DigiMathArt

Connecting Math and Art through Programming

A method of creating new neural networks

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Why new neural networks?

In order to develop our **self-awareness / consciousness**, to cope with the world, no matter if it is “real” or an “illusion”, we need to use all our senses – more than those 5 still considered in the textbooks (**sight**, **hearing**, **taste**, **smell** and **touch**):

- **feromonal**,
- **blue light**,
- **temperature** (thermoception),
- **kinesthetic** (proprioception),
- **pain** (nociception),
- **balance** (equilibrioception),
- **visceral** (the perception of internal organs),
- **chemical** (such as chemoreceptors for detecting salt and carbon dioxide concentrations in the blood)
- …and others…

and to “discover” **all the gifts we were packed with** form the very first moment we were “designed”.

- **Less than 2% of the DNA is responsible with protein production.** The 98% non-coding DNA sequences (*introns* or “junk” DNA) it was believed to represent evolutionary waste. Studies from the past 15 years have contradicted this presumption.

The complexity of an organism is weakly correlated to the number of protein-producing genes, whilst strongly associated with the number of non-coding genes.

(Perkins, Jeffries and Sullivan, 2005)

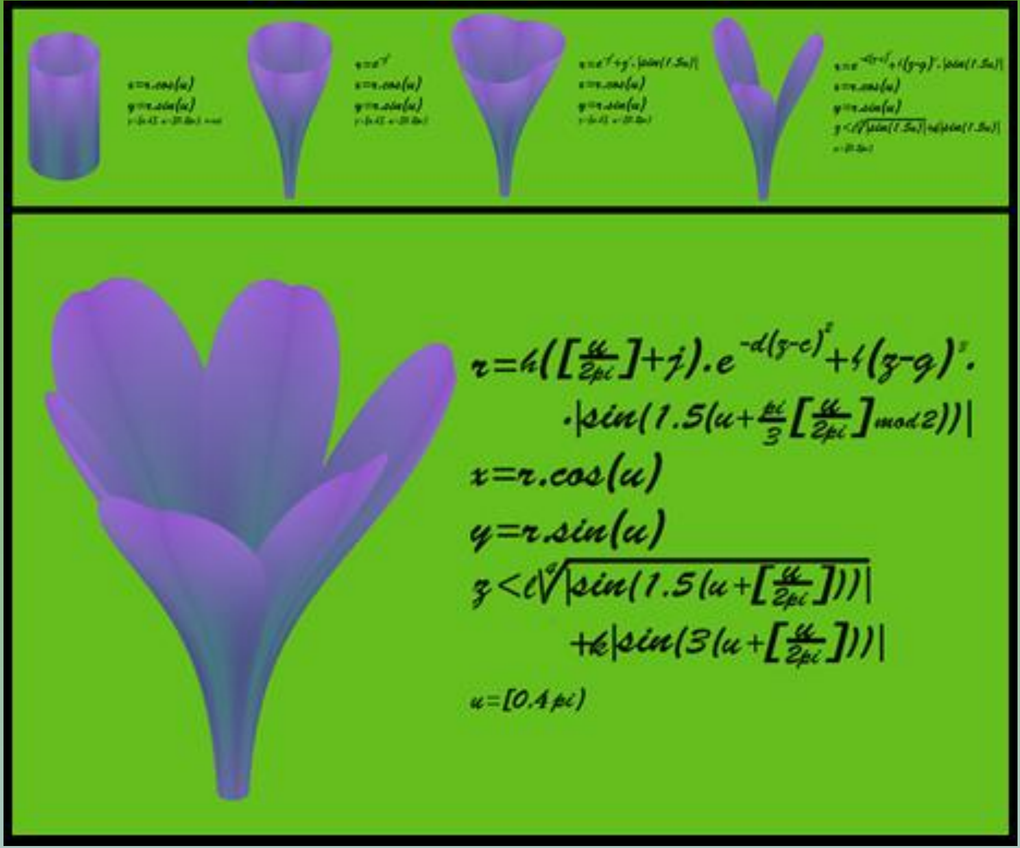
The non-coding DNA sequences tend to be localized next to the genes involved in neuronal functions, triggering the expression of these genes.

The “junk” DNA could orchestrate our brain’s “wiring”.

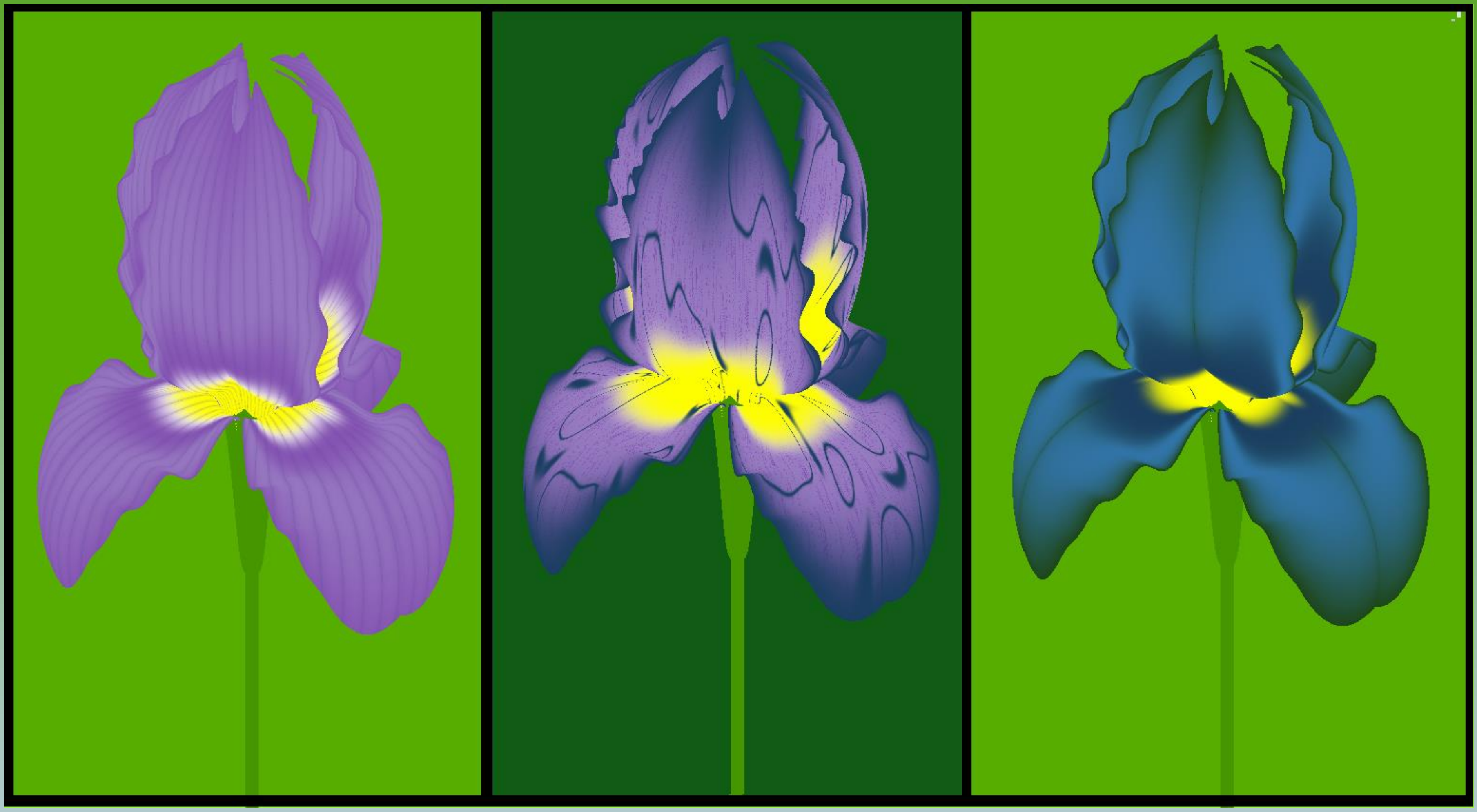
(Prabhakar, Noonan, Paabo and Rubin, 2006)

What is DigiMathArt?

- method of studying, learning and teaching mathematics through programming and computer graphics



- method of generating Fractals and a Fractal Art



- method of generating Fractals and a Fractal Art

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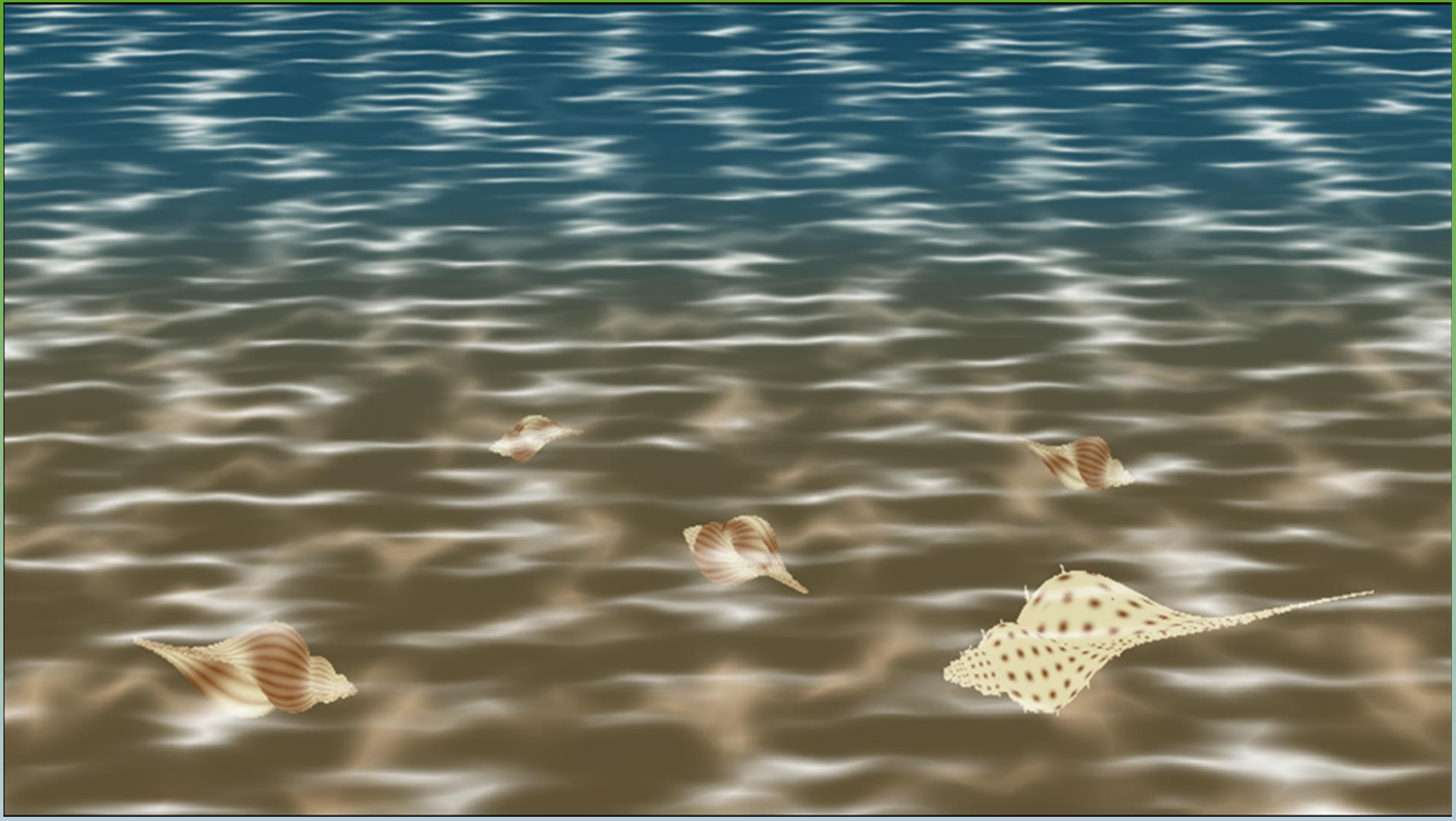
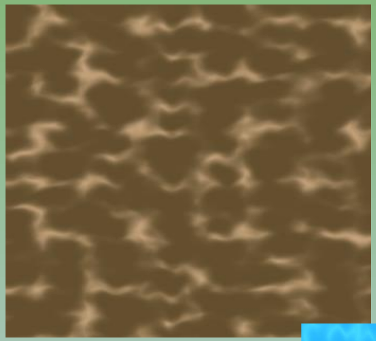
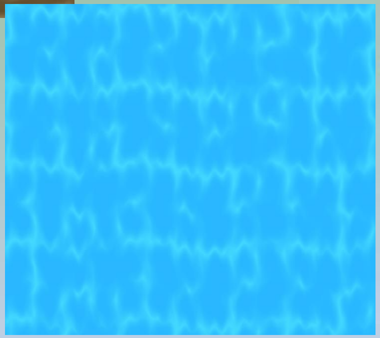


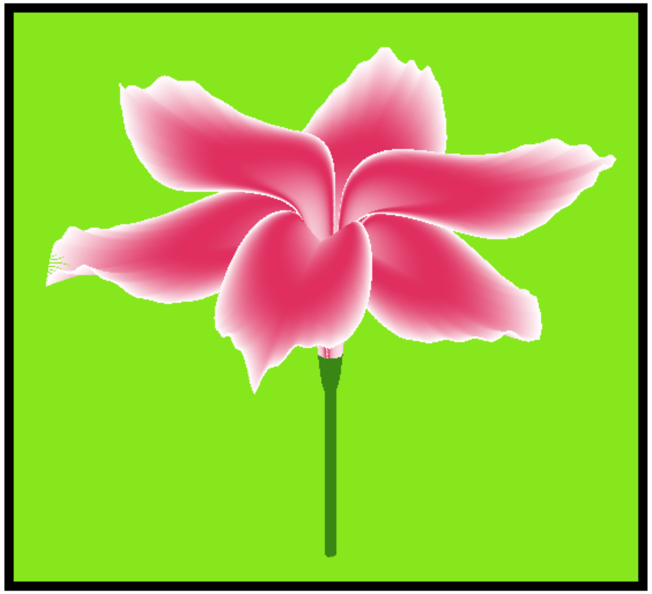
Method of generating Fractals

- **Realistic Fractal Images**
- **One set of Parametric Equations**
- **Function Composition**
- **Geometric Transformations**





The objects are fractals because they are generated starting from very little information.



$$r = a \cdot \frac{|\sin(1.5 \cdot u)| + 1 - |\cos(1.5 \cdot u)|}{2}$$

$$x = r \cdot \cos\left(u + \frac{\pi}{3}\left[\frac{u}{2\pi}\right]\right), \quad y = r \cdot \sin\left(u + \frac{\pi}{3}\left[\frac{u}{2\pi}\right]\right)$$

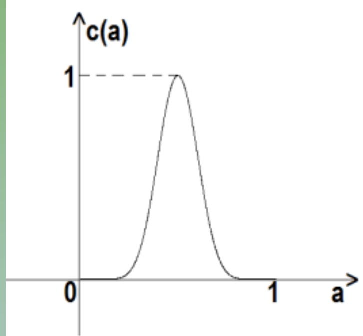
$$z = -\frac{7}{(10 \cdot r)^{40}} - \frac{4}{(3 \cdot r)^4 + 1} + 0.6 \cdot \sin(r) \cdot |\cos(1.5 \cdot u)| +$$

$$0.2 \cdot \left(\frac{r}{6}\right)^6 \cdot \left(0.7 \cdot \sin(10 \cdot u) + \sin(23 \cdot u) + 7 \cdot |\sin(6 \cdot u)|\right) -$$

$$0.35 \cdot \left[\frac{u}{2\pi}\right]$$

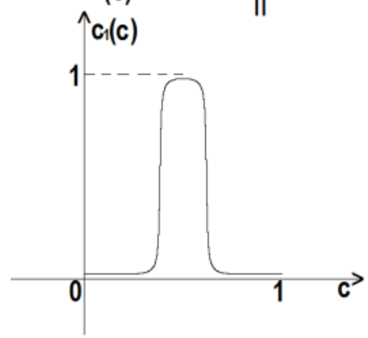
$$a \in [0,6], \quad u \in [0, 2\pi]$$

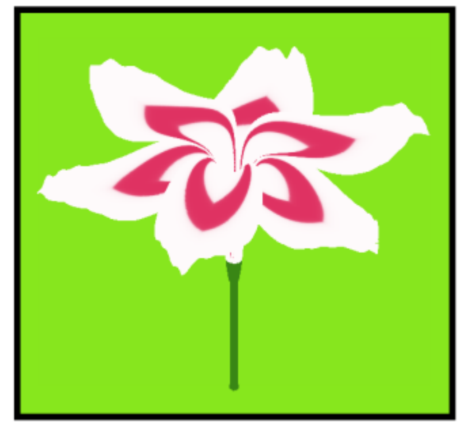
$c(a) = \sin^{10}(\pi a)$



$c(a) = \sin^{10}(\pi a)$

$$c_1(c) = \frac{\mathrm{arctg}(30 \cdot (c - 0.5)) + \frac{\pi}{2}}{\pi}$$



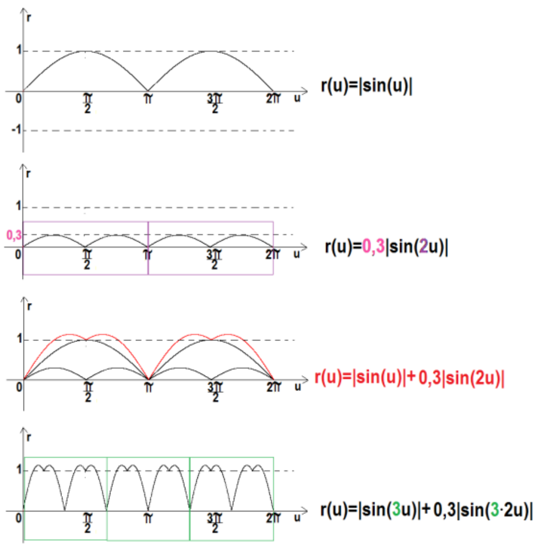


All points of the objects are represented by one single set of parametric equations. The parametric equations of the coordinates are determined through operations on functions and function composition. The same method is applied for the components of the color

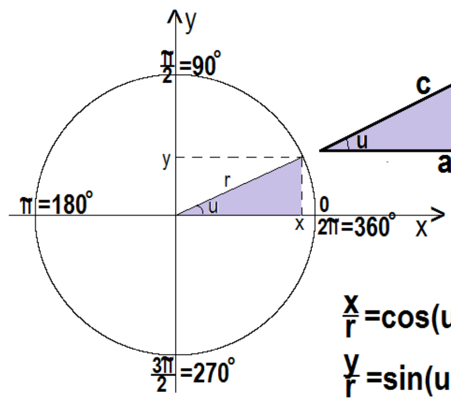
$$R = R_1 \cdot (1 - c_1) + R_2 \cdot c_1$$
$$G = G_1 \cdot (1 - c_1) + G_2 \cdot c_1$$
$$B = B_1 \cdot (1 - c_1) + B_2 \cdot c_1$$

Trigonometric functions

RADIAL FUNCTION



PARAMETRIC EQUATIONS OF THE CIRCLE



$\sin(u)=\frac{b}{c}$

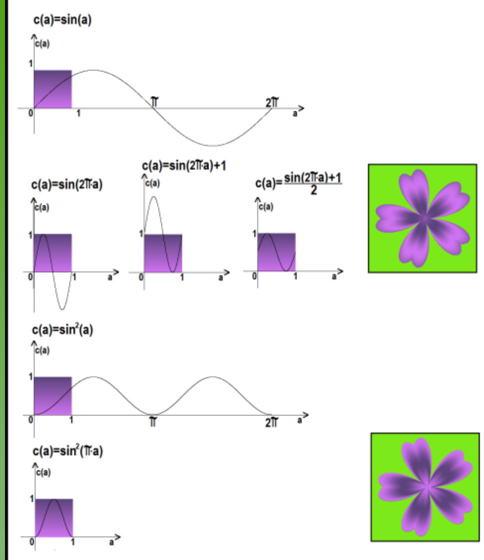
$\cos(u)=\frac{a}{c}$

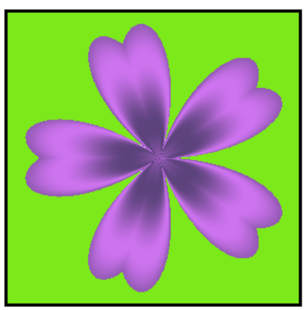
$\frac{x}{r}=\cos(u) \Rightarrow$ $x=r\cdot\cos(u)$

$\frac{y}{r}=\sin(u) \Rightarrow$ $y=r\cdot\sin(u)$

Ecuatiile parametrice ale cercului

COLOR GRADIENT FUNCTION





DOMAIN

$u\in[0, 2\pi)$

$a\in[0,1]$

RADIAL FUNCTION

$r(u)=a\cdot(|\sin(2.5\cdot u)|+0.3\cdot|\sin(5\cdot u)|)$

PARAMETRIC EQUATIONS OF THE CIRCLE

$x=r(u)\cdot\cos(u)$

$y=r(u)\cdot\sin(u)$

COLOR GRADIENT FUNCTION

$c=c(a)$

COLOR GRADIENT

$R=R_1\cdot(1-c)+R_2\cdot c$

$G=G_1\cdot(1-c)+G_2\cdot c$

$B=B_1\cdot(1-c)+B_2\cdot c$

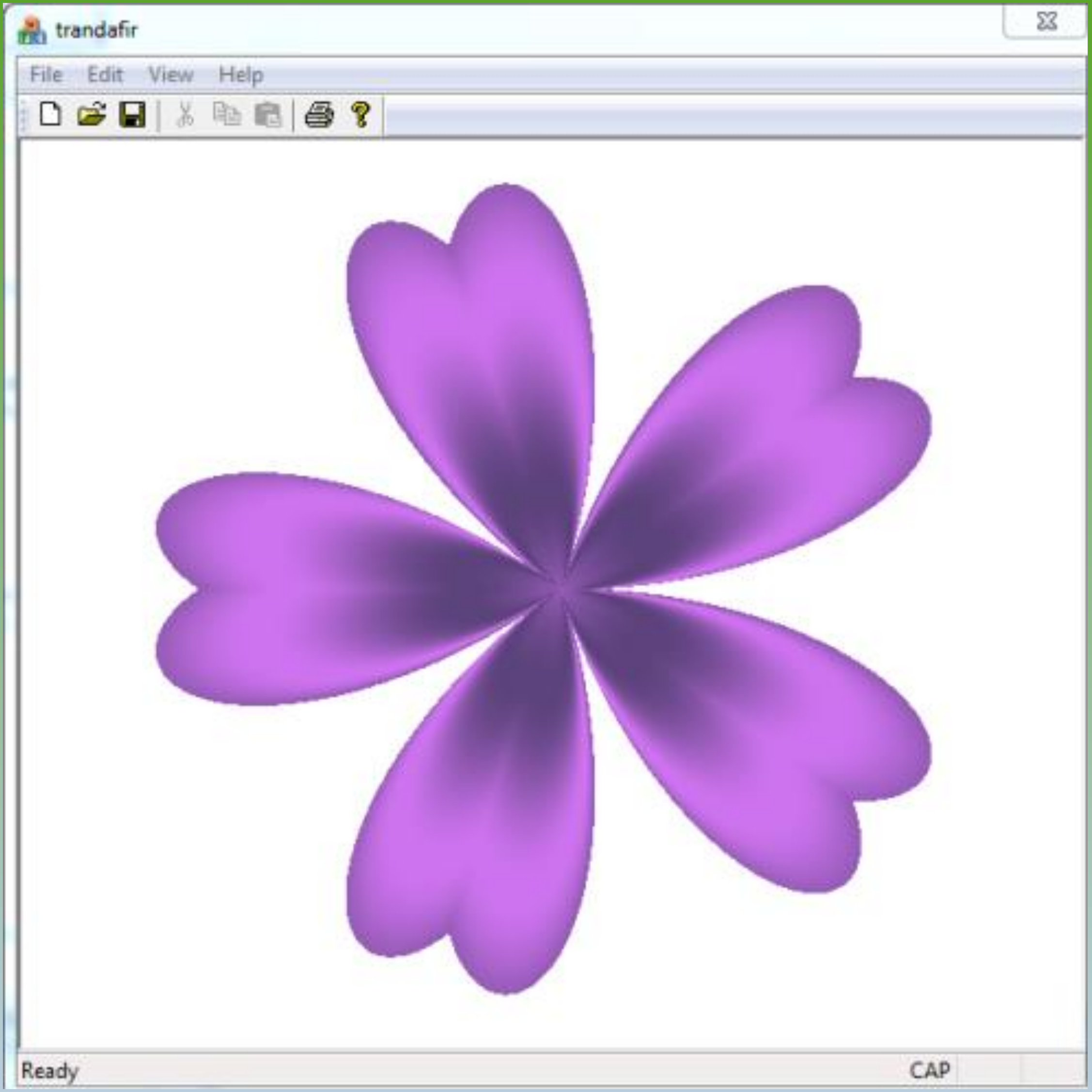
c++ PROGRAM

```
//VARIABLES
float x,y,r,a,u,d=200,cx=300,cy=250,pi=3.14,
R1=207,G1=116,B1=241,R2=92,G2=67,B2=124,R,G,B,c;
//DOMAIN
for(u=0;u<=2*pi;u+=0.0005)
    for(a=0;a<=1;a+=0.002)
    {
        //RADIAL FUNCTION
        r=a*(abs(sin(2.5*u))+0.3*abs(sin(5*u)));
        //PARAMETRIC EQUATION OF THE CIRCLE
        x=r*cos(u);
        y=r*sin(u);
        //COLOR GRADIENT FUNCTION
        c=(sin(2*pi*a)+1)/2;
        //COLOR GRADIENT
        R=R1*(1-c)+R2*c;
        G=G1*(1-c)+G2*c;
        B=B1*(1-c)+B2*c;
        //PLOT
        pDC->SetPixel(x*d+cx,-y*d+cy,RGB(R,G,B));
    }
```

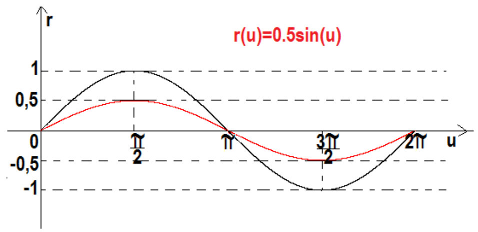
How does the method work?

- create an application
- chunk the theory into steps
- get all the formulas
- switch the formulas to the source code.

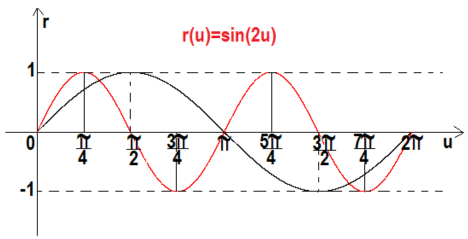
Each step has its correspondent in the source code.



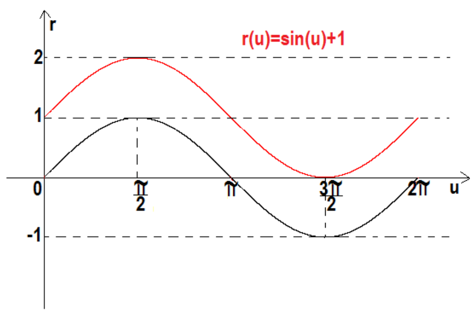
Theory



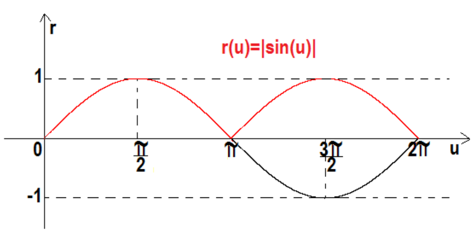
u	sin(u)	0,5sin(u)
0	0	0
$\frac{\pi}{2}$	1	0,5
π	0	0
$\frac{3\pi}{2}$	-1	-0,5
2π	0	0



u	sin(u)	2u	sin(2u)
0	0	0	0
$\frac{\pi}{4}$		$\frac{\pi}{2}$	1
$\frac{\pi}{2}$	1	π	0
$\frac{3\pi}{4}$		$\frac{3\pi}{2}$	-1
π	0	2π	0
$\frac{5\pi}{4}$			
$\frac{3\pi}{2}$	-1		
$\frac{7\pi}{4}$			
2π	0		

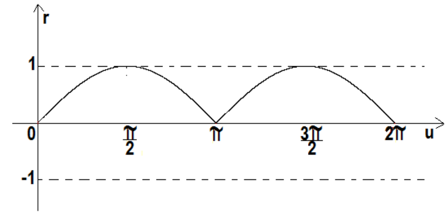


u	sin(u)	sin(u)+1
0	0	1
$\frac{\pi}{2}$	1	2
π	0	1
$\frac{3\pi}{2}$	-1	0
2π	0	1

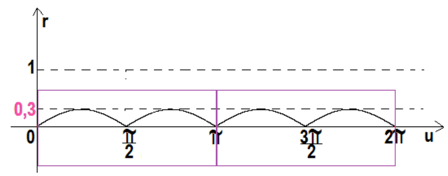


u	sin(u)	|sin(u)|
0	0	0
$\frac{\pi}{2}$	1	1
π	0	0
$\frac{3\pi}{2}$	-1	1
2π	0	0

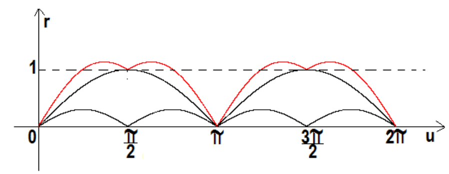
Application



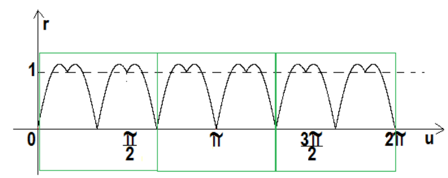
$r(u)=|\sin(u)|$



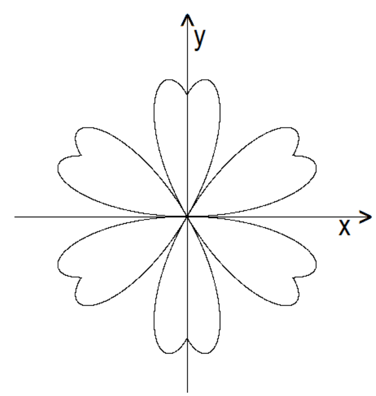
$r(u)=0,3|\sin(2u)|$



$r(u)=|\sin(u)|+0,3|\sin(2u)|$



$r(u)=|\sin(3u)|+0,3|\sin(3\cdot 2u)|$



$r(u)=|\sin(3u)|+0.3|\sin(6u)|$

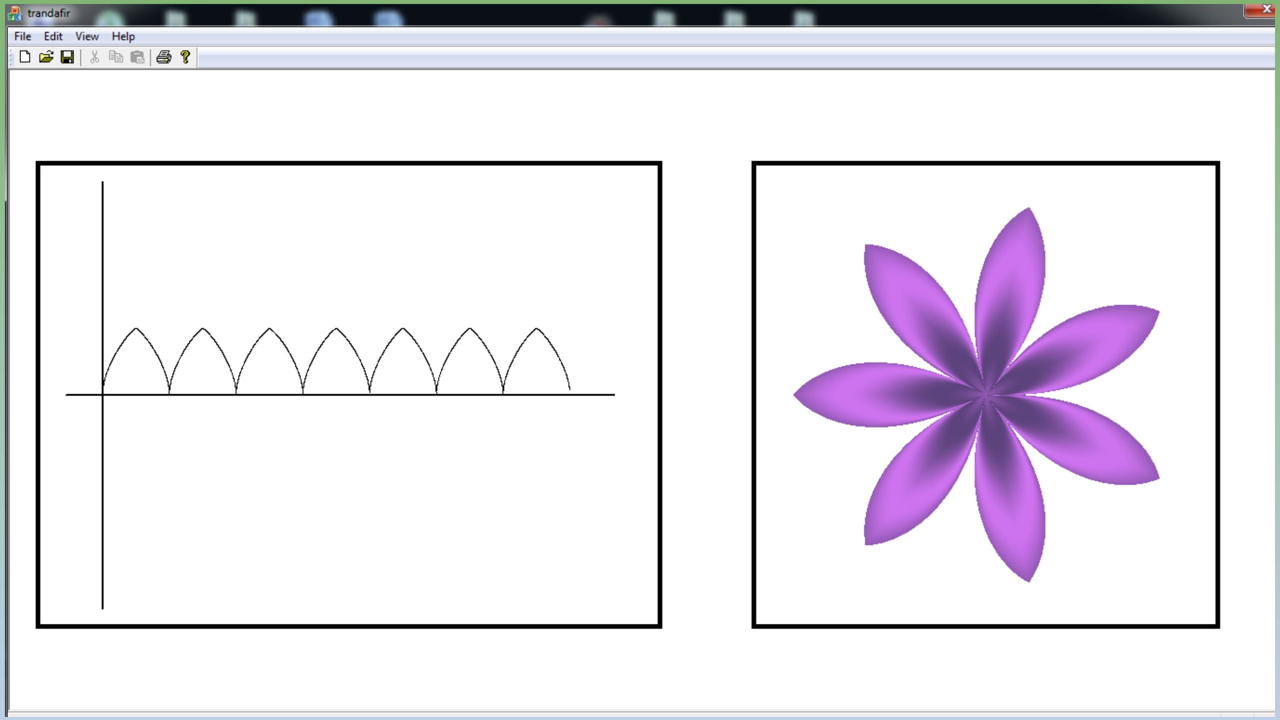
$x=r(u)\cdot\cos(u)$

$y=r(u)\cdot\sin(u)$

$u\in[0, 2\pi)$

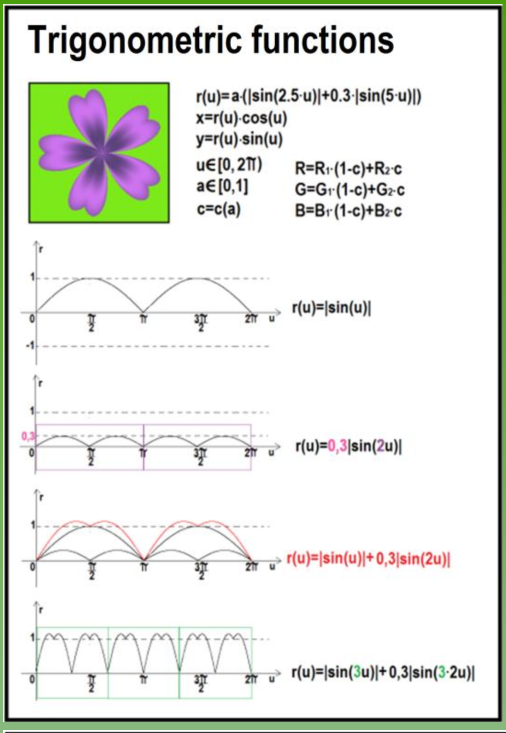
E.g.: detailed study of operations with trigonometric functions.

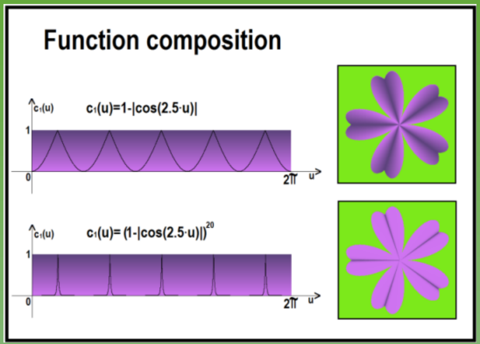
First, some theory, like what happens if I multiply the function with a number, or what if I multiply the argument with a number, and so on.
The result is a pattern I use to create different models of petals.

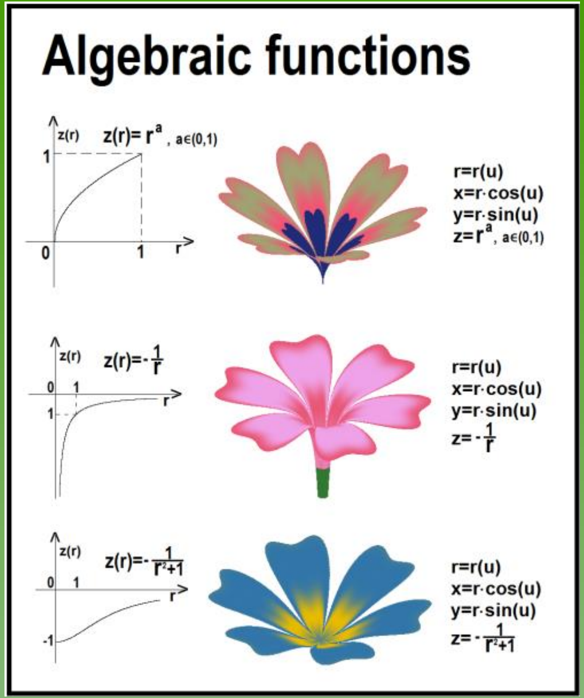


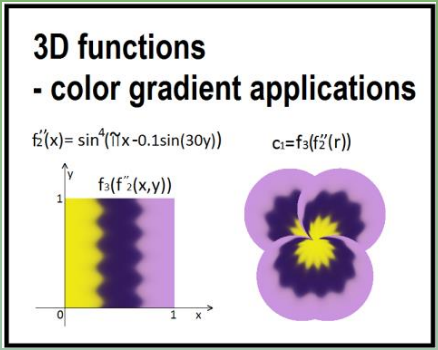
Functions

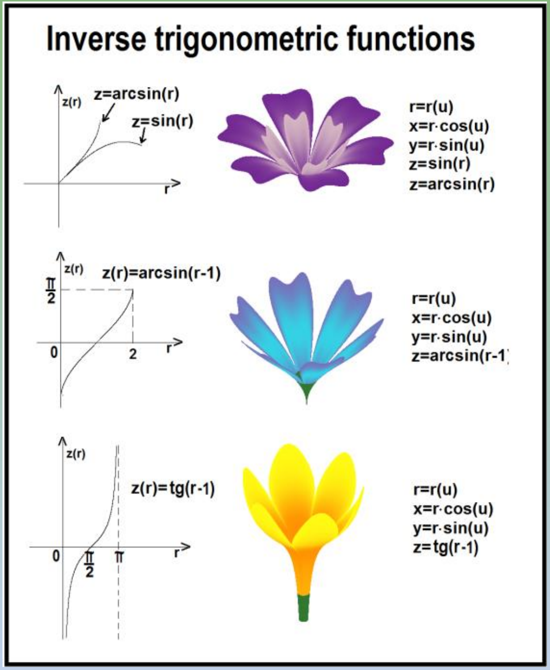
Trigonometric functions represents a part of a whole chapter about functions. Here are some examples:

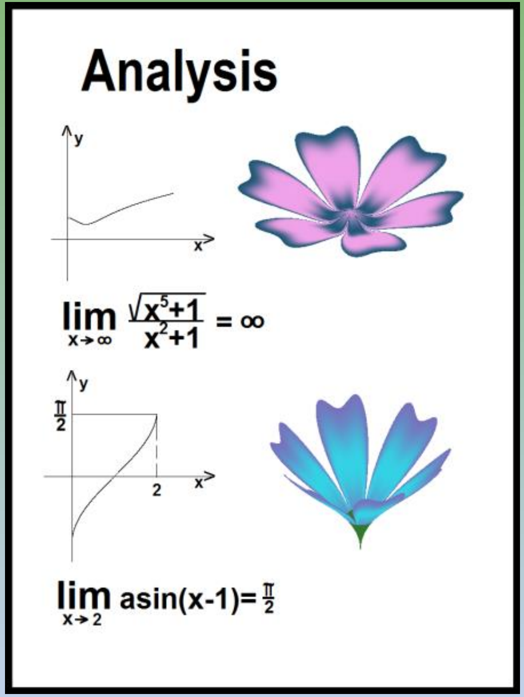








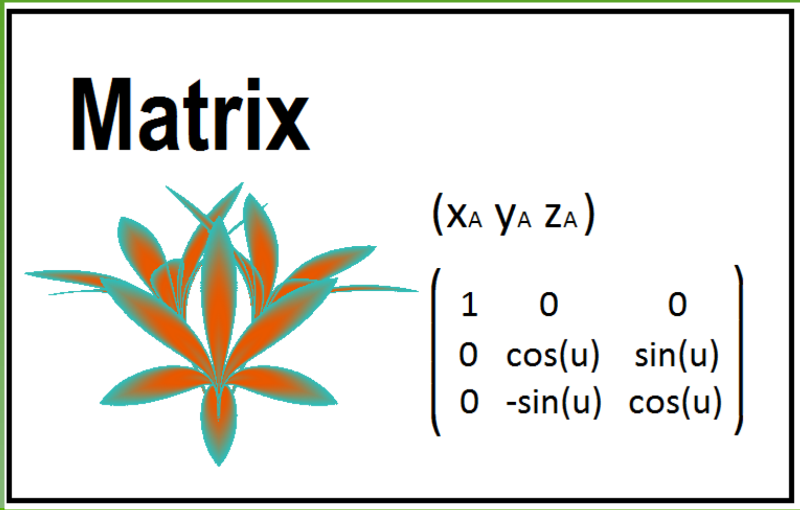


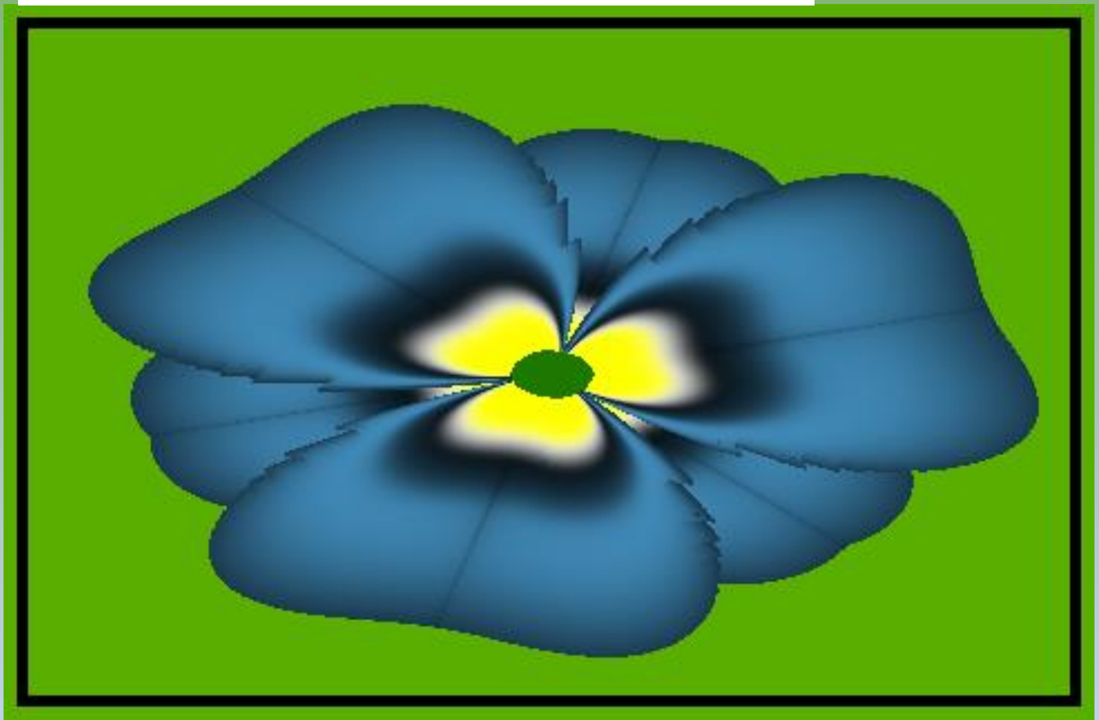


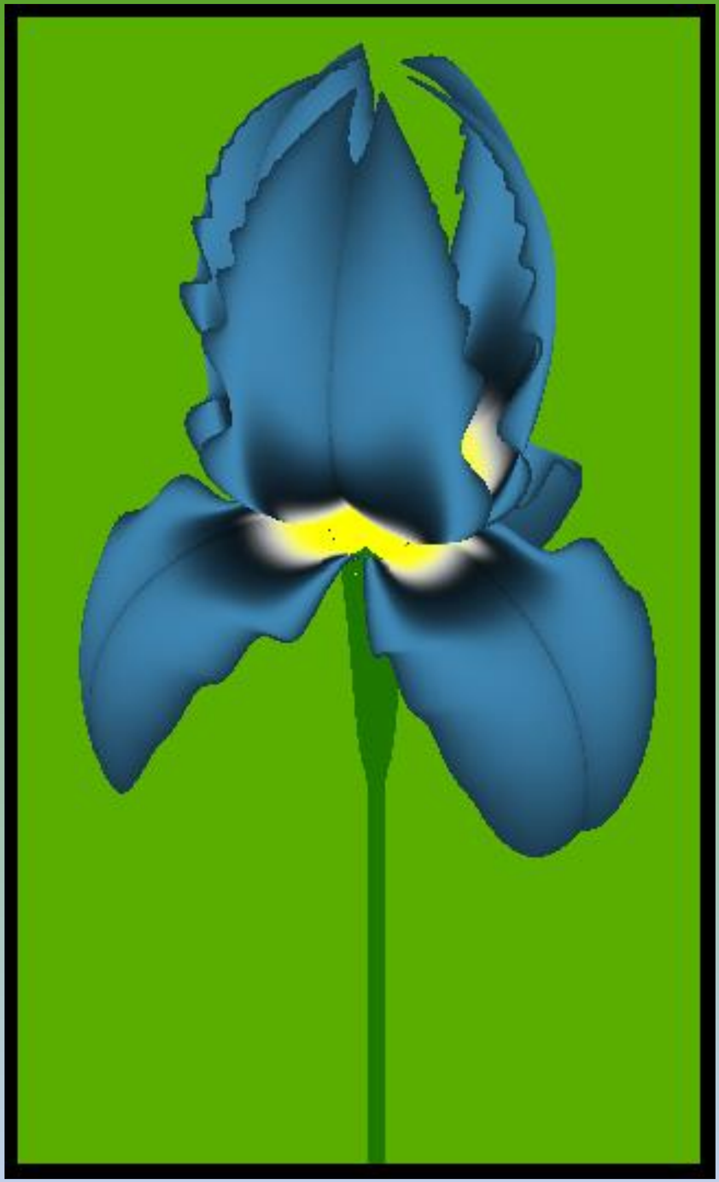
Other mathematical concepts, used to transform the objects already created :

- reflection of a flower onto a surface by means of analytic geometry,

- lightning effect by means of vectors or differential geometry…







Analytic geometry - Reflection application

$\mathcal{P}$: $ax+by+cz+d=0$

d: $\frac{x-x_A}{a}=\frac{y-y_A}{b}=\frac{z-z_A}{c}$

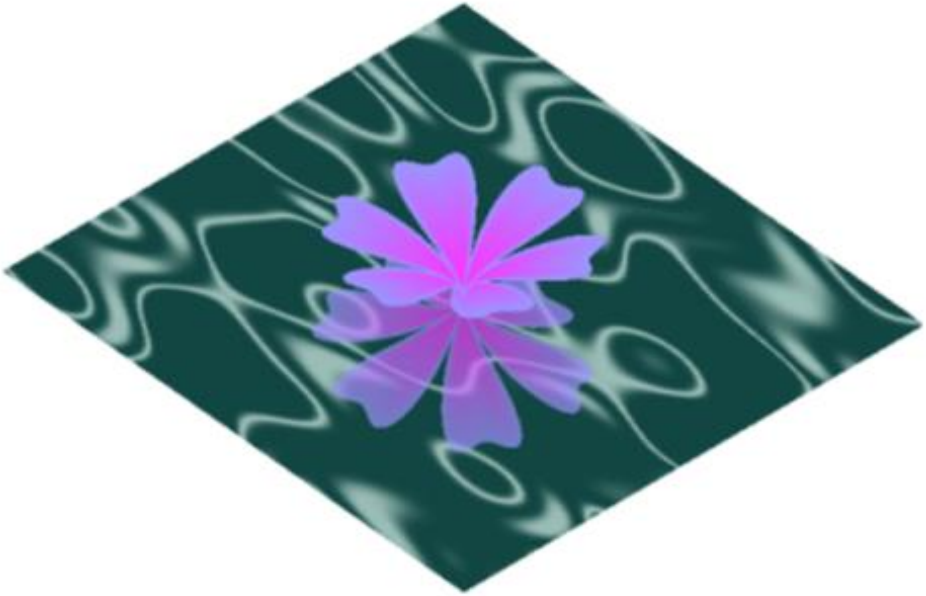
$\{B\}\in d$: $\frac{x_B-x_A}{a}=\frac{y_B-y_A}{b}=\frac{z_B-z_A}{c}$

$\{B\}\in \mathcal{P}$: $ax_B+by_B+cz_B+d=0$

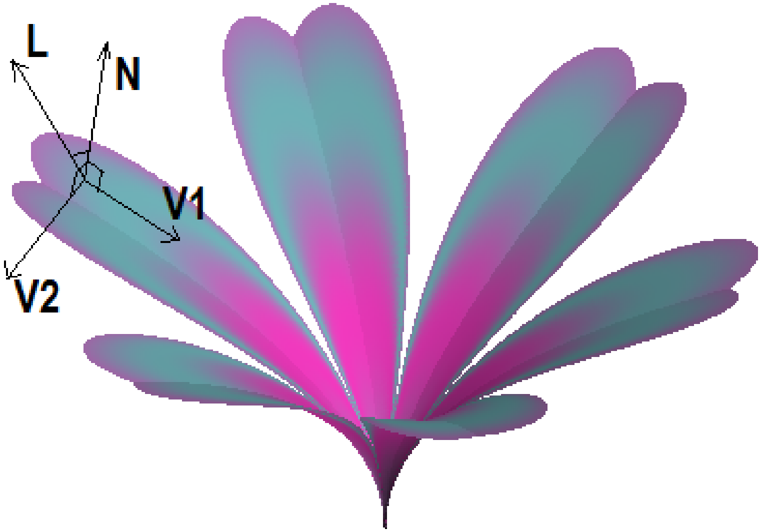
$x_C=2\cdot x_B-x_A$

$y_C=2\cdot y_B-y_A$

$z_C=2\cdot z_B-z_A$



Vectors -illumination application



$x1=x(r1,u),\ y1=y(r1,u),\ z1=z(r1,u) \quad r1=r+dr,\ u1=u+du$

$x2=x(r,u1),\ y2=y(r,u1),\ z2=z(r,u1)$

$\overrightarrow{V1}\ (V1x, V1y, V1z) \quad V1x=x1-x,\ V1y=y1-y,\ V1z=z1-z$

$\overrightarrow{V2}\ (V2x, V2y, V2z) \quad V2x=x2-x,\ V2y=y2-y,\ V2z=z2-z$

$\overrightarrow{N}\ (Nx,Ny,Nz)$

$\overrightarrow{N} \perp \overrightarrow{V1}:\ Nx\cdot V1x+Ny\cdot V1y+Nz\cdot V1z=0$

$\overrightarrow{N} \perp \overrightarrow{V2}:\ Nx\cdot V2x+Ny\cdot V2y+Nz\cdot V2z=0$

$\|\overrightarrow{N}\|=1 \quad \sqrt{Nx^2+Ny^2+Nz^2}=1$

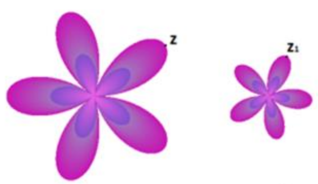
$Nx=V1z\cdot V2y-V1y\cdot V2z$

$Ny=V1z\cdot V2x-V1x\cdot V2z;$

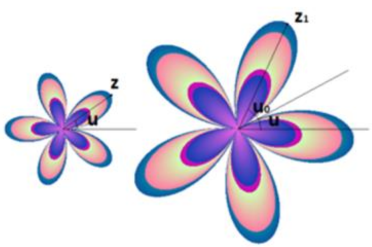
$Nz=V1x\cdot V2y-V1y\cdot V2x;$

$$\cos(\widehat{N,L})=\frac{Nx\cdot Lx+Ny\cdot Ly+Nz\cdot Lz}{\sqrt{Nx^2+Ny^2+Nz^2}\sqrt{Lx^2+Ly^2+Lz^2}} \qquad il=\frac{\cos(\widehat{N,L})+1}{2}$$

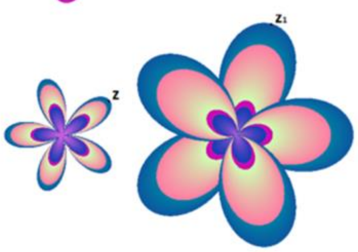
Complex numbers



$$\begin{aligned} z_1 &= z\cdot z_0 \\ &= (x+y\cdot i)(x_0+y_0\cdot i) \\ &= xx_0+xy_0\cdot i+yx_0\cdot i+yy_0\cdot i^2 \\ &= xx_0-yy_0+(xy_0+yx_0)i \end{aligned}$$

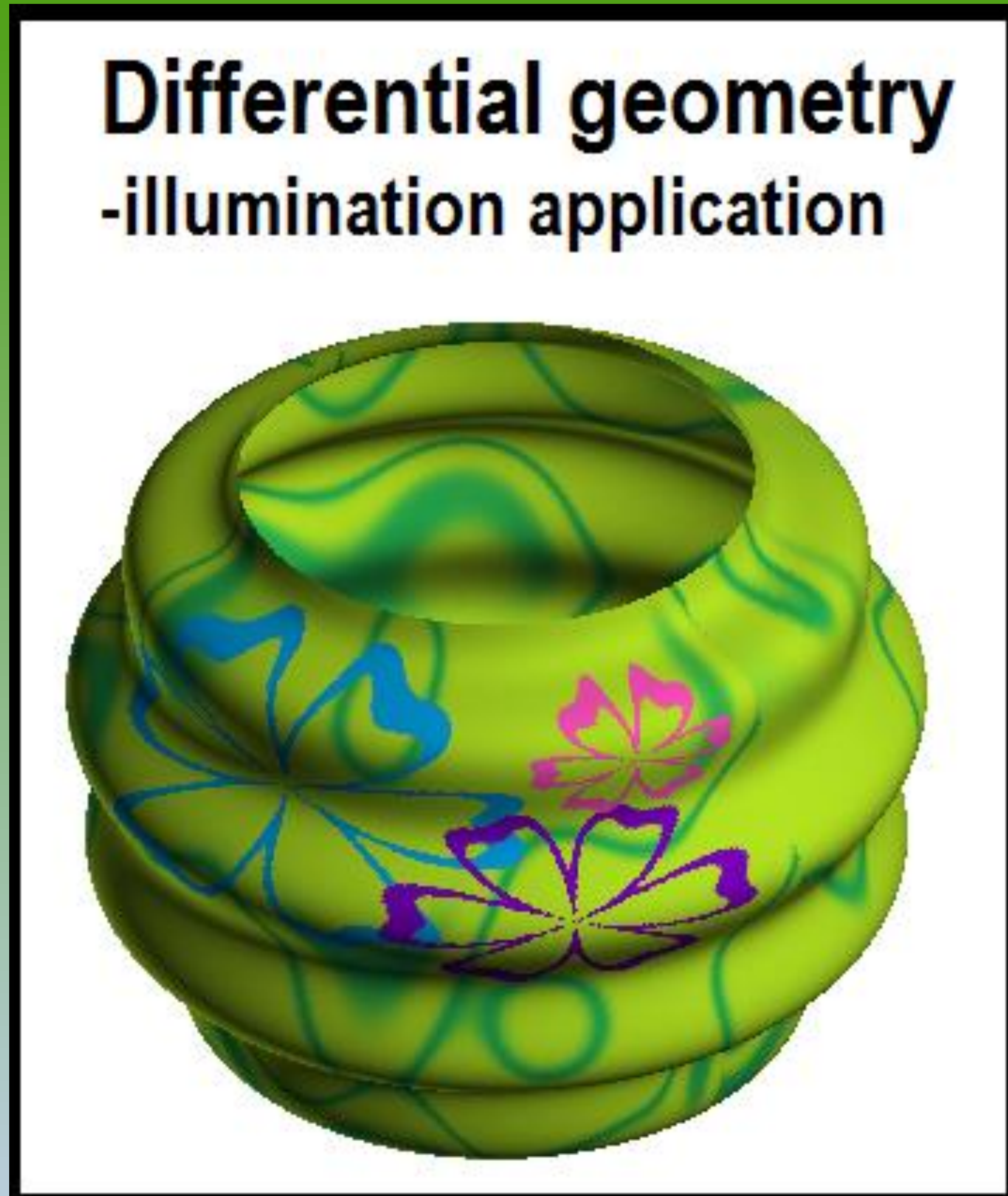


$$\begin{aligned} z1 &= z\cdot z_0 \\ &= \{r[\cos(u)+i\ \sin(u)]\}\{r_0[\cos(u_0)+i\cdot\sin(u_0)]\} \\ &= rr_0\{\cos(u)\cos(u_0)-\sin(u)\sin(u_0) \\ &\qquad +i[\cos(u)\sin(u_0)+\sin(u)\cos(u_0)]\} \\ &= rr_0[\cos(u+u_0)+i\cdot\sin(u+u_0)] \end{aligned}$$



$$\begin{aligned} z_1 &= z^2 \\ &= (x+y\cdot i)^2 \\ &= x^2+2xy\cdot i+y^2\cdot i^2 \\ &= x^2-y^2+2xy\cdot i \end{aligned}$$

$$\begin{aligned} z_1 &= \sqrt{z} \\ &= \sqrt{r}[\cos(u/2)+i\ \sin(u/2)] \end{aligned}$$


Differential geometry
-illumination application

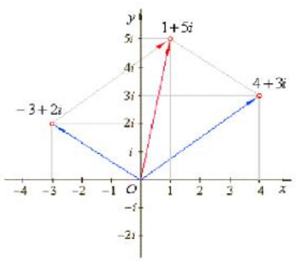
Most softwares use these methods, but kids only learn how to use the softwares.

The aim of DigiMathArt method is to teach mathematics by applying these methods and to create applications through computer programming.

This learning system is so motivating that even a 13 years old teenager can understand, and even prove those formulas.

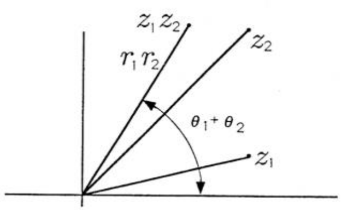
Classical method versus DigiMathArt

Classical method

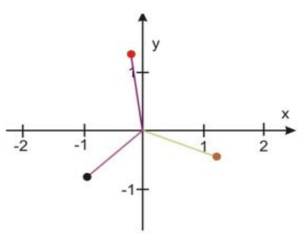


$z_1 = z \cdot z_0$
$= (x + y \cdot i)(x_0 + y_0 \cdot i)$
$= xx_0 + xy_0 \cdot i + yx_0 \cdot i + yy_0 \cdot i^2$
$= xx_0 - yy_0 + (xy_0 + yx_0)i$

$z_1 = z^2$
$= (x + y \cdot i)^2$
$= x^2 + 2xy \cdot i + y^2 \cdot i^2$
$= x^2 - y^2 + 2xy \cdot i$



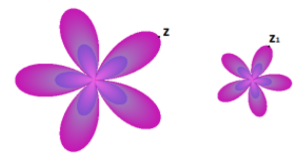
$z1 = z \cdot z_0$
$= \{r[\cos(u) + i \sin(u)]\}\{r_0[\cos(u_0) + i \cdot \sin(u_0)]\}$
$= rr_0\{\cos(u)\cos(u_0) - \sin(u)\sin(u_0)$
$+ i[\cos(u)\sin(u_0) + \sin(u)\cos(u_0)]\}$
$= rr_0[\cos(u + u_0) + i \cdot \sin(u + u_0)]$



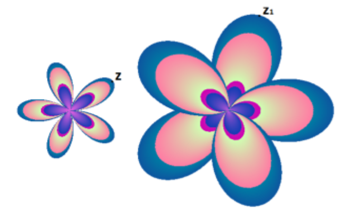
$z1 = z^n$
$= \{r[\cos(u) + i \cdot \sin(u)]\}^n$
$= r^n[\cos(n \cdot u) + i \cdot \sin(n \cdot u)]$

$z_1 = \sqrt{z}$
$= \sqrt{r}[\cos(u/2) + i \sin(u/2)]$

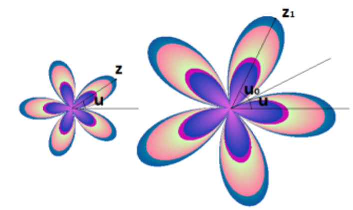
DigiMathArt method



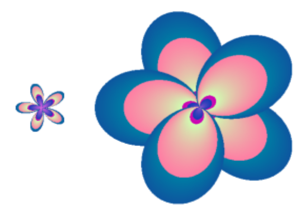
$z_1 = z \cdot z_0$
$= (x + y \cdot i)(x_0 + y_0 \cdot i)$
$= xx_0 + xy_0 \cdot i + yx_0 \cdot i + yy_0 \cdot i^2$
$= xx_0 - yy_0 + (xy_0 + yx_0)i$



$z_1 = z^2$
$= (x + y \cdot i)^2$
$= x^2 + 2xy \cdot i + y^2 \cdot i^2$
$= x^2 - y^2 + 2xy \cdot i$



$z1 = z \cdot z_0$
$= \{r[\cos(u) + i \sin(u)]\}\{r_0[\cos(u_0) + i \cdot \sin(u_0)]\}$
$= rr_0\{\cos(u)\cos(u_0) - \sin(u)\sin(u_0)$
$+ i[\cos(u)\sin(u_0) + \sin(u)\cos(u_0)]\}$
$= rr_0[\cos(u + u_0) + i \cdot \sin(u + u_0)]$



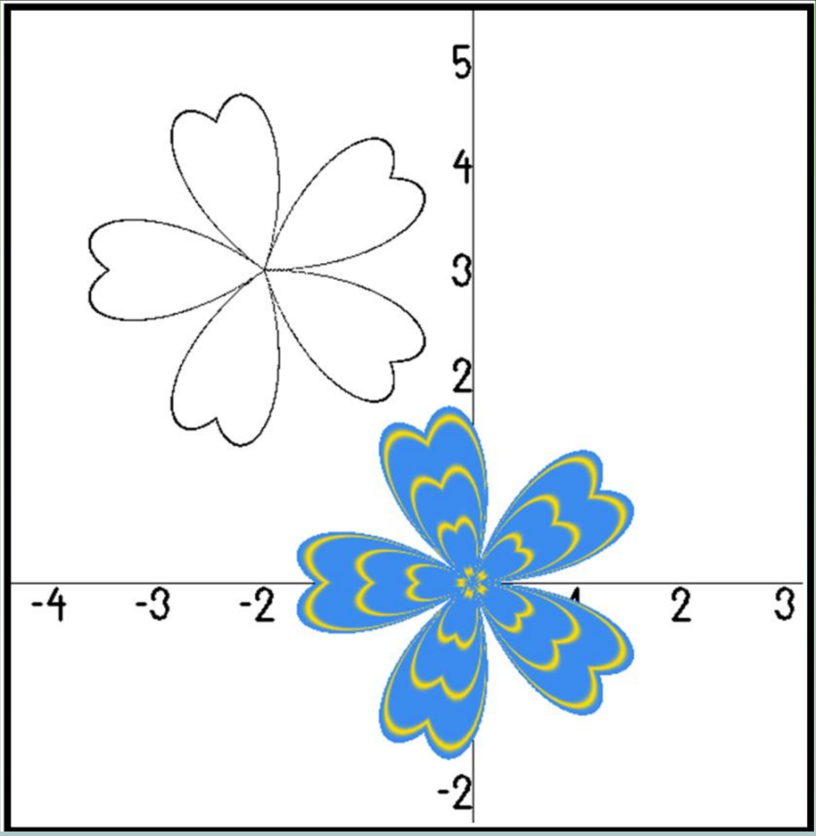
$z1 = z^n$
$= \{r[\cos(u) + i \cdot \sin(u)]\}^n$
$= r^n[\cos(n \cdot u) + i \cdot \sin(n \cdot u)]$

$z_1 = \sqrt{z}$
$= \sqrt{r}[\cos(u/2) + i \sin(u/2)]$

The theory is the same. In the classical method, the transformation is applied to one point, DMA method applies it to an object

Evaluation

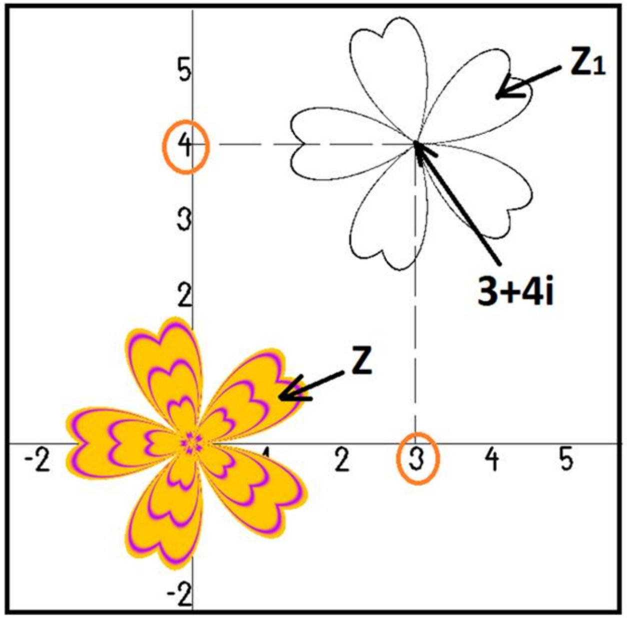
Exercise: **Move the flower using operations on complex numbers**




```
for(r=0;r<=1.5;r+=0.005)
    for(u=0;u<=2*pi;u+=0.003)
    {
        r1=r*(fabs(sin(2.5*u))
            +0.3*fabs(sin(5*u)));
        x=r1*cos(u);
        y=r1*sin(u);

        c=pow((sin(14.5*r)+1)/2,15);
        R=R1*(1-c)+R2*c;
        G=G1*(1-c)+G2*c;
        B=B1*(1-c)+B2*c;
        pDC->SetPixel(x*d+cx,-y*d+cy,
            RGB(R,G,B));
    }
```

- do the math calculus and modify the source code



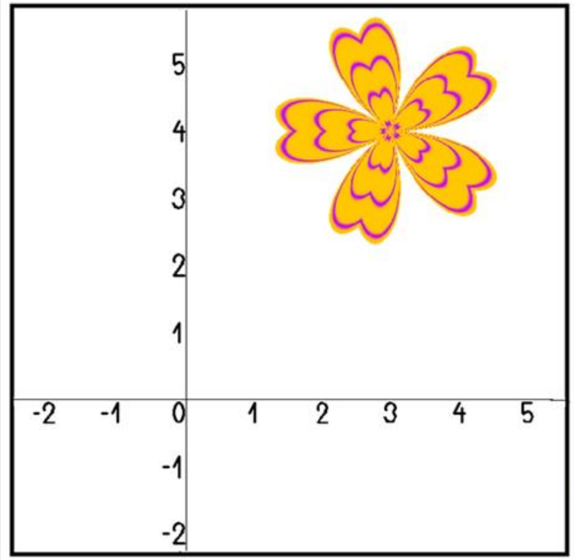
$Z = X + Yi$

$Z_1 = X_1 + Y_1 i$

X1= ?

Y1= ?

```
for(r=0;r<=1.5;r+=0.005)
    for(u=0;u<=2*pi;u+=0.003)
    {
        r1=r*(fabs(sin(2.5*u))
            +0.3*fabs(sin(5*u)));
        x=r1*cos(u);
        y=r1*sin(u);
        c=pow((sin(14.5*r)+1)/2,15);
        R=R1*(1-c)+R2*c;
        G=G1*(1-c)+G2*c;
        B=B1*(1-c)+B2*c;
        pDC->SetPixel(x*d+cx,-y*d+cy,
            RGB(R,G,B));
    }
```



$Z = X + Yi$

$Z_1 = X_1 + Y_1 i$

$Z_1 = Z + (3+4i)$

$= X + Yi + 3 + 4i$

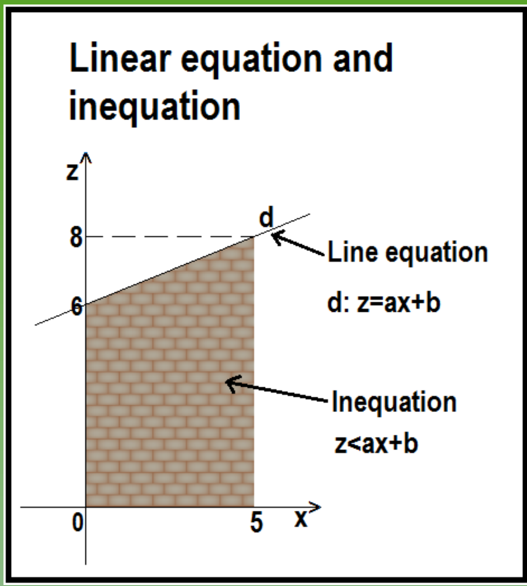
$= X+3 + (Y+4)i$

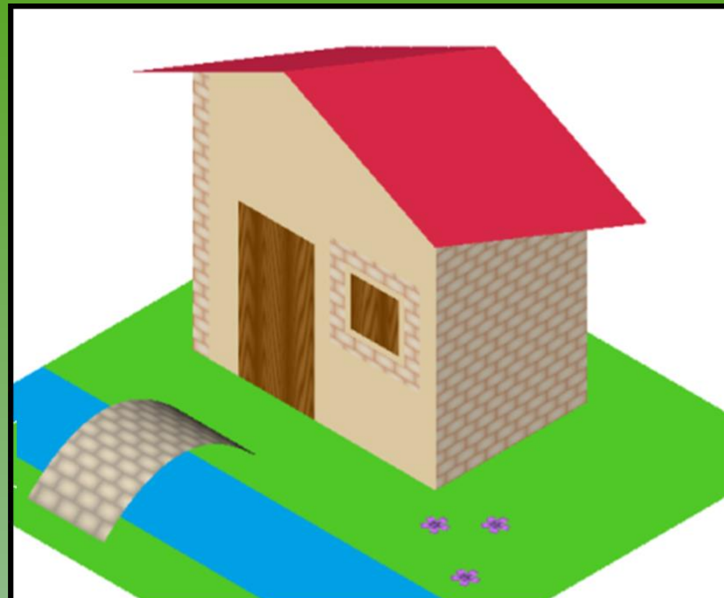
X1=X+3

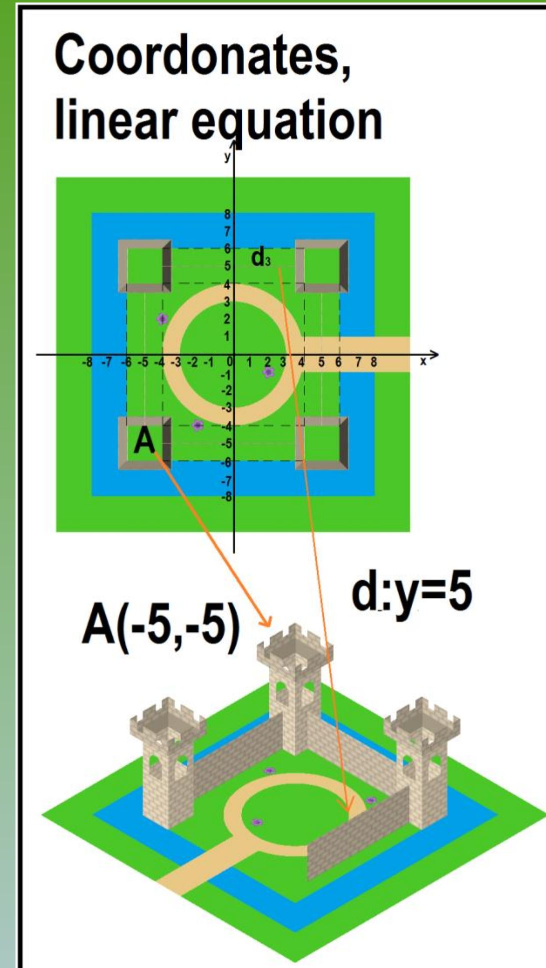
Y1=Y+4

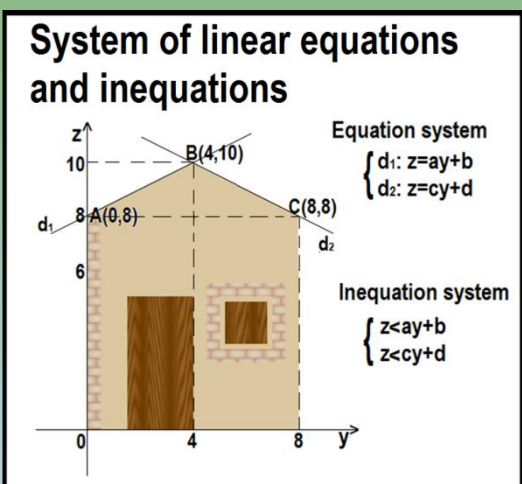
```
for(r=0;r<=1.5;r+=0.005)
    for(u=0;u<=2*pi;u+=0.003)
    {
        r1=r*(fabs(sin(2.5*u))
            +0.3*fabs(sin(5*u)));
        x=r1*cos(u);
        y=r1*sin(u);
        x1=x+3;
        y1=y+4;
        c=pow((sin(14.5*r)+1)/2,15);
        R=R1*(1-c)+R2*c;
        G=G1*(1-c)+G2*c;
        B=B1*(1-c)+B2*c;
        pDC->SetPixel(x1*d+cx,-y1*d+cy,
            RGB(R,G,B));
    }
```

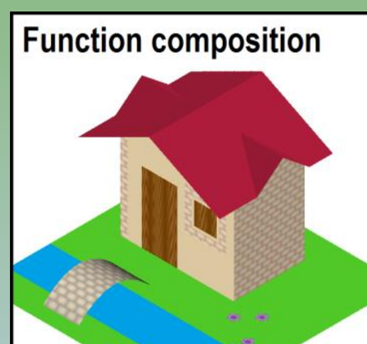
Gymnasium - Algebra

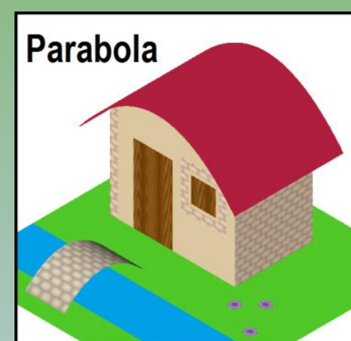






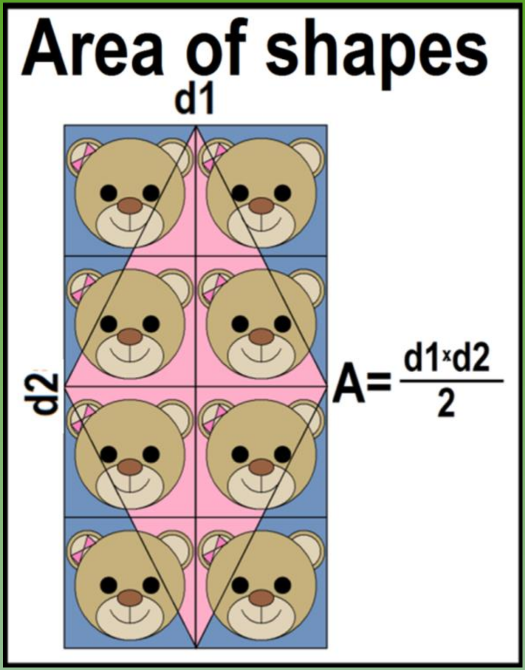


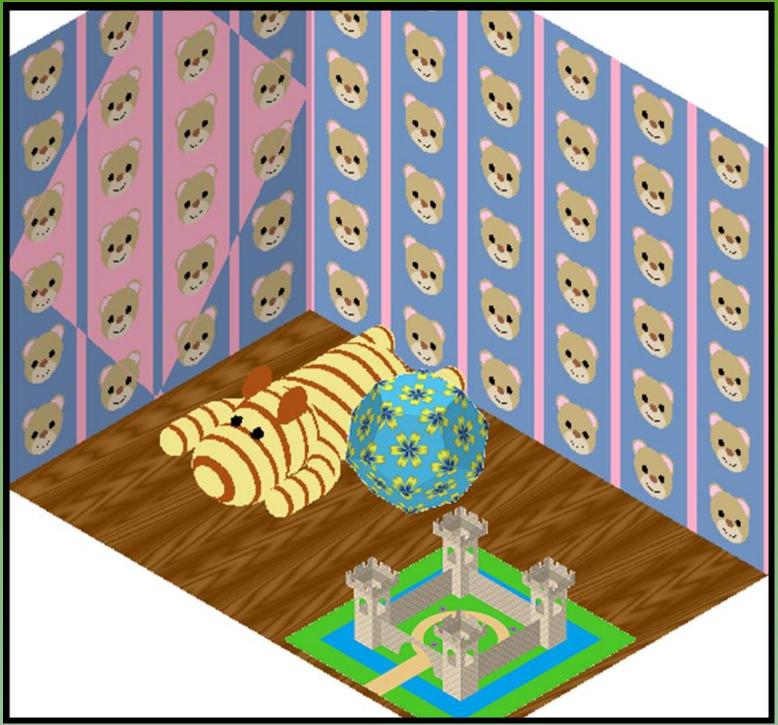


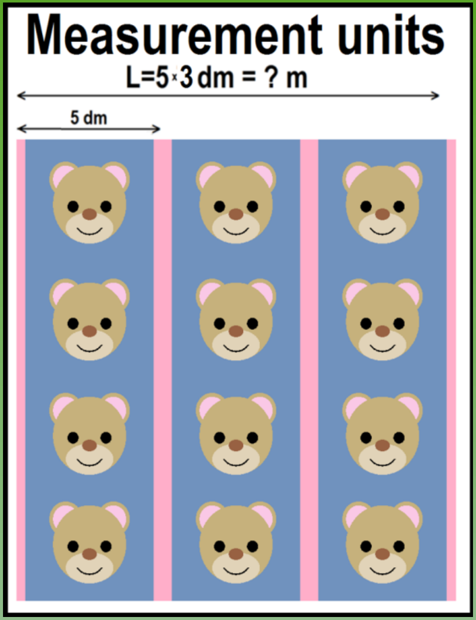


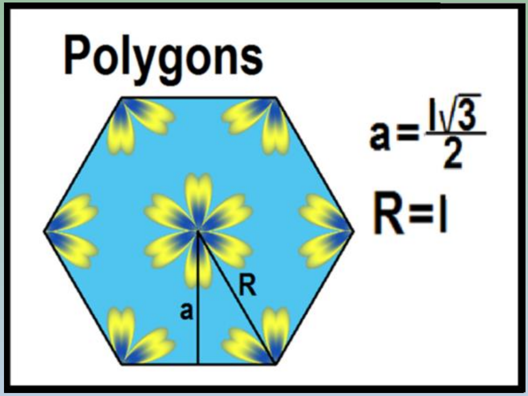
Examples of gymnasium algebra lessons,
as simple as for a 7 years old child to understand

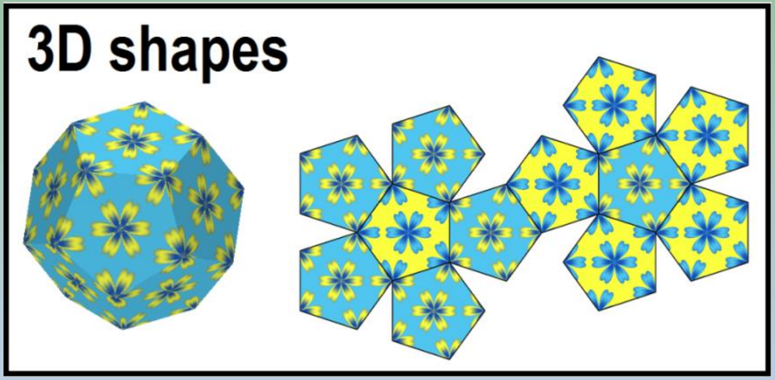
Geometry

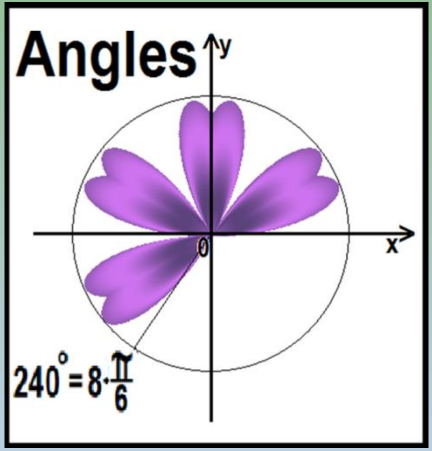

Area of shapes
d1
d2
A= d1×d2 / 2



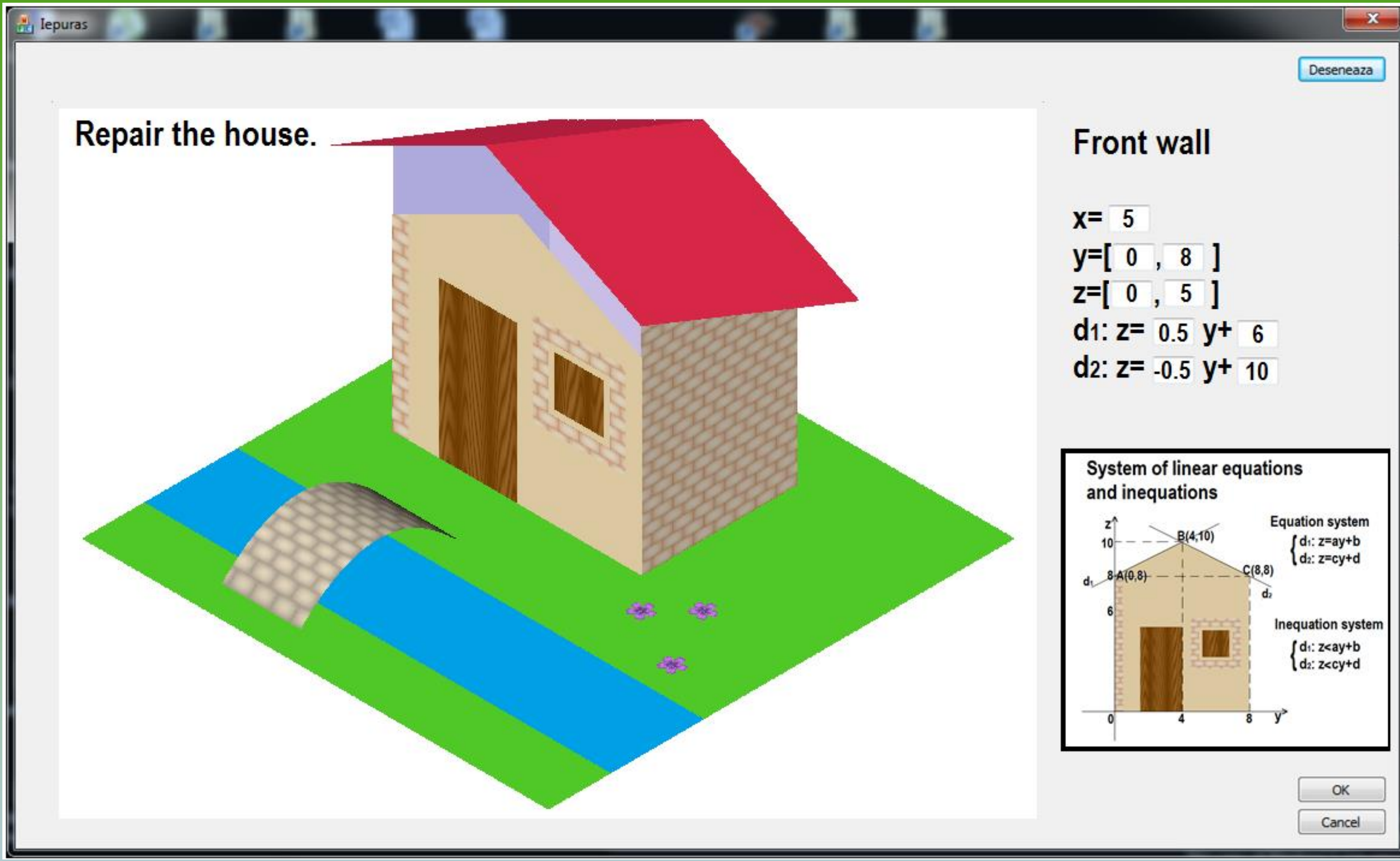

Measurement units
L=5×3 dm = ? m
5 dm


Polygons
a= l√3 / 2
R=l
R
a


3D shapes

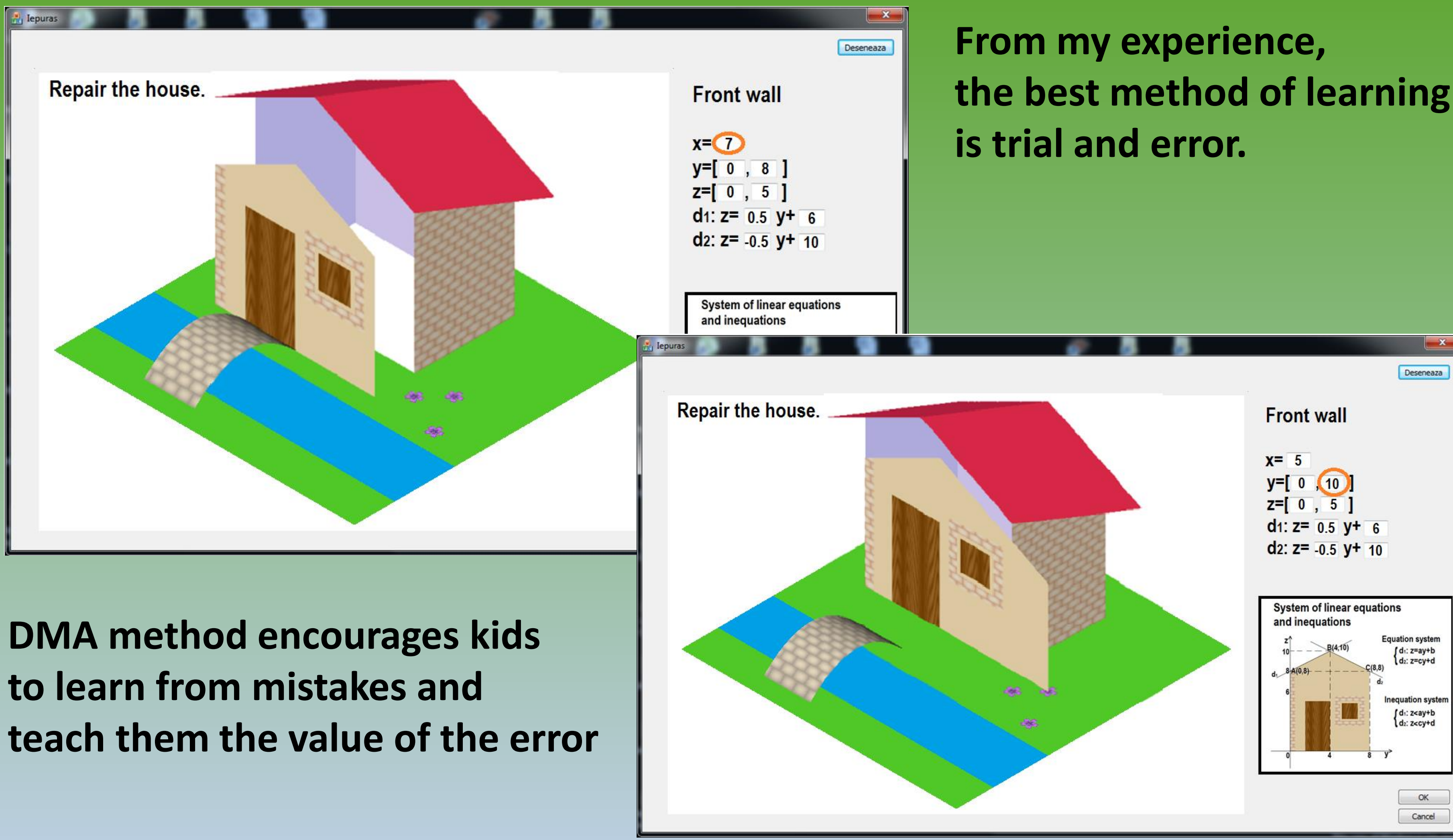

Angles
y
x
0
240°=8·π/6

Evaluation



For gymnasium, I use applications with an interface for evaluation and for practice as well

Learning from mistakes



From my experience, the best method of learning is trial and error.

DMA method encourages kids to learn from mistakes and teach them the value of the error

DMA = “Enriched environment”

An enriched environment increase:

cortical thickness and weight,

dendritic branching,

neurogenesis and newborn neuronal integration within the already existing neuronal networks,

expression of 41 genes involved in learning and memory, synaptic plasticity, neuro / vasculogenesis, cellular growth excitability, synaptic transmission, neurotrophic factors , dopaminergic, serotonergic and noradrenergic systems

(Baroncelli, Braschi, Spolidoro, et al., 2010