COMPARISON OF MATHEMATICAL ACTIVITIES WITH PRESERVICE TEACHERS: MANIPULATIVES VS. PAPER AND PENCIL

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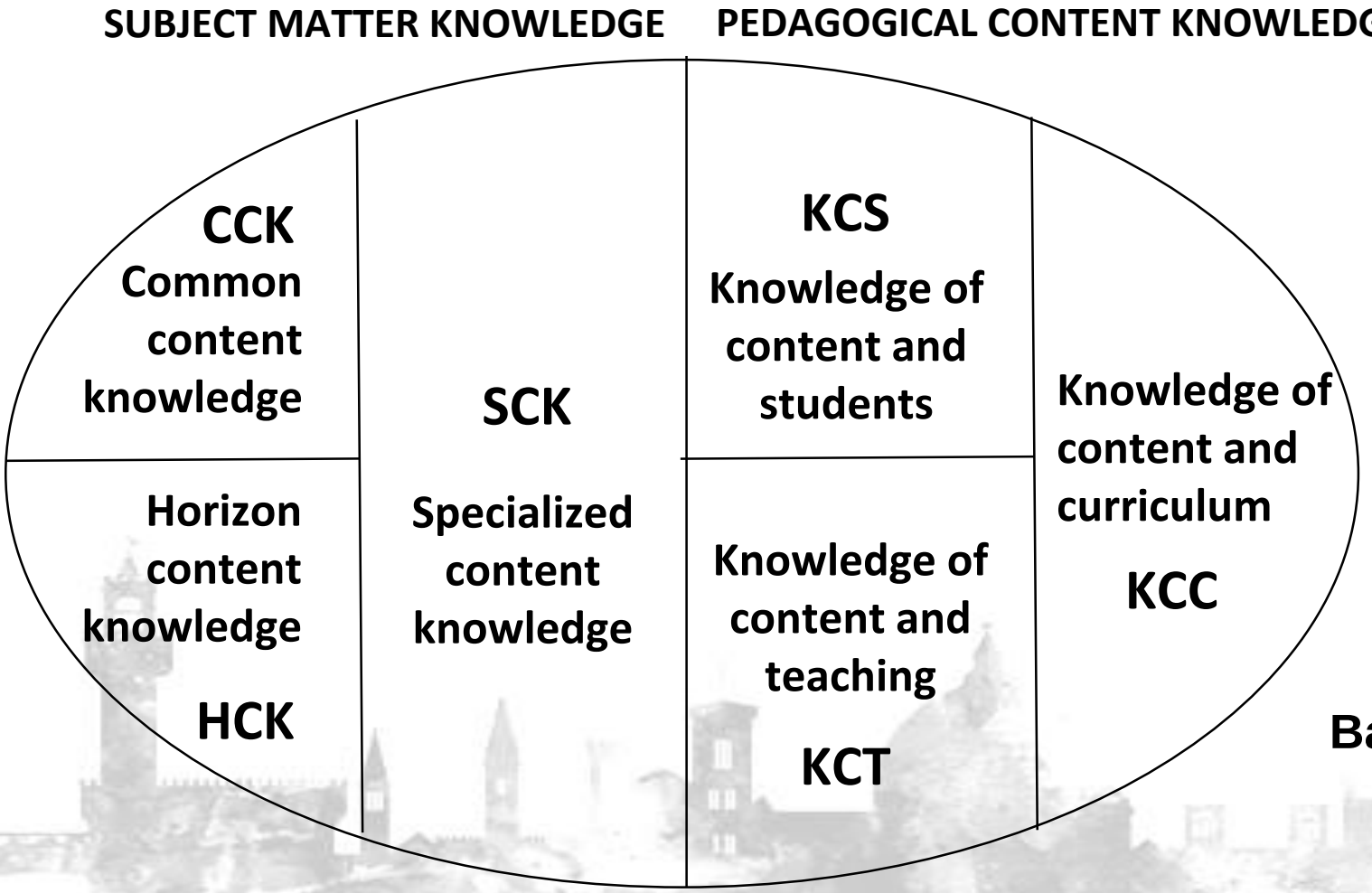
Introduction

- Describe the difficulties associated with the skills, mathematical and didactic, shown by PST in relation to the teaching of arithmetic properties as a first step in the search for their solution.

a:(b:c)=(a:b)xc … 20:(10:2)=(20:10)x2 … 4=4

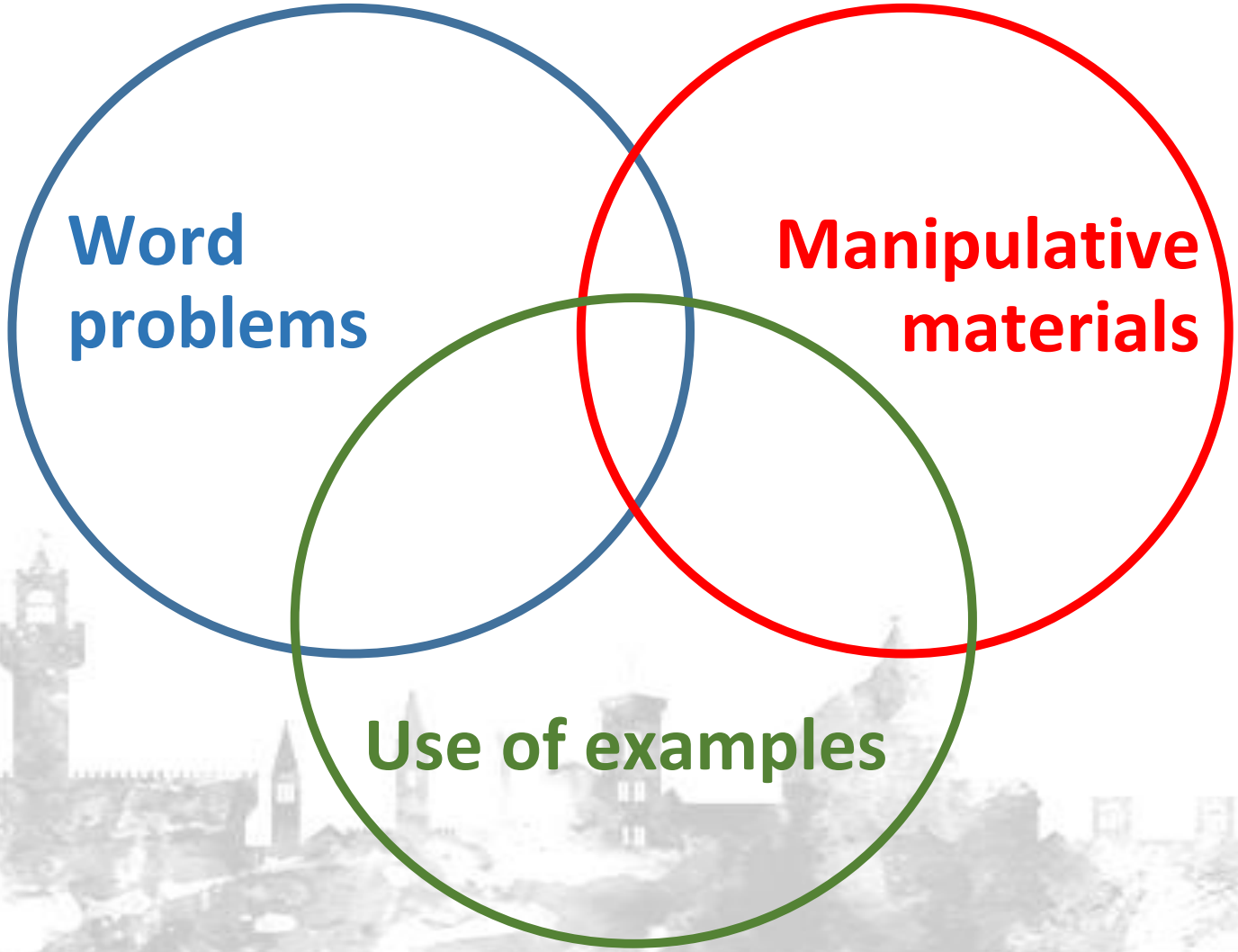
- PST's training of arithmetic properties should include, at least:
  - The learning of specialized mathematical content (SCK) (Ding et al., 2013).
  - The design and implementation of activities for students (KCT) (Butterfield & Chinnapan, 2011; Hill et al., 2008).
  - The construction of statements, contexts, ... to set what Borasi (1986) called word-problems.

THEORETICAL FRAMEWORK



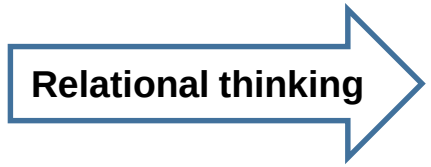
Ball et al. (1998)

THEORETICAL FRAMEWORK



Arithmetic Properties and Word Problems

Understanding the equal sign as a balance between both sides.



Focusing on the relationships between arithmetic operations and their properties, rather than on their calculation

The understanding of the equal sign presents difficulties in primary school students since they tend to consider it as a means to answer.

Arithmetic Properties and Word Problems

One of the options when trying to explain an arithmetic property is to pose a contextualized situation in which an element is unknown (word problem).

It can be different from the idea of mathematical problem in the sense of Carrillo (1998): a meaningful (not mechanical) application of mathematical knowledge to unfamiliar situations.

Ding et al. (2013) propose word problem statements that can be solved in two ways as one of the strategies to present arithmetic properties.

Use of Manipulative Materials…

…to **establish connections** between mathematical ideas and procedures in teaching and learning (Hodgen et al., 2018; Maboya, 2014) . In particular, **to** facilitate **to be able to use arithmetic properties** and the existing relationships between them (Bartolini & Martignone, 2020).

…**to reverse previous arithmetic misconceptions** and facilitate increases in arithmetic knowledge of PSTs (Green et al., 2008).

The **synergy between the student's internal representation and the manipulative representation fosters a deeper understanding** (Moyer, 2001).

Use of Manipulative Materials (MM)

The **selection and use in the classroom** of MM will depend on the **teacher's knowledge** of the mathematical concepts in question (Hiebert, 1997).

Many teachers use MM to change the pace of the subject, provide a more visual model or make it more fun, **misinterpreting the potentiality of the materials** (Moyer, 2001).

A proper selection of material allows word problems to be solved in two different ways (Borasi, 1986) and to justify their solutions on the basis of manipulation (Baroody, 1989).

Use of Examples

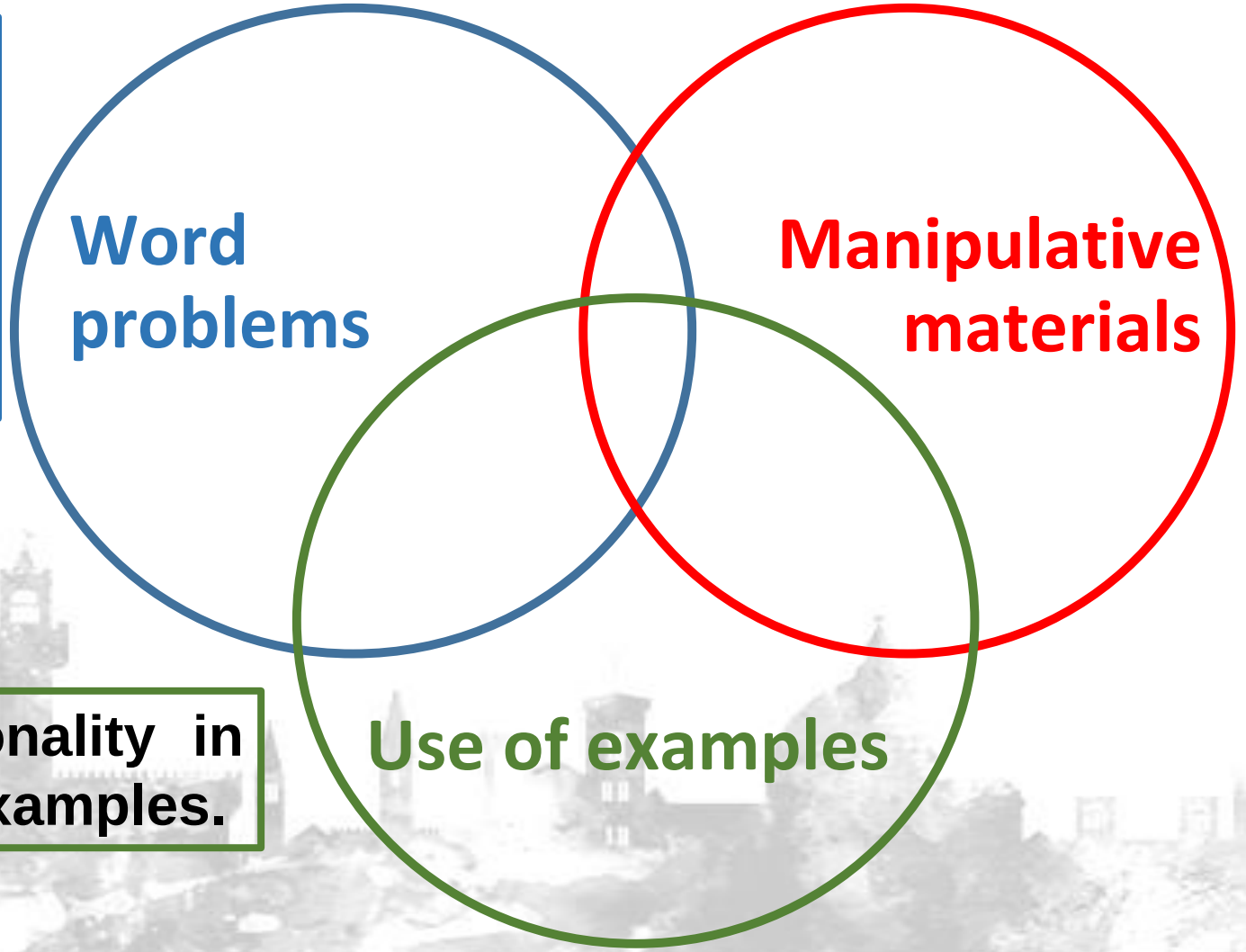
An example is **a particular case** of a broader class of mathematical objects **from which it is possible to generalize** mathematical knowledge (Zodik & Zaslavsky, 2008), i.e., there must be **a didactic intentionality in the choice** of the example.

Possible problems when formulating examples (Rowland, 2008):

- **they hide the role of the variables (two variables take the same value).**
- that the example is not appropriate to illustrate the procedure.
- the examples are randomly generated (with a die for example).

THEORETICAL FRAMEWORK

primary school students tend to consider the equal sign as a means to answer.



To solve wp in two different ways and to justify their solutions via manipulation

a didactic intentionality in the choice of the examples.

Synergy student's internal repr. - manipulative repr. fosters a deeper understanding

Objectives

What aspects of the MKT are employed when PSTs explain arithmetic properties using manipulative materials?

1st study:

1. **To describe aspects of subject matter knowledge (SMK)**, in particular specialized mathematical knowledge (**SCK**) displayed by PSTs when explaining arithmetic properties using manipulative material.

2. **To describe aspects of pedagogical knowledge (PCK),** in particular pedagogical knowledge related to teaching (**KCT**) and curriculum knowledge (**KCC**) that PSTs show when explaining arithmetic properties using manipulative material.

2nd study:

To compare the results obtained in this previous study with manipulative materials and those obtained when the task is presented for solving on paper.

METHOD

Study #1: Make a video explaining, with a material you can manipulate, the property a:(b:c)=(a:b)xc being a, b and c natural numbers

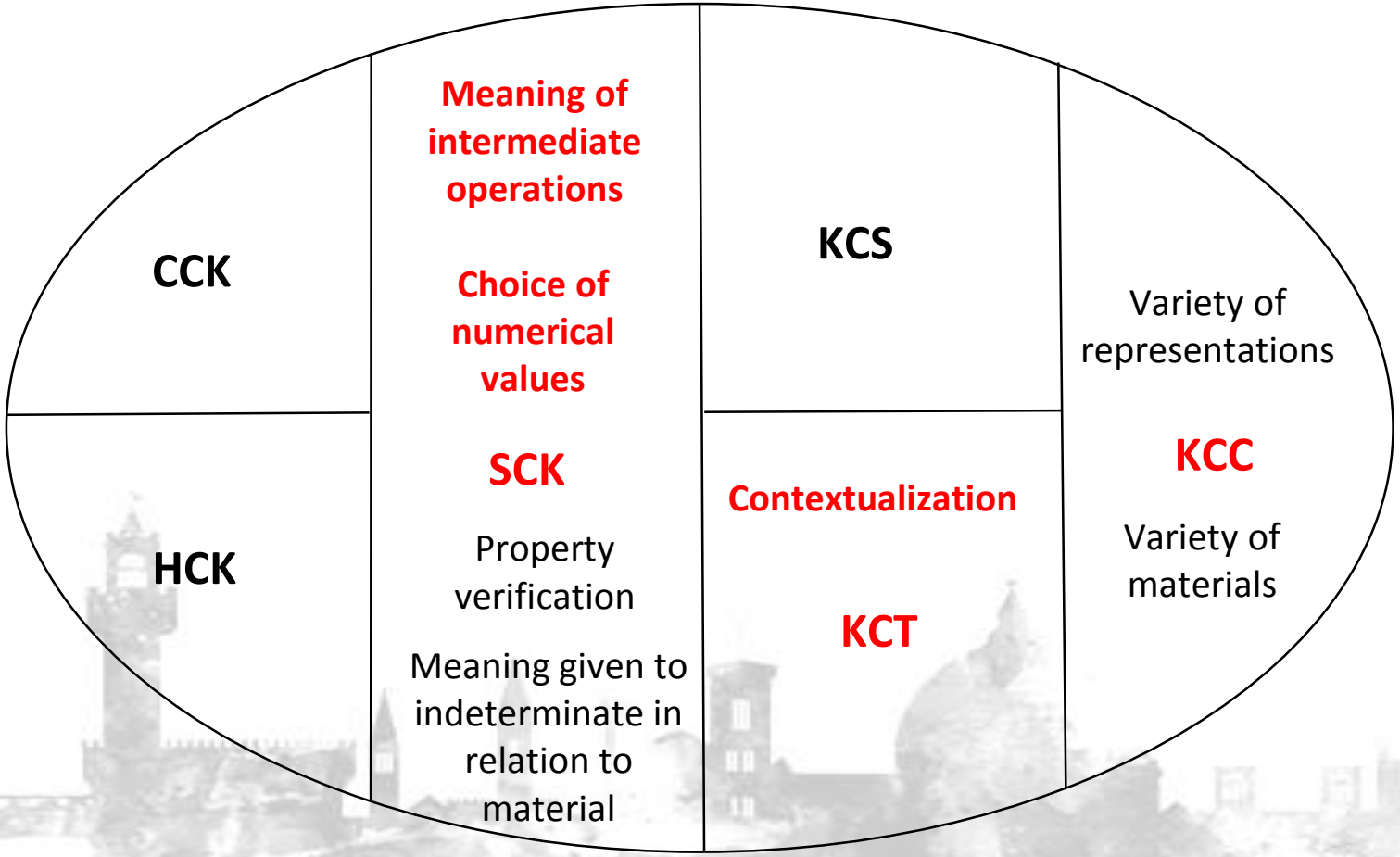
Study #2: Verify the following property a:(b:c)=(a:b)xc being a, b and c natural numbers

27/32 **PSTs** in the 2nd year of the **primary education degree**.
No previous study of any subject that dealt with mathematical content or didactics of mathematics or manipulative materials.

VARIABLES

Variable (subdomain)	Categories written task	Categories video recorded task [6]
Contextualization (KCT)	Includes explicit context/does not include (Borasi, 1986) [7]	
Choice of numerical values (SCK)	No indication/some value is 1/all values are powers/quotient equal to third/different and not powers (Rowland, 2008) [8]	
Meaning of intermediate operations (SCK)	Division: Partitive/quotative (Fischbein et al., 1985) [9] Multiplication: Repeated addition/meaningless	
Variety of representations (KCC)	No representation (only num. expr.) / graphical repr.	NA
Variety of materials (KCC)	NA	Single material/different materials
Meaning given to indeterminate in relation to material (SCK)	NA	Representation only/variable/stable
Property verification (SCK)	NA	Not tested/only with material/only numerically/with material and numerically

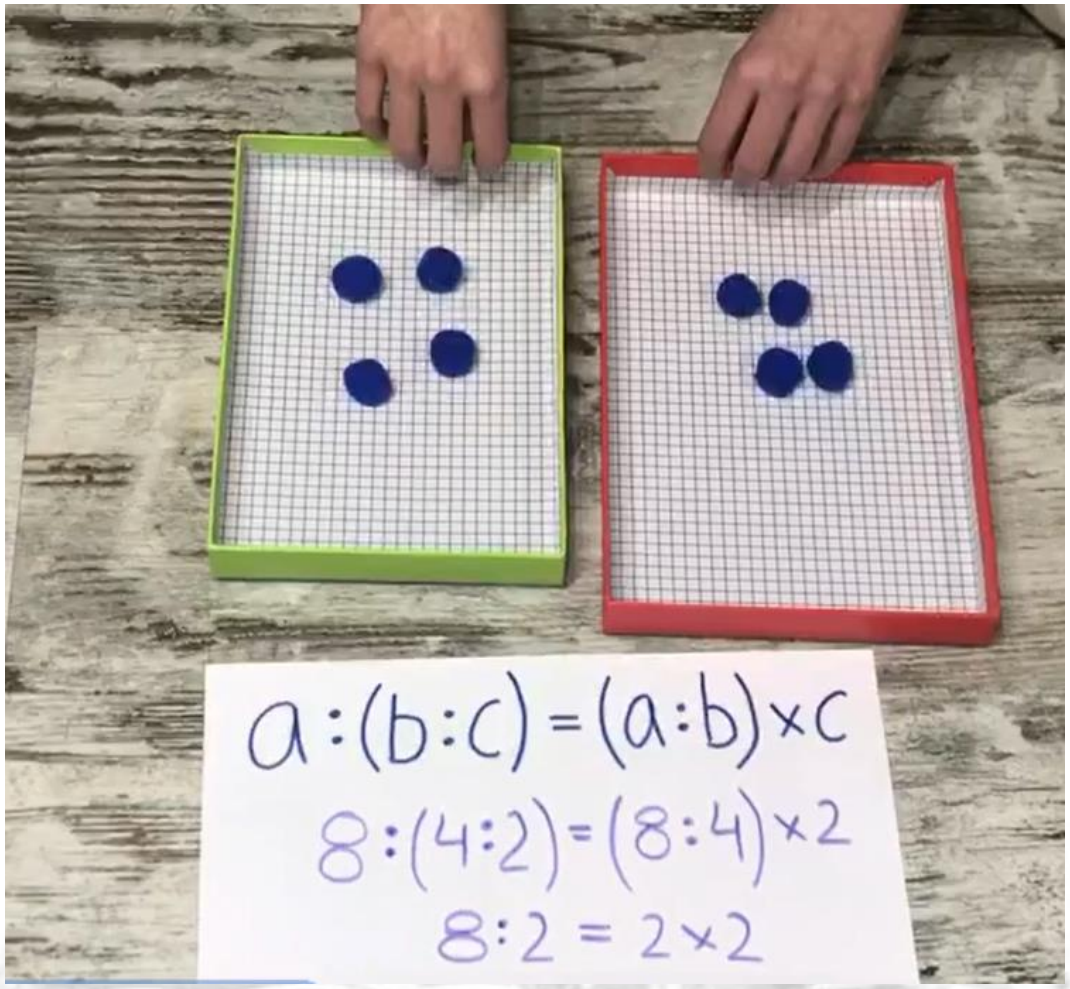
THEORETICAL FRAMEWORK



Choice of numerical values (SCK)

a:(b:c)=(a:b)xc	% written task	% video recorded task (MM)
c = 1	11,8 %	14,8 %
Two of the numbers are powers of the smallest	29,4 %	40,8 %
Quotient equal to the third number	17,7 %	25,9 %
Different choices	41,2 %	14,8 %

Choice of numerical values (SCK)



Student #7: **Two of the numbers are powers of the smallest**

Student #1: **Quotient equal to the third number**

Choice of numerical values (SCK)

$a : (b : c) = (a : b) \cdot c$

$a = 20$

$b = 10$

$c = 5$

$20 : (10 : 5) = (20 : 10) \cdot 5$

$20 : 2 = 2 \cdot 5$

$10 = 10$

Different choice of values (20,10,5)

$a = 12$

$b = 6$

$c = 3$

$12 : (6 : 3) = (12 : 6) \times 3$

$12 : 2 = 2 \times 3$

$6 = 6$

Si se cumple.

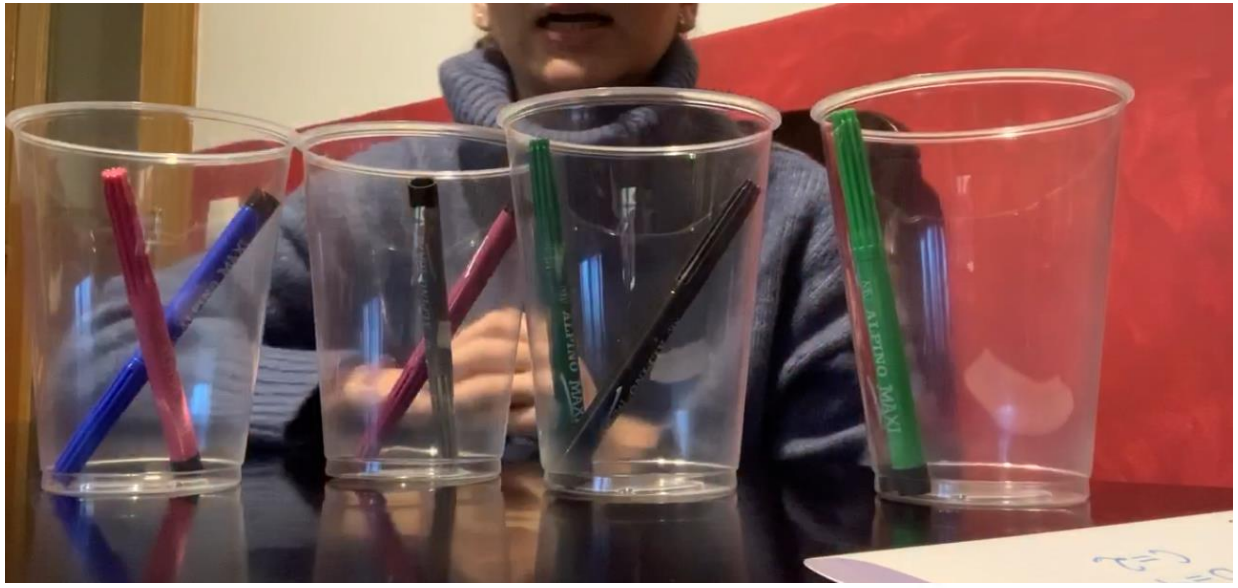
Different choice of values (12, 6, 3)

Meaning of intermediate operations (SCK)

a:(b:c)=(a:b)xc

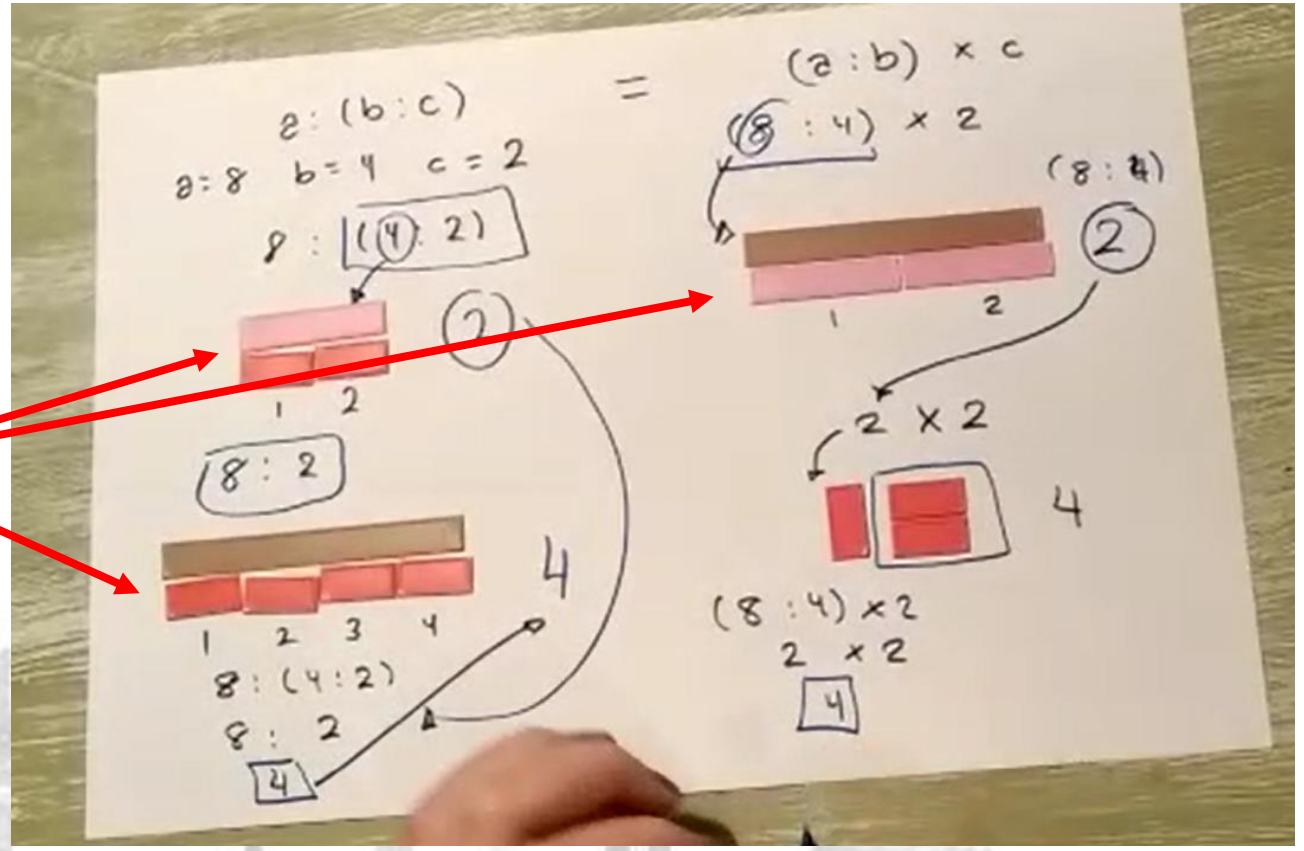
Intermediate operation	written task			video recorded task (MM)		
	Distribution	Grouping	Total	Distribution	Grouping	Total
$b:c$	12,5%	3,1%	15,6 %	51.9%	25.9%	77,8 %
$a:(\square)$	21,9%	0	21,9 %	55.6%	18.5%	74,1 %
$a:b$	15,6%	0	15,6 %	55.6%	18.5%	75 %
	Repeated addition			Repeated addition		
$()xc$	6.3%		6,3 %	59,3 %		59,3 %

Meaning of intermediate operations (SCK)



Student #2: **Distribution (in both sides of the expression)**

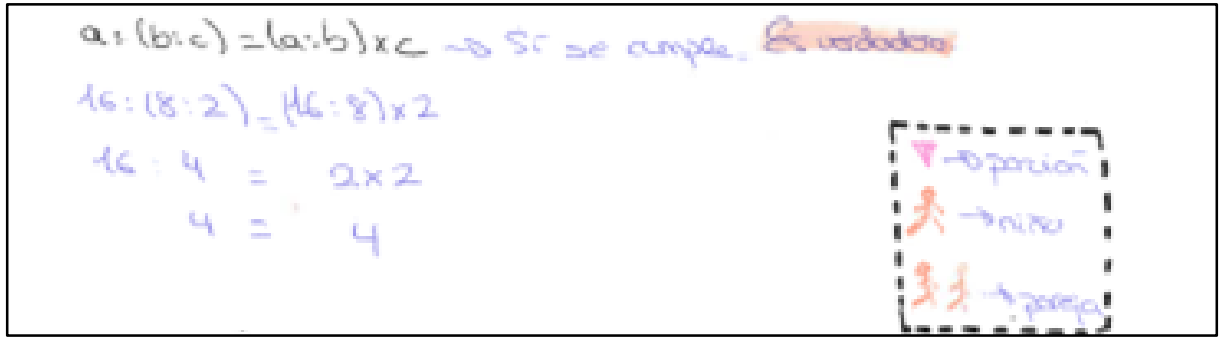
Meaning of intermediate operations (SCK)

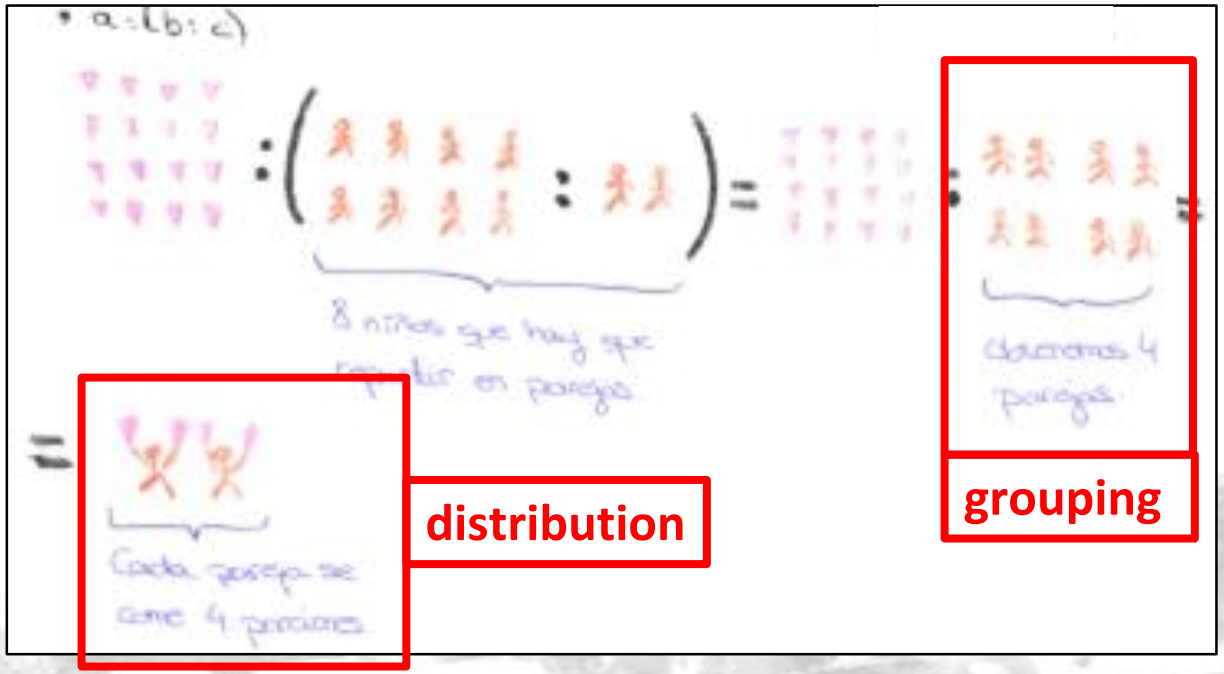


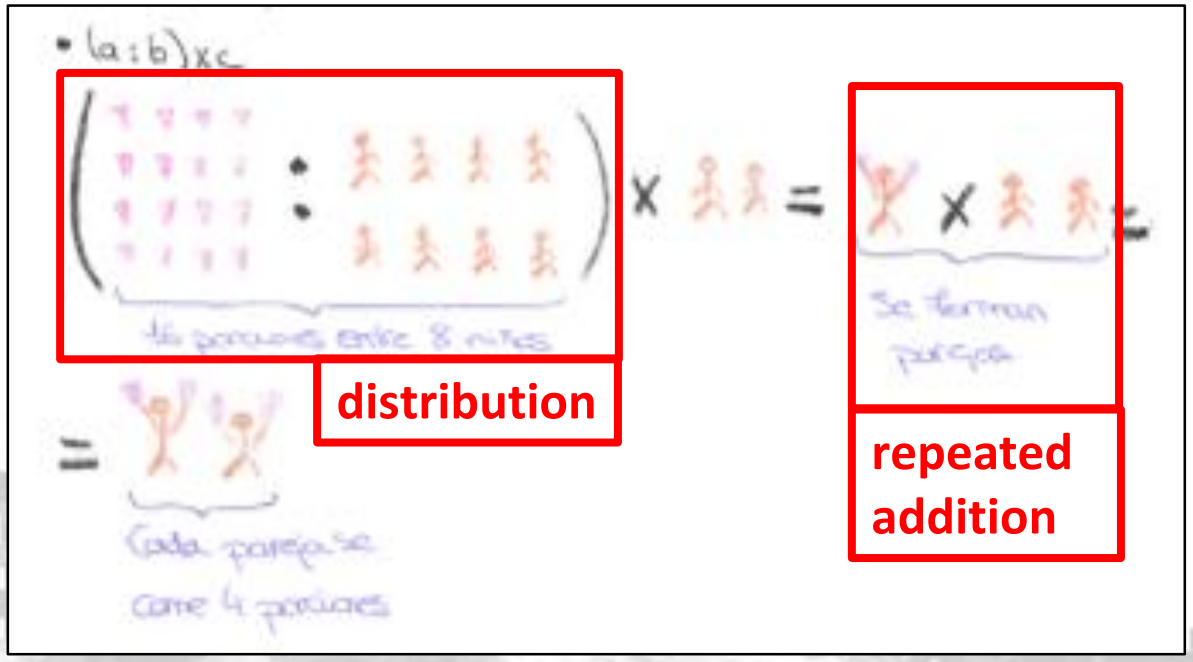
grouping

Student #5: **Grouping (in both sides of the expression)**

Meaning of intermediate operations (SCK)







PST #27 **combining the meanings of grouping and distribution**

Contextualization (KCT)

In the task with MM, **18.5% of the PSTs proposed a word problem** to solve in order to explain the task, usually distributing candies among children as a context.

None of the PSTs carried out any written contextualized task solving a **word problem**, however student #27 used implicitly a context without writing a word problem.

PST #16 **showing material to introduce a word problem**

Conclusions

Three out of the seven analysis categories were suitable for analysis when the task is presented in written form.

Contextualization (KCT): merely asking for the verification of a property is not sufficient to prompt students to create a context in which that property becomes visible.

Choice of numerical values (SCK), richer selection in the **written task**, generating examples with more internal relationships among the data.

Meaning of intermediate operations (SCK), higher percentages of operations with **manipulative materials** showed meanings beyond merely formal.

Conclusions

The video along with the use of manipulative materials proved to be a suitable tool to understand the property.

As a future perspective: repeat this same didactic proposal, incorporating as an additional task a reflection on the context, the choice of numerical values and the meaning of the operations involved, as we consider that these three aspects are the ones that favour a richness in the proposal made by preservice teachers.

THANK YOU

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