Capture the King: Using Analogies to Teach Mathematics to Adults

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Abstract

The vicissitudes in higher education internationally has resulted in universities changing the focus of their undergraduate degrees, increasing enrolments and broadening participation. Non-traditional students, who would have once been excluded from university studies, are now being accepted. Given the resulting social and educational diversity of these students, how do we, as educators, prepare them for undergraduate study?

Central Queensland University (CQU) is well known for accepting non-traditional students and has therefore provided services and courses to ensure that these students are prepared. Preparatory mathematics courses, for students wishing to gain entry to university, follow adult learning principles and can both provide content knowledge and increase confidence [1].

Students' confidence in their ability in mathematics is important; as confidence in their ability increases so do their grades [2]. Over half of the students entering preparatory mathematics courses at CQU expressed a fear of mathematics [1]. Given that so many students have a fear of mathematics, reducing the fear and increasing confidence is therefore a vital part of teaching mathematics.

Analogies are an excellent way to teach mathematics to adults. They enable connections to be forged which increases understanding, thus increasing confidence and reducing fear. Analogies enable mathematics concepts to be conveyed in a form that students can relate to, thus, increasing their understanding and confidence.

One mathematics topic that the majority of preparatory students fear is algebra, which is often due to it being perceived as more abstract and thus irrelevant to the 'real world'. Using chess as an analogy in assisting students to understand the rearranging of an equation and especially the order in which to solve algebraic equations is extremely beneficial. Students are excited that mathematics can be viewed in a manner far removed from those tedious repetitious learning methods many learnt in school. Equation solving then becomes analogous to role playing as students metaphorically eliminate the 'guards' and capture the 'king'. Many students have commented "I always hated algebra in school if only they had taught it like this".

Anecdotal evidence suggests that students better relate to course content when analogies are used to simplify the concepts and provide contextual connections. It also suggests a reduction in cognitive overload and increased engagement.

1. Introduction

The vicissitudes in higher education internationally has resulted in universities changing the focus of their undergraduate degrees, increasing enrolments and broadening participation. Non-traditional students, who would once have been excluded from university studies, are now being accepted. A greater number of adults are returning to study due to policymakers worldwide focusing on social inclusion and subsequent government incentives. Given the resulting social and educational diversity of these students, how do we, as educators, prepare them for undergraduate study?

Central Queensland University (CQU), is primarily located in regional Queensland, Australia. It is well known for accepting non-traditional students and caters to approximately 7000 fulltime equivalent domestic students per year. As CQU has one of the highest rates of low socio-economic students in Australia, services and courses are provided to ensure that students are prepared and supported. The CQU Skills for Tertiary Education Preparatory Studies (STEPS) programme commenced in 1986. Initially funded by a government grant, it aimed to bridging the gap between tertiary education and underrepresented groups – which included Aborigines, migrants, women, and people from low socio-economic backgrounds and those from isolated areas [3]. The programme has evolved to meet the changing needs of people who have not been able to attain their educational goals through traditional educational pathways, often referred to as 'second chance learners' [4].

The STEPS course enables students to tailor their preparatory studies to suite their undergraduate degree aspirations. Therefore, three levels of mathematics, Transition Mathematics (TM) courses, are

offered to cater for various undergraduate entry requirements. Preparatory mathematics courses that follow adult learning principles can both provide content knowledge and increase confidence [1].

2. Mathematical confidence

Over half of the students entering TM courses expressed a fear of mathematics [1]. Given that so many students fear mathematics, reducing this and increasing confidence is a vital part of teaching mathematics. “Students’ confidence in their ability in mathematics does matter” and as confidence increases so do students’ grades [2, p53]. Increased confidence in one’s ability leads to improved capability to self-assess and learn independently. Students who are independent learners and are capable of self-assessing are more assured in their ability to both their determine knowledge gaps and learn new concepts. Graven [5, p177] describes “confidence as both a product and a process of learning”. A study conducted by Adams, Elliott and Dekkers [1] revealed that over 30% of students enrolling in TM courses felt they would never be able to do mathematics. Upon completion of at least one of the courses less than 8% of students still felt the same.

The courses are designed to follow the principles of adult learning and recognise the importance for students to have a sense of empowerment and ownership of their learning. Each student, regardless of study mode, receives: a study guide; textbook; and instructional videos. These resources facilitate self-paced study, encouraging the student to work ahead if they have mastered a concept and take control of their learning. Activities and resources designed to make learning as interactive and engaging as possible are provided for all students. From the commencement of their TM course students are encouraged to discuss their backgrounds and career aspirations. TM course lecturers assist students to be goal driven by attempting to relate explanations and examples to students’ goals as well as using common scenarios. Students find more meaning in the mathematics if they can relate it to familiar situations and understand the practical application with regards to their own goals. Encouraging students to take control of their learning experience and strive to achieve goals increases their confidence.

There are always a number of students in the class who feel they will never be able to ‘do’ maths. Using their life experiences to view the world from a mathematical perspective assists students in improving their understanding of mathematical concepts and thus their confidence. Analogies make excellent teaching tools through which to link mathematics with everyday lives, thus enabling mathematical concepts to be better understood.

3. Mathematical Analogies

The use of analogies for teaching mathematics has long been debated. In 1981 Pimm [6], against convention, proclaimed the benefits of using analogies to improve mathematical understanding.

The initial inclination is to view analogy, and metaphor in particular, as uncertain methods of working and therefore unsuited to mathematics. “Analogy, however, is not proof, but illustration” I hope to show by means of examples culled from arithmetic, algebra and the calculus that this is far from the case and in fact these processes are as central to the expression of mathematical meaning as they are to the expression of meaning in natural language. [6, p47].

For adult students, illustration is important. No longer satisfied to just believe what they are told and requiring reason, analogies enable mathematical concepts to be conveyed in a form that adults can relate to. Analogies assist the simplification of concepts through the benefit of connection to already understood concepts or situations. To effectively incorporate analogies into a class it is important that it is associated with the encouragement of active reasoning [7].

Encouraging active reasoning requires students to participate in the analogy building or process. Analogy can be used to demonstrate the concept of division by zero. Many students assume the answer of a division by zero is zero. We need to demonstrate that it is impossible to divide by zero and that the answer is not zero. In mathematics we say that a function does not exist when dividing by zero or ‘tends toward infinity’. This is a concept that students do not naturally perceive. Using the metaphor of zero being an imaginary ball of paper we can ask “How many imaginary pieces of paper do we need throw into the bin to fill it?” It is quickly realised that the bin can never be filled and division by zero is therefore impossible.

The chess scenario is useful in assisting students to understand rearranging of an equation and especially the order in which to solve algebraic equations. This analogy enables students to recode the information given. Recoding increases the amount of information that can be processed and

recalled [8]. Anecdotal evidence has shown that students, especially males, find that tactics used in chess (and other strategy games) can be easily applied to algebraic problem solving. The student needs to consider the variable for which the equation is being solved as the king. All 'pieces' must be removed until the king is on his own. Students are excited that maths can be viewed in a manner far removed from the boring rote learning and abstract methods many endured in school. Equation solving then becomes analogous to role playing as students metaphorically eliminate the 'guards' and capture the 'king'.

Connections to the order of the mathematical process of equation solving are able to be made as the student determines which 'man' to 'kill' first. This makes algebraic equation solving fun and easier to learn. "I always hated algebra in school if only they had taught it like this" is a common student comment. In the chess scenario the student recognises that the 'man' farthest from the king is easiest to 'kill', these are the additions and subtractions. Then any 'man' guarding a door, multiplication or division of brackets, is eliminated. Once inside the castle the same process is followed until just the king and his body guard are left. It is now easy to get the king by himself. Questions related to solving algebraic equations tend to be asked in role play format. For example, "I've killed this guy out here but what do I do with this one under here?" This question relates to division. As with any teaching strategy, this analogy does not work for all students. Different learning styles are therefore considered, including the need for students to ask questions and seek clarification.

Michener [9] describes understanding as having many levels and that the process of understanding is never really completed. Analogies provide a platform on which to build knowledge, allowing students to link concepts and aid memory. Due to the non-linear nature of mathematics a single concept requires many connections. Cognitive load is therefore an important consideration when teaching new concepts.

4. Cognitive theory

Cognitive load theory is concerned with the processing thresholds of the cognitive system and how to manage these thresholds in learning and instruction [10]. According to Miller [8] this threshold is met when attempting to process seven unrelated chunks of data. The size of the chunks is dependent on the familiarity with the content, making recoding extremely important in increasing information that can be processed and recalled.

Learners construct knowledge through connections to what is already known [11]. Therefore, analogical representations provide learners with visualisations that are easy to relate to. They force the learner to compare and contrast to understand the analogy and create new knowledge. Using analogies to help learners predict or solve problems enriches the knowledge base upon which learners draw, and ultimately shape and strengthen mental models [11], thereby increasing the size of the data chunks capable of being processed and aiding learning.

5. Conclusion

Using chess as an analogy in assisting students to understand the rearranging of an equation and especially the order in which to solve algebraic equations is extremely beneficial. Students are excited that mathematics can be viewed in a manner far removed from those tedious repetitious learning methods many learnt in school. Equation solving then becomes analogous to role playing as students metaphorically eliminate the 'guards' and capture the 'king'. Anecdotal evidence suggests that students better relate to course content when analogies are used to simplify the concepts and provide contextual connections. It also suggests a reduction in cognitive overload and increased engagement.

Analogies are an excellent way to teach mathematics to adults. They enable connections to be forged and assist in removing fear and in crease confidence that may have been lost due to boring rote learning. Analogies enable mathematical concepts to be conveyed in a form that students can relate to, thus, increasing their understanding and confidence. The use of analogies enables adult students to form connections between mathematics and real world situations. Connection with the familiar makes mathematical concepts easier to comprehend.

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