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A study on the Reintroduction of Logic in Secondary Schools

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Abstract

This study investigates the possibility to make logic more easily accessible to students in secondary school within the framework of math classes and explores students' opinions about logic. Do they think it is helpful for mathematics? And how about everyday life? Do they expect it to be useful for their further education? In collaboration with math teachers we developed a series of lessons and course material on propositional and predicate logic for secondary education which focus on logic in everyday life and logic for mathematics. The students can learn the lessons autonomously but other teaching methods can also be applied.

In this study, three different secondary schools and about forty students were involved. The research is based on a pretest and posttest, observations during the courses and a questionnaire. In this paper we report on the results of the study. It is clear that the participants favor logic. They do think it is useful for their further education and in mathematics. However, they are not convinced of the usefulness of logic in everyday life.

1. Introduction

In the 1970's heyday of New Mathematics in Flanders mathematics education focused on the internal structure of pure mathematics. Logic formed a part of the curriculum. In the decades that followed, mathematics education has changed a lot and logic is no longer taught now. However, there are still a lot of mathematics teachers that favor the idea of teaching logic in secondary mathematics classes. Moreover, the rise of IT adds to its importance.

We believe that logic should be reintroduced in the mathematics curriculum, not only because of its link with mathematical proofs, but also in view of the ability to reason logically in everyday life and the importance of logic in further education (e.g. IT, mathematics, philosophy…).

In this study, we investigated how to make logic more accessible to secondary school students and explored their opinions about logic.

2. Research results on teaching logic

In general, literature shows that teaching logic helps students in their mathematical performance. Other studies, however, claim that a lot of students have difficulties in understanding certain aspects of logic.

Lin, Lee and Yu in [4] found that 80% of 202 students, aged between 17 and 20 years old, were unable to negate the quantifier 'only one' and that more than 70% lacked conceptual understanding of proof by contradiction. However, they also showed that it is possible to help students understand this concept and apply it in real world situations.

Proof by contradiction was the worst scoring section in the studies of Suppes [6] and Suppes et al. [7] as well. They also found that students experience difficulties recognizing an invalid proof.

Kossak [3] found that mainly negation and implication make logic difficult. According to his paper, the main reason for this difficulty is the difference between logical and spoken language. For example, when a teacher holding a green pen says 'if this pen is blue, it is red', his statement is true in logic, but makes no sense in spoken language. Also Hoyles et al [2] quote the implication as difficult.

In the research of Corbeil [1], however, the modus tollens, the understanding of logical proofs and designing proofs were experienced as difficult. According to Corbeil, logic is used often in mathematics, not only to prove something, but also to ensure the consistency between theorems and definitions or to perform difficult calculations. Corbeil showed a positive connection between logical and mathematical performance. The latter is confirmed by Nunes et al [5]. They found that learning logic from childhood on improves mathematical performance, on condition that suitable teaching methods and topics are selected.

3. Project goals and research questions

In Flanders, logic used to be a part of the mathematics curriculum but is no longer taught now. We intended to make logic accessible again to secondary school students within the framework of math classes. For that purpose, we developed a series of lessons and course material on logic

Our first research question relates to the quality of the developed material, from the point of view of the students. Secondly, we wanted to investigate whether the students made any progress regarding their understanding of the logical implication. Finally, we wanted to explore students' opinions about logic, more specifically whether the students, after taking the lesson series, perceive logic as a useful topic for mathematics, their further education and everyday life.

4. Method

4.1 Participants

Three secondary schools (*A*, *B* and *C*), three math teachers and 43 12th grade students in three classes were involved in our study. The students are among the best ones for mathematics and are taught 7 or 8 hours of mathematics a week. The lessons in school A, in November-December 2012, were attended by 16 students. A total of 27 students were involved in the lessons in schools *B* and *C* in January-February 2013.

4.2 Development of the course material

The course material was initially designed by the first named author (master student in mathematics education), under supervision of the second named author. Based on the experiences and feedback from students during the try-out in school *A*, the course material was improved a first time. Similarly, the material was revised after the experience in schools *B* and *C*.

The material treats propositional and predicate logic. The difficulties described in literature, were given extra attention. For instance, while the truth tables of the negation, disjunction, conjunction and implication were part of one section, a separate section was devoted to the implication.

We focused on the use of logic in everyday life and in mathematics. We included examples of logic in everyday life and we gave a lot of attention to the difference between spoken and logical language. We also included examples of the use of logic in mathematics. For instance, the definitions of a square root, a logarithm… were given using predicate logic. A logical proof of two mathematical theorems, one by contradiction and one using the law of contrapositive, made the students more comfortable with mathematical proofs.

The connection with IT was made by logical circuits, a topic already studied in the first or second year in secondary school. However, the corresponding logic was never treated before.

Throughout the course material, information on important logicians was included. In each section, exercises were provided.

4.3 Research instruments

To test their prior logical knowledge, the students in schools *B* and *C* were subjected to a pretest (Figure 1), which contained two questions involving a proof (Q3 & Q4) and was mainly based on the logical implication. The test was taken from Hoyles et al [2] (except for the names). The students in school *B* did an almost identical test (questions ordered differently) after taking the course so that we could see whether they really understood the logical implication then.

On average, the students spent weekly about 2 hours during 8 weeks studying the course material, mostly individually and autonomously. The teacher only gave some explanation at the beginning of the course and related to some exercises. Teachers in schools *B* and *C* gave some supplementary explanation at the start of the predicate logic as well.

In the three schools, a total of 9 lessons were observed, which enabled us to see students' reactions on the course material ourselves.

Bart and Peter are thinking about the pair of numbers 3 and 11. They notice that the sum is even ($3 + 11 = 14$) and that the product is odd ($3 \cdot 11 = 33$).

Bart says: 'If the sum of two whole numbers is even, their product is odd.'
Peter says: 'If the product of two whole numbers is odd, their sum is even.'

1. Are Bart and Peter saying the same thing?
2. The product of two whole numbers is 1271. Suppose Peter is right. Which of these must also be right?
   (a) You can be sure that the sum of the two numbers is even.
   (b) You can be sure that the sum of the two numbers is odd.
   (c) You can't be sure whether the sum is odd or even until you know what the two numbers are.
3. Is Bart's statement true? Explain your answer.
4. Is Peter's statement true? Explain your answer.

Figure 1: Pretest based on Hoyles' test [2].

After the course, the students filled in a questionnaire (partly seen in Figures 2 & 3).

Do you agree with the following statements? Mark 5 when you totally agree, 4 when you tend to agree, 3 when you don't have an opinion, 2 when you don't tend to agree and 1 when you totally disagree.

	5	4	3	2	1
1. I think the lessons were instructive.					
2. I think the lessons were interesting.					
3. I understood everything from the course.					

Figure 2: Questionnaire.

5. Results

5.1 Quality of the material

In general, the students of school *A* were positive about the material, both in the questionnaire and during the observations. However, they didn't understand why predicate logic is wider than propositional logic, which showed that they needed a teacher giving additional explanation for this part. Moreover, students found some of the formulations difficult. The opinions of the students of schools *B* and *C* on the modified course are also quite positive (Figure 3). The majority of them thought the lessons were instructive and interesting (Q1-Q2). Some students even preferred logic over other parts of mathematics. They understood most of the course material and were able to solve most of the exercises (Q3-Q4). Most participants thought the text was written clearly (Q6).
From these results, we conclude that the quality of the course material was good.

5.2 Logical implication

Some questions of the pretest (in schools *B* and *C*) proved rather easy. The scores on questions 2 and 3 (Figure 1) were excellent (Q2: 100%, Q3: 94.5%). Reversing an implication (Q1: 60%) and setting up a proof (Q4: 52%) showed more difficult. 6 out of the 27 students did not give any proof at all while 3 students just gave an example. The results of the same test after the course in school *B* (13 students) were much better (Q1:100%, Q4: 92%). This is in accordance with most respondents' agreeing to Q12 in the questionnaire (Figure 3).
The results from the pretest confirm what is claimed in literature about difficulty of the implication and about students being satisfied with an example when a proof is required. After taking the course, however, the majority understood the use of the implication, although a few students still experienced difficulties.

5.3 Students' opinions about logic

During the observations, the students co-operated well. A few (positive) discussions about the solution method of some exercises arose. It was great to see the students helping each other and explaining their ideas to the group.
The questionnaire shows that the students in schools *B* and *C* understood why predicate logic is wider than propositional logic (Q7) and that they were aware of the use of logic in math (Q8-Q9). In a remarkable answer to an open question, one student admitted that he only now fully came to understand mathematical proving techniques. The majority considered logic to be useful for their future education (Q11), but not for other subjects in secondary school or everyday life (Q10-Q13).
We conclude that students' opinion about logic is mainly positive, although they don't think that it is useful in everyday life and other subjects than mathematics.

	5	4	3	2	1
1. I think the lessons were instructive.	19%	74%	4%	4%	0%
2. I think the lessons were interesting.	4%	63%	22%	11%	0%
3. I understood everything from the course.	15%	50%	23%	12%	0%
4. I could solve all exercises.	4%	44%	33%	19%	0%
5. I understood the history of the most important logicians that I could read in the frames.	19%	33%	37%	4%	7%
6. I think the text was written clearly.	19%	59%	19%	4%	0%
7. I understand why predicate logic is wider than propositional logic.	31%	35%	31%	0%	4%
8. After taking this course I understand the use of logic in math.	11%	44%	33%	11%	0%
9. The course is useful for mathematics.	11%	48%	30%	11%	0%
10. The course is useful for other subjects.	0%	11%	30%	41%	19%
11. The course is useful for my further education.	4%	48%	26%	19%	4%
12. I understand the difference between an implication in everyday life and logic.	22%	41%	19%	15%	4%
13. The course is useful for everyday life.	0%	7%	48%	37%	7%

Figure 3: Questionnaire results of schools *B* and *C* (27 students).

6 Discussion

After taking the course, about 19% of the participants still does not fully understand the difference between an implication in everyday life and logic. This implies that the implication may need even more attention in the course material. It is also possible, however, as seen in literature, that the implication is just very hard to understand.

The majority of the participants did not view logic as useful for other school subjects. The answers to the open questions, however, revealed that a lot of students appreciated logical circuits, which are related to IT.
Students consider logic as useful for their future education. They are right. According to White and Sivitanides [8], for instance, logical insight is necessary to be competent in (object oriented) programming. Hence, logic is important for any study containing IT. Furthermore, in [9] many students in higher education, mostly engineering, mentioned logic when asked to name topics they needed in their further education and didn't meet in secondary school. Many of these students felt unprepared for the abstraction of mathematics and mathematical proofs. In Flanders, logic is not only taught in mathematics and IT, but also in studies like philosophy, sociology, political science…

Although our study is quite small, it is clear that students like logic. Nevertheless, there are a number of issues that need to be taken into account when we want to reintroduce logic in secondary schools. All the participants were mathematically very able students that are taught to think more abstractly than others. They will probably choose a field of study that involves mathematics and logic.

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