Innovative Methods in Teaching of Mathematics at the University

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Abstract

The article arose in the frame of Project "Implementation of the internal quality assurance system in education." For assuring the quality of mathematics education we prepared a database of tests from selected parts of mathematics based on requirements summary of students' knowledges and skills. Before test preparation the analysis of the content of the curriculum was needed, the identification of key competencies, setting of standards and also the analysis of students classworks to determine the inappropriate answers. From prepared database, we generated tests appropriate to verify the students' knowledge of the chosen thematic units in terms of continuous verification, proposed an effective test integration into education process and then applied them in the compulsory course of Mathematics 1.

The objective of the this course Mathematics 1 is to provide an understanding of solving algebraic equations, some basics of matrix calculation, of analytic geometry, limit calculation, understanding of differential and integral calculus. The goal of this course is to learn how to apply these knowledges in solving problems, to acquire knowledges and skills necessary for studying technical subjects.

Data were collected from students attending the first year of Bachelors degree at the Faculty of Materials Science and Technology of the Slovak University of Technology. For the analysis of the collected data, the similarity and implicative statistical methods were carried out using computer software called C.H.I.C (Cohesitive and Hierarchical Implicative Classification).

1. Introduction

The project "Implementation of the internal quality assurance system in education" aims to improve the teaching and learning mathematics at the university and to promote innovative teaching methods for assuring the quality of mathematics education.

Results from international surveys suggest that education outcomes are relates not only to students' family background, but also to the quality of teaching and to certain structural and organisation features of education systems [10].

This contribution begins with an overview of the structure and content of the curriculum of compulsory course Mathematics 1 at the Faculty of Materials Science and Technology of the Slovak University of Technology, of the main objective of this course and highlights students' key competencies.

2. Curriculum

The objective of the course Mathematics 1 is to provide an understanding of solving algebraic equations, some basics of matrix calculation, of analytic geometry, limit calculation, understanding of differential and integral calculus with a stronger focus on competencies and skills necessary for studying technical disciplines, so a greater emphasis on the application of mathematics in other courses responding to the needs of learners.

We limit here our considerations to topics Functions, Limits and continuity of function and Applications of differentiation. In the European qualifications framework, learning outcomes are defined as statements of what a learner knows, understands and is able to do on completion of a learning process; these are described in terms of knowledge, skills and competences [10]. The following key competencies were identified and defined:

Functions: definition, domain and codomain of a function, graph of a function, function composition, basic properties of functions (monotonicity, parity and imparity of functions), inverse function, types of Functions (exponential and logarithmic functions, natural logarithm, inverse trigonometric functions, their domain and codomain, graphs and properties);

Limit and continuity of function: the concept of the limit of a function at a point, limit theorems, one-sided limits, limits involving infinity, indeterminate forms, the limit definition of the number e, definition of the continuity of a function of one variable at a point and on an interval, the points of discontinuity of function.

Applications of differentiation: increasing and decreasing functions, the nature of stationary point, function's local extremes, intervals of concavity, inflection points and asymptotes.

In order to ensure effective schooling, learning objectives and outcomes, as defined in the curriculum, need to be aligned with the approaches to teaching and assessment used in the classroom [10]. It is also important to align teaching materials with curriculum. The university mimeographed [4], [5], [6] are recommended and preferentially used by teachers at the course Mathematics 1. This learning materials are designed for students who are interested in learning how to use the freeware mathematical software Winplot and Maxima to solve tasks and become familiar with applications of mathematical knowledge in physics and other technical disciplines. The use of these programs Maxima and Winplot in course Mathematics 1 works as a tool, and changes various problem situations by allowing easy graphics, algebraic and numerical computations and visualizations [8].

3. Method

Appropriate teaching methods can improve students' level of understanding and help them master mathematical rules and procedures. Student assessment is an essential tool for monitoring and improving the teaching and learning process. Two main forms of assessment can be identified: those where results are used for formative purposes, that is to improve future teaching and learning, and those which are used for summative purposes [10].

To ensure that students acquire the essential mathematical skills and competences, we prepared a database of tests from selected parts of mathematics based on requirements summary of students' knowledge and skills. The basic information about this project Mathematics 1, a list and basic information about the participants in the project, a tree containing all folders in the test base and folders contents are available in Academic Information System of Slovak University of Technology [9]. A tree contains 18 folders and each folder contains average 20 questions. All questions in the folders are of the type of selective 1 from 4 options. To enter incorrect options for all questions in the test base, the analysis of students' class works was needed to identify student's inappropriate strategies and misconceptions. Some results were published recently [2], [3].

From prepared test base, we generated tests appropriate to verify the students' knowledge of the chosen thematic units in terms of continuous verification and proposed effective test integration into education process. The prepared tests were used to for formative purposes, to identify the specific learning needs of students and so improve future teaching and learning. As it was mentioned, we limit here our considerations to topics Functions, Limits and continuity of function and Applications of differentiation. Each test was consisted of 5 tasks.

In the test called Functions, students had to find the domain of an inverse trigonometric function, choose a graph for the given line, calculate the value of a function at the given point and recognise the graph of an impaired function and an inverse trigonometric function.

In the test called Limits and continuity of function, students had to choose the correct definition of the continuity of a function of one variable at a point, determine the intervals of continuity of an elementary function, calculate the one-sided limit of rational function at the point of discontinuity of this function, answer to question what does the concept of infinite limits mean and classify an indeterminate form of limit of given function according to type.

In the test called Applications of differentiation, students had to choose the correct statements about stationary point, find the stationary points and local extremes of a power function, determine on what intervals the given function is decreasing, if the first derivate of this function is given, then determine the intervals of convexity and finally find any asymptotes of an rational function if the graph of function is given.

Data were collected from students attending the first year of Bachelors degree at the Faculty of Materials Science and Technology of the Slovak University of Technology. The presented tests were integrated at the end of the educational process; students worked individually and the time proposed to students for solving test was 15 minutes. Accesses to tests have each student through their own username and password to Academic Information System.

4. Test results

Test statistics are available in Academic Information System of faculty [9]. In the overall overview teacher can learn the basic information about selected test, like number of completed tests, attainment, average scores (points), median and below this information there is a test result graph. Teacher can also acquire details about individual folders and questions of the test, learn about the

questions score in the folders and the questions share from a folder in the whole test. For each question teacher can find out how many tests it appeared in and how successful students were at answering it. Average questions score is also displayed to students.

Using a sample of 28 students answering to the first test called Function made the first statistical survey. Attainment of this test was 47,86%, so average score was 2,39 points of 5 (highest possible score). The students were most successful at answering two questions, overage questions score was more than 60%. At the first one, students had to choose a graph for the given line and at the second question they had to recognise the graph of an inverse trigonometric function. Students' great difficulty in calculating the value of a function at the point $(a+1)$ is shown by their low success rate (17,86%).

The number of completed test called Limits and continuity of function was 31. Attainment for this second test was 51,61%, average score was 4,65 points of 9. The biggest rates of success have the question where students had to classify an indeterminate form of limit of given function according to type (70,97%). On the contrary, only 10 students determined the intervals of continuity of an elementary function.

The third statistical survey was made by using a sample of 37 students answering to the test called Applications of differentiation. Attainment of this test was 45,50%, so average score was 2,73 points of 6 points. The students were most successful at determining the intervals of convexity, overage questions score was 78,38%. Students had low success rate (21,62%) in finding the stationary points of a power function.

5. Analysis by CHIC

For the analysis of the collected data, also the similarity and implicative statistical methods were carried out using computer software called C.H.I.C, version 4.2 [7].

CHIC is a data analysis tool based on SIA. Its aim is to discover the more relevant implications between states of different variables. It proposes two different ways to organize these implications into systems: in the form of an oriented hierarchical tree and as an implication graph. It also produces a non-oriented similarity tree based on the likelihood of the links between states [1].

The corresponding codification of the variables used for the analysis of the data is shown in Table 1. All the variables are binaries, so with value 1 if it is manifested for a student and with value 0 in the contrary case. So we obtain a matrix presence-absence of dimension $n \times m$ where n is the number of the subjects (here $n=30$) and m is the number of the variables binaries ($m=15$).

Relabelled as	Meaning
1Fdf	Domain of inverse trigonometric function
1Fgl	Graph of given line
1Fvf	Value of function
1Fgif	Graph of impaired function
1Fgtf	Graph of inverse trigonometric function
2Ldc	Definition of continuity of function
2Lic	Intervals of continuity
2Losl	One-sided limit of rational function
2Lil	Concept of infinite limits
2Lifl	Classification of indeterminate form of limit
3Dsp	Statement about stationary point
3Dsple	Stationary points and local extremes of power function
3Did	Intervals of decrease
3Dic	Intervals of convexity
3Daf	Asymptotes of rational function

Table 1. Relabeling of tests items.

For this study, a similarity diagram was produced. Figure 1 illustrates the similarity diagram of students' responses to the questions of the tests. The similarity diagram allows for the arrangement of students' answers to the tasks into groups according to their homogeneity.
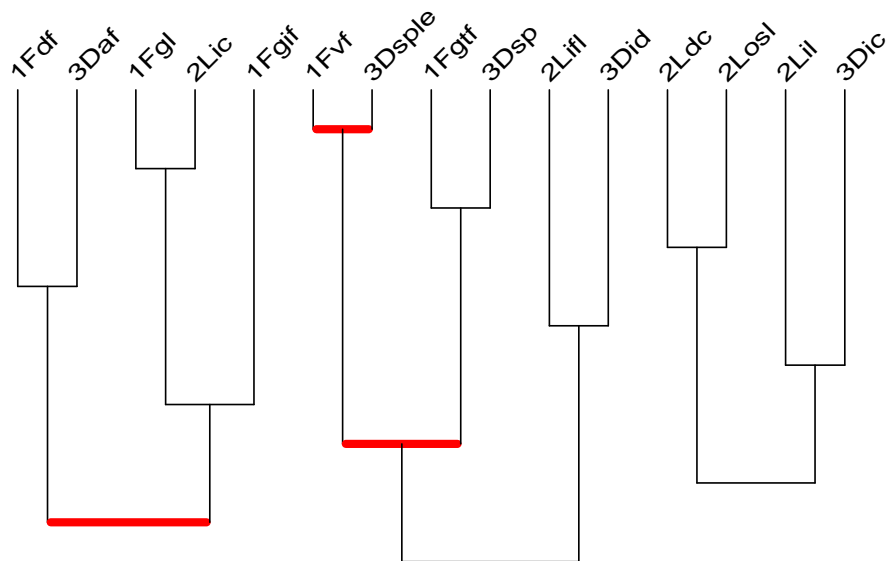
Fig. 1. Similarity diagram

Three clusters of tasks are identified in the similarity diagram. The first cluster involves the determination of domain of function, the intervals of continuity and so the points of discontinuity of the rational function, finding all asymptotes of rational function and the recognition of the graph of given line and of the power function. The total similarity of this first cluster, represented by red line, is statistically significant that demonstrates the relation between the vertical asymptotes and the domain of function, so the points of discontinuity of elementary function and also that the representation of asymptotes is related to recognition of the graph of given line. The second cluster represents the close connection between statements about stationary points, the nature of stationary points and the value of function and intervals of decrease of function. The strongest similarity at level more than 84% occurs between variables 1Fvf and 3Dsple. The third cluster suggests the close similarity relation between the definition of continuity, concept of infinite limit and calculating the one-sided limits of function.

6.Conclusion

In order to be successful, strategies to address low achievement need to be embedded within all aspects of learning and teaching, including curriculum content and organisation, classroom practices and teacher education and training [10].

The results obtained can be useful to help us in organizing our future didactic situation and can be important in improving the students' learning process. In what follows, tests prepared should be re-evaluated, standards should be revised and test tasks completed by questions which will serve to better ownership of those parts of the curriculum, of which we have seen less success.

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