



Latex as an Ideal Tool for Integration of Ict and Mathematics in Science Teaching

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Abstract

Our research shows that the desktop publishing system LaTeX is the ideal tool for the integration of ICT and mathematics in the science teaching. LaTeX is best known as a system of computer typesetting and is popular among scientists because of its possibility to produce the complex scientific texts (mathematical, physical, technical, chemical and even musical) of the highest typographical quality. But the ideal means of integration of ICT and mathematics it is due to its numerous extensions appeared in the last 15–20 years.

One of such extensions is a package **TikZ** and its extension **tkz-euclid**. Features of these packages are described in detail by their authors Till Tantau and Allan Mattes (see references). From these descriptions we know that the packages allow to produce and describe planimetric drawings in the TeX notation in much the same way as in the ordinary course of geometry: **tkzDrawPoint** (take a point), **tkzLabelPoint** (denote it), **tkzDrawLine** (draw the line), and so on.

Recently, we found that the package **tkz-euclid** permits comfortably and quite naturally produce not only planimetric but also stereometric drawings of the school type. To these findings is devoted our paper.

1. Sliding tangent

In this first section, we demonstrate the interaction of ICT and mathematics with the help of the example given not from geometry but from beginning calculus. Suppose we want to prepare a dynamical model of the tangent line sliding on the waves of the sinusoid. What must we know to do it? We need to know the equation of the tangent line to the graph of a differentiable function at the arbitrary point, the derivation of the sine function and something from LaTeX, namely, that in LaTeX there exist the drawing tool **mfpic** and the package **multido**, which permits to repeat easily calculations by one and the same formula in many points.

More concretely, we take the equation of the tangent line to the graph of differentiable function f at the arbitrary point x_0 ,

$$y = f(x_0) + f'(x_0) \cdot (x - x_0),$$

replace in it " x_0 " by " t ", " f " by " \sin ", get

$$y = \sin(t) + \sin'(t) \cdot (x - t),$$

remember that $\sin' = \cos$, and obtain the equation of the tangent line to function \sin :

$$y = \sin(t) + \cos(t) \cdot (x - t).$$

This means that our tangent line consists of all the points of the form

$$(x, (\sin(t) - \cos(t) \cdot t) + \cos(t) \cdot x)$$

with $x \in \mathbb{R}$. Of course, we cannot draw the whole line, we only can draw some interval of it. We choose the interval of length 2 centred at the point $(t, \sin t)$. Simple calculations show that this interval has ends

$$\left(t \pm \frac{1}{\sqrt{1 + \cos^2 t}}, \sin t \pm \frac{\cos t}{\sqrt{1 + \cos^2 t}} \right).$$

Varying t , we obtain the family of intervals sliding on the sinusoid. Again, the whole sinusoid we cannot draw, we only can draw some its portion. For the picture below we chosen portion over the segment $[-5, 5]$. Once again, it is impossible to draw such intervals for all $t \in [-5, 5]$. We only can draw them for some more or less big *finite* set of points. We take points of the form $t = i \cdot \pi/16$ with natural i from -23 to 23 and apply the package **multido**. With some more details, by placing a page

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with the picture inside `\multido{...}` and putting after pictures (but inside `\multido!`) the command `\newpage` for the transition to a new page, we obtain a series of replacing one other pages with the same sine waves but varying segments of tangents.

Here is the whole code.

```
\documentclass[12pt]{amsart}
\usepackage[metapost]{mpic}
\usepackage{multido}
\usepackage{color}
\usepackage[pdfTeX]{web}
\margins{0pt}{0pt}{0pt}{0pt}
\screensize{1.68in}{5.76in}
\pagestyle{empty}
\parindent=0pt
\begin{document}
\thispagestyle{empty}
\opengraphsfile{tangraphsin}
\multido{\i=-26+1}{53}{
\begin{mpic}[40]{-5.2}{5.2}{-1.5}{1.5}
\axes
\xmarks{-3/2pi,-2pi,-pi,-pi/2,0,pi/2,pi,3/2pi}
\tlabel[cl]{.1,1.41}{\mathit{y}}\tlabel[bc]{5.1,.1}{\mathit{x}}
\tlabel[tc]{4.5,-1.2}{\mathit{y} = \sin x}
\pen{.8pt} \drawcolor{blue}\function{-5,5,.2}{\sin x}
\drawcolor{red}
\lines{(\i*pi/16-(1+(\cos\i*pi/16)**2)**(-1/2), \sin(\i*pi/16)-\cos(\i*pi/16)*((1+(\cos\i*pi/16)**2)**(-1/2))),
((\i*pi/16+(1+(\cos\i*pi/16)**2)**(-1/2), \sin(\i*pi/16)+\cos(\i*pi/16)*((1+(\cos\i*pi/16)**2)**(-1/2))))}
\point[2pt]{((\i*pi)/16, \sin((\i*pi)/16))}
\tlabel[tc]{-4.71,-.1}{\mathit{-\frac{3}{2}\pi}}\tlabel[tr]{-3.14,-.1}{\mathit{-\pi}}
\tlabel[tc]{-1.57,-.1}{\mathit{-\frac{1}{2}\pi}}\tlabel[tl]{.1,-.1}{\mathit{0}}
\tlabel[tc]{1.57,-.1}{\mathit{\frac{1}{2}\pi}}\tlabel[tr]{3.14,-.1}{\mathit{\pi}}
\tlabel[tc]{4.71,-.1}{\mathit{\frac{3}{2}\pi}} %
\end{mpic}
\newpage
}
\end{document}
```

And here is one slide of the whole model consisting with 53 replacing one other slides.

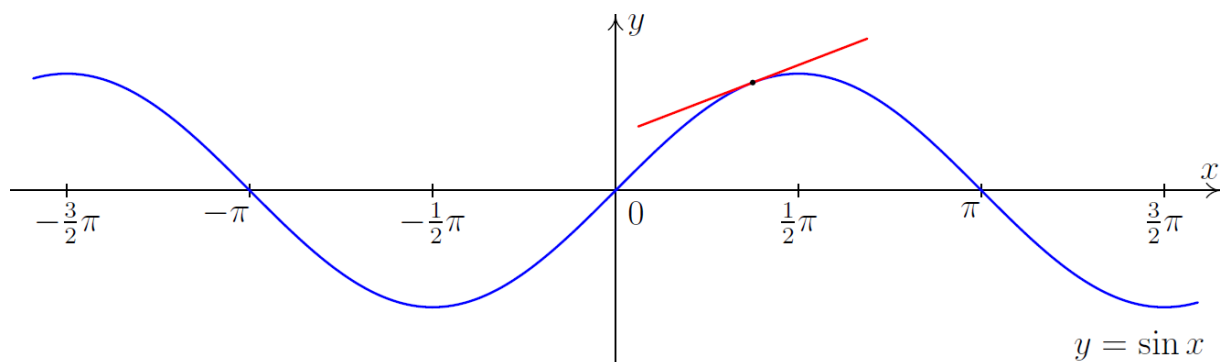


Fig. 1. Sliding tangent line to the graph of sin

2. Drawing a right circular cone

Now we turn to drawings in the stereometry. First of all, we look at drawing a cone. In the literature and the Internet one can find a lot of incorrect drawings of cones similar to the following:

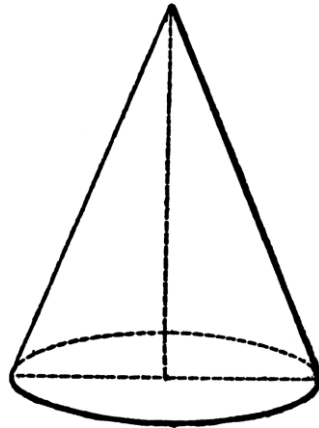


Fig.2. Incorrect drawing of a cone (from the Internet).

What is incorrect in this picture? The base of a cone must be represented by an ellipse. Contour generators of the conic surface must be tangential to this ellipse. But the tangent lines to the ellipse in the ends of any diameter are parallel. So, contour generators of the conic surface (which both go through the point representing the vertex of the cone) cannot go through the ends of a diameter.

Now we must draw a correct picture of a cone. In order to do this, remember that ellipse is compressed or stretched circle. Let us choose compression. Draw a circle, select a point for the vertex of a cone, draw two tangents to the circle through this point and compress this picture along the height. We obtain a correct picture of a cone! To do this we use the package **tkz-euclide** [3].

Here is the code:

```
\begin{tikzpicture}
\tkzSetUpPoint[size=6]
\begin{scope}[yscale=.5]
\tkzDefPoint(0,0){O_1}\tkzDefPoint(0,2.5){R}\tkzDefPoint(0,10){A}
\tkzTangent[from = A](O_1,R) \tkzGetPoints{B}{C}
\tkzDrawArc[dashed](O_1,C)(B)
\tkzDrawArc(O_1,B)(C)
\tkzDefPointBy[symmetry=center O_1](B)\tkzGetPoint{B'}
\draw[dashed](B)--(B');
\draw (A)--(B');
\draw[dashed] (A)--(O_1); \draw (A)--(B) (A)--(C);
\end{scope}
\tkzDefPoint(0,3.055){P}
\tkzDrawPoints(B,C,B',O_1)
\end{tikzpicture}
```

And here is the resulting picture:

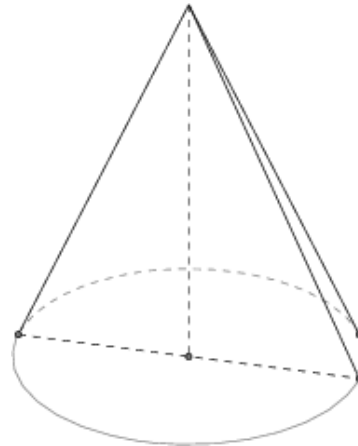


Fig. 3. Correct drawing of a cone

3. Drawing a sphere inscribed in right circular cone

Due to lack of space, we only present the final code and the resulting picture. Here is the code:

```
\begin{document}
\thispagestyle{empty}
\begin{tikzpicture}
\clip (-3,-1.5) rectangle (3,7);
\tkzSetUpPoint[size=6]
\begin{scope}[xscale=2]
\tkzDefPoint(0,0){O_1}\tkzDefPoint(0,1.25){R}\tkzDefPoint(0,6){T}\tkzDefPoint(-1.25,0){A}
\tkzDefPoint(1.25,0){B}\tkzDefPoint(0,1.25){C}\tkzDefPoint(0,-1.25){D}
\tkzTangent[from = T](O_1,R)\tkzGetPoints{t_1}{t_2}
\tkzDrawArc[dashed](O_1,t_2)(t_1)
\tkzDrawArc(O_1,t_1)(t_2)
\tkzDefPointBy[symmetry=center O_1](t_1)\tkzGetPoint{t_3}
\draw[dashed](t_1)--(t_3); \draw[dashed] (T)--(O_1); \draw[dashed] (C)--(D);
\draw (T)--(t_1); \draw (T)--(t_2); \draw (T)--(t_3);
\tkzLabelPoints[above right](O_1)
\end{scope}
\tkzDefPoint(-2.5,-1.25){X1}\tkzDefPoint(12,-1.25){Y1}
\tkzDefPoint(-2.5,0){X}\tkzDefPoint(12,0){Y}
\tkzDefPoint(-2.5,1.25){X2}\tkzDefPoint(12,1.25){Y2}
\tkzDefPoint(-2.5,6){X3}\tkzDefPoint(12,6){Y3}
\tkzDefPoint(3,1.25){C'}
\tkzInterLC[R](X1,Y1)(C',5 cm) \tkzGetPoints{D'}{E'}
\tkzInterLL(X,Y)(C',D') \tkzGetPoint{O_1'}
\tkzDefLine[mediator](C',D') \tkzGetPoints{F'}{G'}
\tkzInterLL(X3,Y3)(F',G') \tkzGetPoint{T'}
\tkzDefCircle[in](C',T',D')
\tkzGetPoint{I} \tkzGetLength{rIN}
\tkzTangent[from = T'](I,O_1') \tkzGetPoints{G'}{H}
\tkzInterLL(T',O_1')(G,H) \tkzGetPoint{J}
\tkzDefLine[parallel=through J](X,Y) \tkzGetPoint{i}
\tkzInterLL(T,O_1)(J,i) \tkzGetPoint{O_2}
\tkzDefLine[parallel=through I](X,Y) \tkzGetPoint{i}
\tkzDefLine[parallel=through G](X,Y) \tkzGetPoint{g}
\tkzInterLL(T,O_1)(G,g) \tkzGetPoint{C_1}
\begin{scope}[xscale=2]
\tkzDefCircle[radius](O_2,C_1)
\tkzTangent[from = T](O_2,C_1) \tkzGetPoints{t_4}{t_5}
\end{scope}
\end{tikzpicture}
```



```

\tkzDrawArc[dashed](O_2,t_5)(t_4)
\tkzDrawArc(O_2,t_4)(t_5)
\end{scope}
\tkzInterLL(T,O_1)(I,i) \tkzGetPoint{O}
\tkzDrawCircle[R,dashed](O,\rIN pt)
\tkzInterLC[R](T',O_1')(I,\rIN pt) \tkzGetPoints{N'}{Z'}
\tkzDefLine[parallel=through N'](X,Y) \tkzGetPoint{n}
\tkzInterLL(T,O_1)(N',n) \tkzGetPoint{N}
\tkzDrawPoints(O,O_1,O_2,t_1,t_2,t_3,N)
\tkzLabelPoints[left](O)
\tkzLabelPoints[right](O_2,N)
\end{tikzpicture}
\end{document}

```

And here is the resulting picture. In it O_1 is the south pole and the centre of the base, O the centre of the sphere, O_2 is the centre of the circle on which the sphere touches the side surface of the cone and N is the north pole.

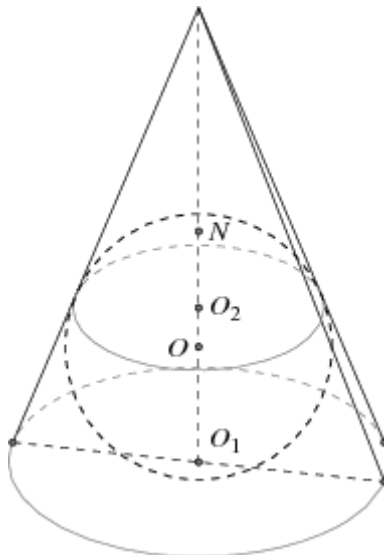


Fig.4. Correct drawing of the sphere inscribed in a cone

Underline that this drawing is accurate; all points in it are in their proper places! Compare with the following incorrect picture (Fig. 5, see [4]). Do you see what is incorrect in this picture?

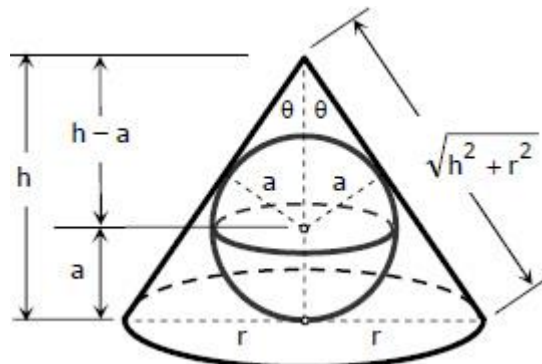


Fig.5. Incorrect drawing of the sphere inscribed in a cone (from the Internet)

References

- [1] <http://www.ctan.org/pkg/> CTAN (Comprehensive TeX Archive Network)



- [2] Tantau, T. "TikZ & PGF: Manual", <http://sourceforge.net/projects/pgf>, 2011.
- [3] Matthes, A. "tkz-euclide 1-16c", <http://altermundus.com>, 2013.
- [4] <http://www.mathalino.com/sites/default/files/users/Mathalino/differential-calculus/065-sphere-inscribed-in-cone.jpg>.