importance to learn how to find the maximum and minimum points of the polynomial function. To find the maximum and minimum points they need to use the tool “functions inspector” (as presents Figure 1 (a)) and then the function and range once we used the Extreme Value, which the final result is presented on Figure 1 (b). This example had an interesting discussion because at this time, there were still a lot of students with a great difficulty to construct and understand elementary functions graphics. After a discussion of around thirty minutes, the learning was reached when the students were able to find the maximum and minimum points of the function and also to conclude the reason why de derivative is zero in these points, which led them to better understand the Fermat’s Theorem for stationary points.

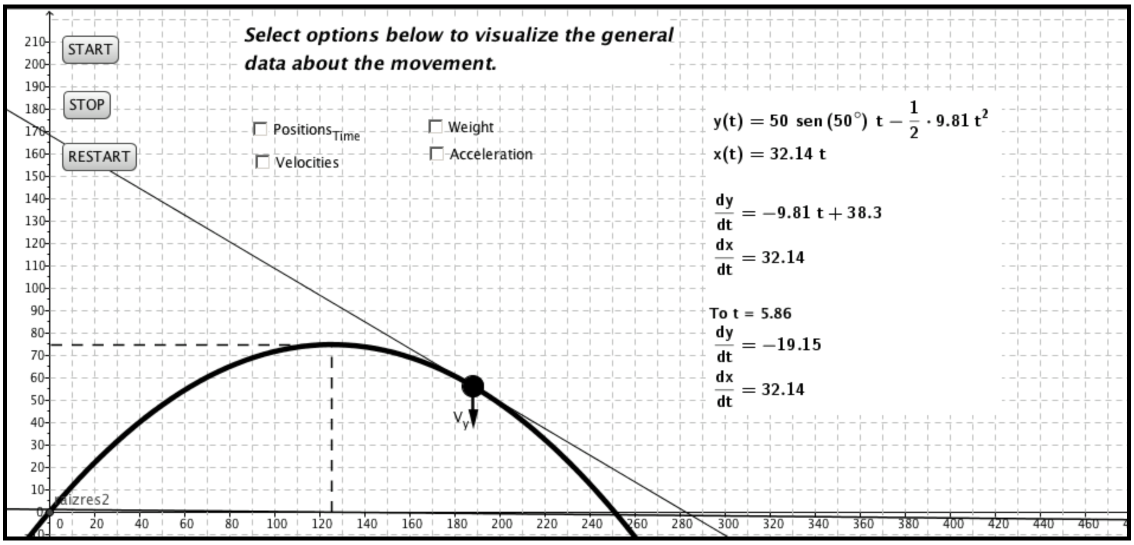
As was initially proposed in Silva, Jardim and Carius in [5], the teacher goal using the mathematical model for a projectile launching presented in the Figure 2 was to make the students understand the natural phenomena using derivatives concepts as an application to solve it.



(a) Tangent line with derivative positive



(b) Tangent line with derivative equal zero



(c) Tangent line with derivative negative

Figure 2: Lauching a projectile.

The tangent lines show the velocity behavior as a vertical component and observing Figure 2 (a), (b) and (c) is possible to make the student build a knowledge bridge connecting the mathematical equations learned at Physics classes on High School with a simple use of the derivation concept. This model is also very important because connects the maximum and minimum point's concepts with the maximum and minimum high and also presents the instant velocity concept in each point inside the dominium function. By the end, as the example on Figure 1 presents, this model also shows how the Fermat's Theorem can be used to conclude that the vertical component of the velocity is zero (Figure 2 (b)). It is important to notice that sometimes, differences between what the teacher expects and what is accomplished by the students are only perceived after the task is completed [6].Then, according to Morrison in [7], the students experienced the fundamental elements of a science education: facts, abstractions and comparing facts with abstractions. It is important to say that this is a crucial experience to develop the mathematical modeling thinking. As Kaiser and Sriraman pointed out in [8], the social-critical perspective on modeling in mathematics education is widespread in the world and also in Brazil as cited by Araujo in [9]. With these simple examples, teacher and GESE could promote in the students interactions between "elementary" and "advanced" mathematical thinking through the cognitive processes of representation, abstraction and generalization, as proposed by Palharini and Almeida in [10].

3. Discussions

It is important to point here that with this teaching technology methodology the students are able to develop the ability to find the responses by themselves using definitions and theorems and also by the use of technology using the GeoGebra software. In this sense, these approaches complement each other. Therefore, to achieve the teacher goals, it is essential to search an appropriate methodology for the teaching of mathematics is essential. The *a posteriori* analysis process of what was proposed *a priori* based on the Didactic Engineering steps construction assumes internal validation. Therefore is crucial to find an efficient validation method so the research reaches a positive result considering the teaching goal. The research group had the participation of an observed researcher and recording of the student's dialogue while they were realizing the activities. This observed researcher noted the important points, recorded the most relevant information and discussed them with the GESE group immediately after the activities conclusion. The reflection about the didactic sequence to be applied at the experimentation moment led the researcher group to a very careful analysis since the number of participating students was quite large. As a teacher strategy and a goal to promote a co-operative environmental only between students, the GESE group worked with the students without the presence of the teacher. In the future, we will insert the teacher in a negotiation environment and evaluate his presence in the students thinking, considering his influence as a negotiator of knowledge that can lead the students to their conclusions. This is an important conclusion because these students in particular prefer to work with the presence of the teacher in the workshop given that they feel more secure about their mathematical thinking conclusions.

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