Enhancing Conceptual Understanding of Fractions with New Methodological Tools (MERLO)

Marien Alet Graham[1], Lydia Omowunmi Adesanya[2]

Department of Early Childhood Education, University of Pretoria, South Africa[1]
Department of Science, Mathematics and Technology Education, University of Pretoria, South Africa[2]

Abstract

In the South African context, the state of mathematics education is a pressing concern, with both national and international assessments indicating significant underperformance among South African learners [1]. Recognising this challenge, Adesanya and Graham [2-4] pioneered the introduction of a novel tool called Meaning Equivalence Reusable Learning Objects (MERLO) to enhance the teaching and learning of mathematical concepts. Prior to our intervention, MERLO had only been implemented in countries such as Australia, Canada, Israel, Italy, Russia, and the Netherlands. Our initiative marked the first integration of MERLO into African schools, with the conviction that our educational system stands to benefit significantly, given the positive outcomes observed in other nations. This particular study focuses on addressing the issue of understanding fractions, which has been identified as a significant hurdle in mathematics education. The aim was to demonstrate to South African educators how MERLO could be effectively utilised to improve students' grasp of fractions. Conducted in Grade 8 and 9 classrooms within Tshwane South Gauteng public schools, the research drew upon the Theory of Didactic Transposition to analyse the transformation of mathematical content into pedagogical practices. This paper is part of a broader study that adopted participatory action research and employed qualitative data collection methods. This study showcased the efficacy of integrating MERLO assessment items as pedagogical tools, enabling teachers to strategically plan and pose questions throughout their lessons on fractions. Moreover, the study highlighted how these assessment items were instrumental in gauging learners' depth of understanding of quantitative concepts, particularly fractions, thereby facilitating their learning process. However, it's important to note that the research faced challenges due to the impact of COVID-19, leading to a reduced number of participating teachers, with only five remaining by the study's conclusion. Future endeavours in this line of inquiry should aim to involve a larger cohort of teachers to ensure broader insights and more robust conclusions.

Keywords: *mathematics education, assessment, fractions*

1. Introduction

Mathematics education is important because it provides a foundation for understanding and solving problems in many sectors of life, including science, engineering, finance, and everyday decision-making. It also helps develop critical thinking and problem-solving skills, which are beneficial in any career or field of study. Many international large-scale assessment studies are being conducted globally, and regarding mathematics assessment, the TIMSS "Trends in International Mathematics and Science Study" is the most well-known and has been running since 1995 every four years. Low (under 400), middle (under 475), high (under 550), and advanced (under 625) criteria comprise what is referred to as the TIMSS benchmarks and learners that perform above the low benchmark are described as having acquired basic mathematical knowledge which is the case for only 41% of South African learners [1]. These results are based on the most recent TIMSS 2019 cycle and indicate that the situation of mathematics education in South Africa is dire and urgent interventions are needed to improve South African learners' mathematics achievement. MERLO "Meaning equivalence reusable learning objects", is a new form of pedagogical tool that teachers can use to engage students in rigorous mathematical reasoning, exploration, and discussion. MERLO has evolved and been verified, tested, and deployed in numerous countries and across a variety of curriculum areas and disciplines, but not in African countries prior to this study. Grade 8 and 9 South African mathematics teachers were introduced to the pedagogical tool MERLO as an assessment method to teach and promote the conceptual understanding of fractions. Our focus of this article is on the use of MERLO assessment items in the T&L of fractions at Grade 8 and 9 levels as a recent South African study conducted on fraction knowledge of South African Grade 9 learners concluded that "Analysis of the data revealed gaps in essential aspects of fraction knowledge that should have been dealt with at an earlier stage in these learners' education" [5].

2. Literature Review

Fractions are crucial for improving learners' mathematical reasoning; however, understanding fractions remains challenging [6]. Scholars proposed interventions to address these barriers [7]. The MERLO methodology, tested in multiple countries and disciplines, helps teachers assess deep understanding of quantitative concepts and aids learners in evaluating their own understanding in formative assessments [7 - 9].

Meaning equivalence (ME) is "a construct that denotes commonality of conceptual meaning across different representations within and across sign systems. The related construct of representational competence is the ability to recognize and to trans-code equivalence-of-meaning in multiple, multi-semiotic representations within and across sign systems … and to re-represent equivalent meaning by incorporating higher-order relations within and/or across different sign systems" [10, p. 315]. MERLO is a multi-dimensional database that allows for the sorting and mapping of key concepts via multi-semiotic representations in different sign systems, such as exemplary target statements of specific conceptual contexts and relevant other statements [11, 12]. Each MERLO database node is an item family with five statements: one target statement (TS) that represents a conceptual sequence and encodes various components of an essential idea; and four other statements that are sorted by two groups of criteria: Shared meaning equivalence (ME) with the TS; shared surface similarity (SS) with the TS. In the context of a mathematical concept, for instance, a component of MERLO items should include an element of a TS and other statements that share or do not share the same meaning in different kinds of representations (graph, table, symbolic code) with the TS. The pattern for designing MERLO items is embedded in a single TS. The quadrant statements (Q1, Q2, Q3, Q4), represented in Figure 1, are related to the TS that supports the specific item. They are categorised into two criteria: SS and ME, which are linked to the TS. Based on perspectives rather than facts or proof, we can say the statement "5 + 3" and "5 + 2" appear the same but do not share the same meaning, while the statement 3^2 and 3×3 share the meaning but do not share SS.

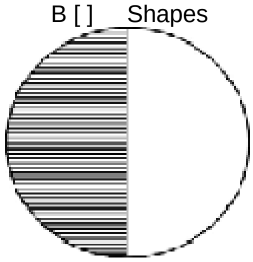
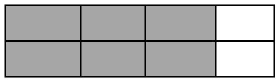
Fig 1. MERLO Item Framework

Q1 Meaning equivalence (yes) Surface similarity (yes)	**Q2** Meaning equivalence (yes) Surface similarity (no)
Q3 Meaning equivalence (no) Surface similarity (yes)	**Q4** Meaning equivalence (no) Surface similarity (no)

Note: Self-developed figure

An example of a MELO item is given in Figure 2. The MERLO approach is designed to allow learners to discuss and exchange ideas while considering a specific MERLO item; to share and contrast their views, to prompt and refresh each other's memory regarding important details of the conceptual context, and to compare notes on possible responses. The rationale for presenting the MERLO approach was that we highlight the usefulness of MERLO items as a teaching tool, both to address a pervasive didactical phenomenon - the duplication problem - and to assist learners in working with many semiotic representations and their mutual relationships of the same mathematical object [7, 13-14].

Fig 2. Example of MERLO Item on Fraction

Instructions	**TS**	**Q2**
1. Mark all statements with the same mathematical meaning 2. Write down the thoughts that guided your decisions	A [ ] Fraction $\frac{1}{2}$	B [ ] Shapes 
Q2	Q3	Q4
C [ ] Decimal 0.5	D [ ] Fraction $\frac{2}{5}$	E [ ] Shapes 

Note: Self-developed figure

3. Theoretical Framework

The TDT "Theory of Didactical Transposition" [15-17] were used to support this study. The TDT stresses the integration of MERLO pedagogy approaches as important in the mathematical learning situation. A carefully planned action is an object that supports the learner's solutions in the mathematical concept of fraction. The TDT, like planned MERLO assessment items, include phases when the teacher is active, and the learners listen. They also include moments of shared reflection (peer-assessment) and learners working on their own (self-assessment). In addition, TDT is essential to understand how a learner learns. The framework was used to describe the process of didactic transposition that consists of different steps on how teachers need to transform the MERLO knowledge and skills acquired into didactic knowledge; the knowledge to be taught with the research question "How, then, can mathematics teachers integrate the use of MERLO assessment items into the teaching of fractions"? MERLO items are the context for the didactic transposition of T&L of mathematics in the senior phase, i.e., the taught knowledge in the classroom context. The knowledge accessible for learners to learn is determined by their prior experiences, how the teacher designs and teaches the topic, and whether the classroom context is conducive enough to learn. A scholar describes mathematical praxeology as a mathematical and didactical development, as well as the outcome of the creation process. The required knowledge to solve a given task contains the theory, techniques and technology [15-17]. The practicality of interaction in the eighth and ninth grades has an impact on praxeology. The purpose of this study is to understand how knowledge to be taught is changed into knowledge taught in the classroom (learning to learn). On the other hand, how knowledge is taught in the mathematics context is supported by the components of praxeology model [15-17].

4. Research Methodology

Participatory action research was used for this study because it tries to combine action and reflection, theory and practice, and engagement with others on practical solutions to issues of interest to individuals and communities. public schools were selected conveniently in the Tshwane South in Gauteng, as these schools were close to the university where the researchers are based. The teachers were selected purposively since they had to be full-time permanent staff members, teaching mathematics at the eighth or ninth-grade level for at least two years and holding all the necessary qualifications for this role. At the commencement of the study, 12 teachers agreed to participate, but due to the outbreak of COVID-19, which was during the time the current study was conducted, many teachers pulled out from the study, leaving only five participants. Thus, a total of five teachers and 90 learners (38 from Grade 8 and 52 from Grade 9) were involved in the MERLO assessment items. The dynamic process of implementing the pedagogical tool MERLO in the context of teaching and mathematical praxeology (i.e., classroom observation) was supposed to be conducted in a three-cycle process, but due to Covid-19, only a two-cycle process was conducted. Although both eight- and ninth-grade classes were observed, in this manuscript, for conciseness, we opted only to describe one class in detail, and we opted for a Grade 9 class, because of South Africa's overall low performance in previous rounds of TIMSS [1].

The teacher combined MERLO learning, question and answer sessions, class discussions, self-assessment, feedback and assessment tasks to involve learners in the learning process. During the presentation of MERLO methods, the entire class debates the answer and the reasons for choosing (or not choosing) each of the five representations in the MERLO based on the shared equivalence of meaning criterion. This teaching technique differs greatly from the typical classroom scenario in which learners are given class activities and asked to do them independently or the teacher simply shows the learners how it is done on the board.

For data analysis, praxeology was used to understand the knowledge about fractions and how the knowledge taught was planned and implemented. A praxeology is made up of a task and a technique, or "know-how," for solving the work. It also incorporates "knowledge" of technology and theory [15-17]. To be concise, all data from the five participating teachers were gathered, but only one learning situation/occurrence from one of the classrooms was chosen and analysed using TDT. Learners' worksheets, exit tickets, teachers' reflections and classroom observations were the main data in the analysis. The classes were audio-taped, and the recordings were transcribed. The teachers' aim and objectives of the activity would promote learners' level of understanding and conceptual thinking of fractions in mathematics classrooms. During the classroom activities, learners had to: (i) identify the equivalent form of common fractions, decimal fractions, shapes, and percentages in the MERLO patterns that share the equivalence of meaning with the TS, (ii) write out their reasons that guided the choice of answer, (iii) participate in MERLO problem-solving examples, (iv) know how to identify and relate the TS with Q2A, Q2B, Q3C, Q4D, (v) have a good knowledge of how to solve MERLO activities. The intended learning outcome is for learners to identify one or two statements of MERLO items on fraction that share the same meaning equivalence with the TS.

5. Findings

As mentioned earlier, each teacher was observed twice. The goal of the first cycle observed lesson was for teachers to gain a thorough understanding of how to use the pedagogical tool MERLO in their classrooms, including assessing learners' prior knowledge of the concept fraction, asking open-ended questions about MERLO, time management and control, assessing learners' understanding of fraction, classroom involvement and providing meaningful feedback. In addition, the first cycle lesson also aimed at observing how learners reacted to the new teaching methodology (i.e., their body language, performances, and active participation), which could elicit the need for teaching planning modification for the next class. All teachers were given feedback on areas where they needed to improve, such as how they could manage their time while teaching. The first cycle lesson also assisted the learners in becoming familiar with MERLO. This process allowed the teachers to spend less time in the second cycle lesson explaining the MERLO items to the learners, which resulted in learners practicing self- and peer-assessment in the mathematics classroom. The second cycle lesson was observed, a follow-up after the classroom observation in terms of teachers reflecting writing on their experiences and challenges were collected, as well as learners' class activities and exit tickets. The purpose of all the data collection mentioned above was to explore whether there was any improvement in the implementation of the pedagogical tool MERLO in class.

The findings of teachers' observation, learners' worksheets and exit tickets revealed that the implementation of MERLO pedagogy is very different from the usual classroom scenario where learners are given an exercise and are asked to solve the problem individually. The MERLO assessment is designed to allow learners to engage in discussion and exchange ideas related to a specific MERLO item. This process includes sharing and contrasting different points of view, prompting and refreshing each other's memory regarding important details of the conceptual situation, and comparing learners' notes about their practicable responses. It seemed that even though the integration of MERLO pedagogy was successful, both the teachers and learners experienced some

challenges in the context of teaching and mathematical praxeology. This being said, the challenges were progressively addressed. One of the teachers mentioned that:
"Hum, following a proper step, it was a bit difficult because sometimes you know, the order can easier be forgotten. So, there is a specific order to follow when doing the presentations, but I used my workshop guide to remind myself" (TeacherB).

In the mathematics classroom, the didactic transposition from knowledge to be taught to knowledge is from the ideas about so-called rational numbers, which represents a quotient and, more specifically, knowledge about any number of equal parts to knowledge about a fraction that represents a part of a whole [5, 18]. In the analysis of the mathematical classroom through the teaching and mathematical praxeology, the knowledge to be taught is "fraction" and teachers have carefully planned and prepared how they can direct their learners' responsiveness to specific elements and how to identify the content using MERLO items. According to [19], responsive teachers ask questions that elicit learner emergent ideas that encourage learners to provide alternative explanations during classroom instruction. The teachers planned with a question they could ask and how MERLO assessment items could be used to support the learners in discovering the components of fractions. A praxeology consists of tasks, techniques, technologies and theories [15-17, 20]. The knowledge taught is designed in different equivalence forms of objects and their specific features, and the aim of the task is to encourage the learners to identify and reflect on the different representations of the objects. The technique is based on how the learners could identify ME and SS criteria in the mathematics classroom. The observed learners could identify and reflect on the criteria of ME and SS in the mathematics classroom. [21] indicates that learners' reflecting on the learning make self-judgments in order to determine how well their general and specific goals were met. Learners adopted the same strategy as instructed by the teachers. The theory and technology explain and justify the technique employed to solve the actual problem. The design of MERLO assessment items and the context of teaching and mathematical praxeology, as well as MERLO materials, was based on the teacher's notion of how learners understand the concept of fractions and how learners were able to give reasons that guided their decisions. The design, ideas and assumptions make up the praxeology form of the theory.

6. Discussion and Conclusions

As learners' fractional knowledge and understanding have been identified as a problem in South Africa, the current study necessitates a change from a traditional type of question (posed as an open question) to a learner-centered learning environment. The mathematics teachers design MERLO items as a form of an open-ended question and use the teaching context to direct a learner's attention to identify the criteria of ME and SS of fractions in the lessons (i.e., mathematical praxeology). The findings revealed that the implementation of MERLO pedagogy is very different from the usual classroom scenario where learners are given an exercise and are asked to solve the problem individually. The MERLO assessment is designed to allow learners to engage in discussion and exchange ideas related to a specific MERLO item. This process includes sharing and contrasting different points of view, prompting and refreshing each other's memory regarding important details of the conceptual situation, and comparing learners' notes about their practicable responses (shared reflection, peer-assessment). The implementation of MERLO has both pedagogical and assessment implications. From a pedagogical perspective, it places emphasis on teaching and discussion of conceptual elements. From an assessment perspective, it enables the evaluation of the level of conceptual understanding achieved by the learners. This approach to assessment is formative in nature, as opposed to traditional summative assessment approaches that prioritise memorization skills needed to pass tests [22]. Based on MERLO pedagogy, memorisation skills are considered secondary, with a greater emphasis placed on the skills required for conceptual thinking. This shift in focus is particularly beneficial in the T&L of fractions. The finding revealed that implementing MERLO items in the context of T&L fraction brings about a transformation in the analysis of concept equivalence, both cognitively and semiotically, which helps to overcome the obstacle of duplication. The process of transformation and conversion between various representations of an object in fraction is at the heart of mathematical understanding, emphasing the importance of improving learners' abilities in relation to such semiotic transformations. The findings further suggested that the use of MERLO pedagogy seems to be substantial if teachers' knowledge of the content and knowledge taught could be carefully practised often, it would promote learners to solve problems independently continually. Regarding future research, due to the fact that COVID-19 prompted several teachers to withdraw from the study, leaving only five teachers in the end, future research should involve more teachers in similar studies.

7. Ethical Considerations

Approval to conduct this study was obtained from the University of Pretoria (SM 19/05/01) and from the Gauteng Department of Education (8/4/4/1/2). All participants completed consent forms saying their participation was entirely voluntary, and pseudonyms were utilised. The participants' anonymity and confidentiality were protected by not releasing any identifying information about them.

8. Declarations

8.1 Competing Interests
There are no competing interests.

8.2 Funding
This research was funded by the National Research Foundation (NRF) [Ref.: CSRP190415430728, Grant number: 120401].

REFERENCES

[1] Reddy, V., Winnaar, L., Juan, A., Arends, F., Harvey, J., Hanna, S., Namome, C., Sekhejane, P. & Zulu, N. "TIMSS 2019. Highlights of South African Grade 9 results in mathematics and science", 2021, Human Sciences Research Council.

[2] Adesanya, L.O., & Graham, M.A. "Effective communication of learning intentions and success criteria in the mathematics classroom: MERLO pedagogy for senior phase South African schools", Pythagoras, 43(1), 2022, 1-13.

[3] Adesanya, L.O., & Graham, M.A. "Promoting formative assessment practices in senior phase mathematics classrooms using meaning equivalence reusable learning objects", South African Journal of Education, 43(3), 2023, 1-21.

[4] Graham, M.A., & Adesanya, L.O. "Teacher professional development for the teaching of mathematics: MERLO items design in the South African schools", Conference Proceedings: 13th International Conference the Future of Education, 2023, 106-110. Filodiritto Editore.

[5] Moyo, M., & Machaba, F.M. "Grade 9 learners' understanding of fraction concepts: Equality of fractions, numerator and denominator", Pythagoras, 2021, 42(1), 1-13.

[6] Tzur, R. "On natural numbers and fractions: A reorganization (no bias) stance", In Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education (PME), 2016, 3-7.

[7] Arzarello, F., Kenett, R. S., Robutti, O., Shafrir, U., Prodromou, T., & Carante, P. "Teaching and assessing with new methodological tools (MERLO): A new pedagogy?", In IMA International Conference on Barriers and Enablers to Learning Maths: Enhancing Learning and Teaching for All Learners. Institute of Mathematics & its Applications, 2015.

[8] Prodromou, T. "Teaching and assessing with new methodological tools (MERLO)", In 2015 MANSW Annual Conference Presentation Summaries. Mathematical Association of New South Wales, 2015.

[9] Robutti, O., Carante, P., Prodromou, T., & Kenett, R.S "Teachers involved in designing MERLO items", In M. Etkind & U. Shafrir (Eds.), Pedagogy for conceptual thinking and meaning equivalence: Emerging research and opportunities. IGI Global, 2020, 61-85.

[10] Etkind, M., Kenett, R.S., & Shafrir, U. “Learning in the digital age with meaning equivalence reusable learning objects (MERLO)”, In Handbook of research on applied learning theory and design in modern education. IGI Global, 2016a, 310-333.

[11] Etkind, M., Shafrir, U., Kenett, R., & Roytman, L. “Pedagogy for conceptual thinking in the digital age: Enhancing learning outcomes with meaning equivalence reusable learning objects (MERLO) formative assessments”, In 2nd. International Conference on Higher Education Advances (HEAD’16). Editorial Universitat Politècnica de València, 2016, 105-133.

[12] Etkind, M., & Shafrir, U. “Teaching and learning in the digital age with pedagogy for conceptual thinking and peer cooperation”, Proceedings of the 7th International Technology, Education and Development Conference, INTED, 2013, 5342-5352.

[13] Robutti, O., Arzarello, F., Carante, P., Kenett, R., Prodromou, T., & Shafrir, U. “Meaning equivalence: A methodological tool for assessing deep understanding”, In L. G. Chova., A. L. Martínez & I. C. Torres (Eds.), Proceedings of the INTED-2016 Conference. International Association of Technology, Education and Development, 2016, 7358–7367.

[14] Robutti, O., Prodromou, T., & Aldon, G. “Teachers’ involvement in designing MERLO items: Boundary crossing”, Digital Experiences in Mathematics Education, 2021, 7, 276-300.

[15] Chevallard, Y. (1992). “Fundamental concepts in didactics: Perspectives provided by an anthropological approach”, Research in didactique of mathematics: Selected papers. La Pensée sauvage, Grenoble, France, 1992, 131–167.

[16] Chevallard, Y. “Didactique? Is it a plaisanterie? You must be joking! A critical comment on terminology”, Instructional Science, 1999, 27(1/2), 5-7.

[17] Chevallard, Y. “Introducing the anthropological theory of the didactic: An attempt at a principled approach”, Hiroshima Journal of Mathematics Education, 2019, 12, 71-114.

[18] Kärki, T., McMullen, J., & Lehtinen, E. “Improving rational number knowledge using the NanoRoboMath digital game”, Educational Studies in Mathematics, 2022, 110(1), 101-123.

[19] Lindstrom, D., & Selmer, S. “Responsive teaching and the instructional reasoning of expert elementary mathematics teachers”, Education Sciences, 2022, 12(5), 1-16.

[20] Bosch, M., & Gascón, J. “Twenty-five years of the didactic transposition”, ICMI Bulletin, 2006, 58(58), 51-65.

[21] Barana, A., Boetti, G., & Marchisio, M. “Self-assessment in the development of mathematical problem-solving skills”, Education Sciences, 2022, 12(2), 1-27.

[22] Kenett, R.S. (2021). “Conceptual thinking in statistics and data science education: Interactive formative assessment with Meaning Equivalence Reusable Learning Objects (MERLO)”, Available at SSRN 3862006.