Abstract
Nowadays the integration between contents and new technologies can be considered as a founding and qualifying aspect of teaching from primary school to university. It is then very important, in order to ensure high educational and learning standards, to design and use new technology teaching tools able to integrate scientific and methodological precision with easy of use. However all the digital teaching product today on the market as, in particular, e-books or LIM – books, are nothing more that copies, in digital format, of the traditional paper books, sometimes including only some interactive figures (including examples pre - defined by author and not modifiable by teachers). Furthermore, especially when referring to Physics and Mathematics subjects, these tools don’t contain serious references to recent experimental results, innovative fields (as environmental or health protection) or to every day applications. In this paper the prototype of a novel e-book, to be used in Physics and Mathematics courses at secondary school and University, is proposed. It is realized by using, as interactive and calculation interface, the famous and powerful software Mathematica, produced by Wolfram. The flexibility of this software together with the novel design of theoretical and applied teaching approach have allow us to realize a completely interactive product in which every important features is fully customizable and upgradable by the teacher. In particular, in the theoretical sections, mathematical formulas are not static elements but “living” calculation tools (symbolic and / or numeric) in which teachers and students could insert numerical values and / or analytical expressions in order to very or modify a numerical or symbolic result and then can be also used for the exercises resolution. The exercise section contained in our project, founded on GUI interface, represents a virtually infinite source of training since both teacher and student can create new quizzes and more or less complex exercises by changing the input and / or output requirements with a few clicks or by the introduction of new numerical parameters. But the most innovative feature of this projects is represented by the presence, within the e-book, of a complete virtual mathematical and physical laboratory (the latter able to control, through an hardware programmable interface, real scientific instruments) in which teachers and students can completely customize the proposed experiments or making their own ones, creating true simulation of physical phenomena or animations reproducing algebraic or geometrical concepts. The project, although firstly covering Physics and Mathematics, can be easily extended to all the other scientific subjects. Finally it is important to outline that, due to the use of cdf format, using the e-book doesn’t need for the installation of Mathematica software on all the machines (pc, mac, smartphone or tablets) where it runs.

1. Introduction
The integration between contents and innovative technologies today represents a fundamental feature of any educational methodology in Physics and Mathematics from primary school (where Physics can be limited to a few examples from every day experience) to university. In this sense, the elaboration of an interactive teaching tool able to join methodological and scientific rigour with facility, immediacy and intuitiveness of use, could ensure high learning levels with very low costs.
All the present teaching proposals of Physics and Mathematics courses, especially when referring to secondary school and university as, in particular, e-books or LIM – books, are nothing more that copies, in digital format, of the traditional paper books, sometimes including only some interactive figures (pre - defined examples made by author and not modifiable by teachers). Furthermore, when looking at the contents of these courses, we very often notice a structural lack of references, also as outlines or in – depth studies, to
the most recent result of scientific research, to the emerging theories or issues (as, for example, the environmental and health ones) or, finally, to the application of theoretical concepts to the study of every day cases. In this paper a new idea of fully interactive, customizable and upgradable e – book for teaching Physics and Mathematics will be proposed.

The structure of the proposed e–book also reflects the goal to open a window on the methods of scientific research, in order to present not simply the final and the well – established result but an interactive method to solve the physical or mathematical problem through the different subsequent steps of resolution. In the proposed idea the “container” tool is represented by the very powerful and flexible software MATHEMATICA realized by Wolfram [1].

The result is not only an interactive educational instrument but also a "living" and active tool, customizable and upgradable in all its part as well as simple to use for teachers and students. The particular features of MATHEMATICA make this software perfectly suitable to ensure the perfect integration between content, interactivity and easiness to use. It is in fact characterized by very high calculation powerful, graphics, GUI, animations and simulations capabilities that can be completely customized by user. A distinguishing feature, that makes MATHEMATICA the right software for the creation of a modern interactive e - books lies in the possibility to insert, throughout the textual sections of a document, executable code lines, through command line or GUI interface, doing all sort of mathematical calculation, simulation and graphical representation [2]. The relative outputs can be also inserted in the main text body and are, in turn, interactive elements themselves that can be manipulated through software controls.

2. Main structure of the interactive E-Book

The proposed model of interactive e – book uses the MATHEMATICA notebook (.nb) format as main structure (typically one nb file for each chapter or thematic unit). The main nb files include different sections (completely customizable and upgradable) containing theory in advanced scientific typesetting, images and multimedia contents (audio and video), with hyperlinks to webpages or to other e – book sections and cross – references between different matters as, for example, physics and mathematics. The latter features is very interesting and useful in a teaching / learning framework since it can give, for example, in a very simple and immediate way, the mathematical meaning of a given physical equation and its resolution techniques in a fully interactive mode or, conversely, the physical application of a mathematical equation to a physical problem that can be simulated by animated graphics and dynamical reproductions.

One of the most interesting and innovative features of the proposed e–book is represented, without doubt, by its “living” equations. Every mathematical expression in this e – book is not simply a text but a real and running algorithm that can be used “live” by teacher or student by inserting numerical values or properly modifying the expression itself. In every section there also a lot of applications in the form of MATHEMATICA source code. These can be completely customized and adapted by teacher that, in this way, will have a virtually infinite set of exercises, obtained changing numerical values or modifying mathematical or physical scenarios. As we have anticipated above, the high level of interactive characterize the present idea and is particularly manifest in the simulations [3] (modifiable by teacher) of mathematical and physical systems present in each section. These can be recalled and made be work from any point of the whole e - book. All the commands, from the simulation interface to the exercises and examples controls are very user – friendly and can be used by teachers and students even without a minimal familiarity with the MATHEMATICA core language. The feasible hyperlink structure allows the automatic management of crossref between all the various parts of a given e-book as well as between Physics and Mathematics concepts, so creating a “real – time” bridge between the two matters.

Being written in MATHEMATICA code, all the routines, simulations and the e – book core itself are based on this specific software language, nevertheless in order to “run” the e – book it is not necessary to have installed on our own PC or Mac the main application (so avoiding license questions). In fact, due to the new Wolfram’s file format cfs (conceptually similar to pdf format), the e – book can be considered as a “standalone” tool that can be ran through a free player software, distributed by Wolfram, that allows its use, according to copyright, on all the machines.
Another very interesting feature of the present project, provided by MATHEMATICA capability, concerns the ability to automatically extract from the notebook (nb) one or more sections, slides reporting theory, calculation or simulation results to be used on softwares like Microsoft PowerPoint or with LIM, without losing interactive and calculation features.

Finally, it can be provided for the periodic on–line upgrade of contents and simulation routines contained in the e–book, through the connection to a dedicated website maintained by author.

A conceptual map of the ebook structure is represented in fig. 1.

Fig. 1. Conceptual map of e book structure

3. An example chapter of the novel Physics interactive e - book

3.1 The “notebook”

The notebook represents the main file and contains the all the e-book elements including main text (and hyperlinks in underlined blue text), and interactive features (figures, formulas, simulations, etc.). An explanatory screenshot is shown in fig. 2.

3.2 The “interactive simulations”

The interactive simulation represents one the most innovative features of the present project. It is characterized by a customizable GUI through which teachers and students can simulate physical phenomena. The interface is equipped with one or more controls that allow the user to change the values of the physical parameters characterizing the physical problem and the 3D point of view so producing a potentially infinite quantity of simulations. The interactive simulation routine con also be used to control, via hardware interface, a real laboratory equipment able to reproduce the physical system under study. The fig. 3 shows an example of 3D simulation describing the motion of a charged particle in magnetic and electric fields.

3.3 The “interactive equations and formulas”

Equations and formulas in the e-book are “living” expressions in the sense they are completely interactive. They are constituted by Mathematica computation code able to actually calculate and solve physical equations both symbolically and numerically when the suitable numerical values of physical quantities are inserted. In this sense they also represent a virtual calculator for the exercises and an interactive explanation of the problem – solving method in physics. The fig. 4 shows some examples of these interactive equations in our e-book related to the calculation of angular velocity and radius for a circular orbit related to a motion of a charged particle in a magnetic field.
3.4 Summary and further developments

In this paper we have presented a prototype of a novel e-book, for teaching Physics, based on Wolfram MATHEMATICA software. The key features of the project are represented by the actual and complete interactivity of every component of the e-book, by the presence of several simulations, constituting a complete virtual physics laboratory, and the inclusion of “living” equations and formulas that can be used by teachers and students as a calculator and checking tool for the proposed exercises. These are realized using the same interactive philosophy and can be then considered virtually “infinite” as their number and typology. Moreover, the interactive simulations represent not only an actual and virtual physics laboratory (whose “experiment” can be really modified and upgraded by teacher) but also a source of a true physics laboratory, since the simulation software interface can be connected, through an hardware interface, to real laboratory equipment available in school’s and university’s educational and research laboratories. In addition to these very innovative features, the proposed e-book presents many others force points as, for example, its complete upgradability of textual and interactive contents, via author’s website or other web resources (in order to grabble the latest results of scientific research as well), the possibility to run as a standalone software (because of the Wolfram’s cdf format, similar to Adobe’s pdf for documents and through the possible conversion in html format) on all the software and hardware platform, including tablet and smartphones, also determining a noticeable total cost reduction for the final user. Nevertheless, in the start – up phase of adoption, in order to make its use simpler, it can be provided for a concurrence of the digital product consisting in the e-book software and its printed version.

Finally we must underline that, although the proposed idea has been initially applied to the development of a course in Physics, it could be easily extended to all the other scientific subjects (and, potentially, also to human studies).
1.1 The interaction between electric charges in motion and magnetic fields: the Laplace equations

In this section we will study the fundamental problem of electrodynamics that is the calculation of the magnetic force exerted on a generic element of electric current and, conversely, the magnetic field generated in a given point by a generic current. These expression are given by Laplace equations that will be discussed below.

The empirical Ampere law already discussed in a previous section (cf., par. 9) allows us to express the force per unit of length between two uniform rectilinear conductors. It gives the strength of the reciprocal action of the magnetic field produced by the current flowing in each conductor on the electric current that flows in the other. As we'll see, the electric current circulating in conductor 1 determines, in the surrounding space, a magnetic field \( \mathbf{B} \), exerting a magnetic force \( \mathbf{F} \) in every point of conductor 2; the latter, in turn, determines, in the same space region, a magnetic field \( \mathbf{B} \) that gives a force \( \mathbf{F} \) on conductor 1.

These forces, according to the third principle of dynamics, satisfy the relation \( \mathbf{F}_1 = -\mathbf{F}_2 \) (see interactive simulation 1.1).

![Interactive simulation 1.1. Force per unit of length between two infinite rectilinear currents](image)

The validity of Ampere's equation is nevertheless limited to a very particular conductors configuration (rectilinear, uniform and of finite length) and doesn't hold in general for conductors of any geometry.

It is necessary to generalize it in order to apply it to a generic conductor: this is the fundamental problem of electrodynamics that can be, at least theoretically, solved determining the magnetic field generated by a circuit element \( d\mathbf{A} \) sufficiently small, to be considered as rectilinear, homogeneous and placed in an uniform magnetic. In this way:

- the total magnetic field generated in a given point is obtained by a vectorial sum of the magnetic fields of each element of length \( d\mathbf{A} \);
- the total force exerted on a generic circuit is given by the vectorial sum of the forces exerted on each rectilinear element of length \( d\mathbf{A} \).
Fig. 3. Example of a 3D interactive simulation

\[q \nu B = \frac{mv}{R} = m \omega^2 R\]

from which we have:

\[\nu = \sqrt{\frac{qv}{mR}}\]  \hspace{1cm} (1.8)

we can finally calculate the radius of the circular orbit as:

\[R = \frac{mv}{Bq}\]  \hspace{1cm} (1.9)

Fig. 4. Examples of interactive equations

References
[1] www.wolfram.com