Prospective teachers' knowledge of multiplication

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Task 5
In a forest, there are 4 birdhouses. There are 4 birds living in each birdhouse. How many birds live in the birdhouses?

Background for the study

The **four basic arithmetic operations** are fundamental to further learning in mathematics. It is therefore crucial that students acquire solid knowledge and skills in these operations during primary school. In the Norwegian school system, **multiplication** is introduced in **Grade 3** (ages 7–8) ([1]).

Teachers' competence is important in the challenging and complex process of teaching mathematics. More specifically, **specialized content knowledge** in mathematics is essential for teachers to teach mathematics effectively ([9]). Teachers who have **developed advanced** levels of specialized content knowledge are better equipped to understand students' thinking and to interpret the solutions students produce ([2]).

Several studies have examined how **prospective teachers** apply **multi-digit multiplication algorithms** ([3]) and have concluded that they experience difficulties in understanding these algorithms.

However, there is limited research on how prospective teachers solve multiplication problems that **do not require the use of algorithms**. Therefore, in the present study,

we investigate how 46 prospective teachers enrolled in the primary teacher education program (Grades 1–7) solve multiplication tasks that do not require the use of a multiplication algorithm.

Metodology

In this study, **46 prospective teachers** enrolled in the primary teacher education program (Grades 1–7), during their **first semester** of teacher education, solved **a set of tasks** that included four multiplication problems.

We analyzed the solutions to the multiplication tasks in order **to identify the strategies used** by the prospective teachers.

The strategies were identified by **a descriptive analysis** inspired by **a qualitative thematic analysis**, as described by Braun and Clarke (2013) ([5]).

The four tasks with a multiplicative structure, which constitute the data material of the study, are presented in Table 1.

Task	Task description	Mathematical content
5	In a forest, there are 4 birdhouses. There are 4 birds living in each birdhouse. How many birds live in the birdhouses?	Multiplication The multiplier and the multiplicand are known
6	Two gingerbread cookies cost 8 kroner. You buy 4 gingerbread cookies. How much do you have to pay?	Proportionality The mulitple-batchs perspective
7	Two caterpillars eat 5 leaves per day. How many leaves are needed to feed 12 caterpillars for one day?	Proportionality The mulitple-batchs perspective
8	There are 6 pieces in a pack of chewing gum. How many pieces are there in 3 packs of chewing gum?	Multiplication The multiplier and the multiplicand are known

Table 1. The four tasks included in the study.

Theoretical Framework

Our **theoretical framework** is based on the two situations for multiplication identified by Greer (1992) ([14]) that are present in this study:

- **repeated addition** with equal groups, and
- **multiplicative change of quantities** (proportionality).

Multiplication are defined by

$$a \cdot b = p$$

$$multipler \cdot multiplicand = product$$

the factor $\boldsymbol{a}$ represents the number of groups (the multiplier) and the factor $\boldsymbol{b}$ represents the number of objects in each group (the multiplicand).

Hickendorff et al. (2019) ([6]) identify several solution strategies for multi-digit multiplication, such as

> **sequential strategies, decomposition, variation strategies, column-based methods, and standard algorithms for multiplication**.

Proportionality

Proportionality refers to a multiplicative relationship between quantities represented in a situation and can be expressed algebraically as $y = m \cdot x$ ([7]).

One dog two eyes, two dogs four eyes, ...

Cramer and Post (1993) ([7]) describe four different solution strategies for proportionality problems, two of which are included in our theoretical framework:

- **The unit-rate strategy** involves determining the value of one unit and then scaling it up to find the required quantity.
- **The factor-of-change strategy** involves solving the problem through multiplication; for example: *"It takes 20 minutes to drive four miles. Since Mark is driving three times as far, it should take three times as long. Therefore, the answer is 20 minutes times three, or 60 minutes"* ([7]).

Mathematical knowledge can be divided into two types: ***procedural* and *conceptual knowledge*** ([18]).

Procedural knowledge of multiplication involves understanding multiplication as repeated addition, for example interpreting $4 \cdot 5$ as taking the number 5 four times (5 + 5 + 5 + 5) ([19]).

Conceptual knowledge of multiplication, by contrast, involves:

1) understanding the **concepts and mathematical content** associated with multiplication;
2) the ability to solve multiplication problems **using a variety of strategies and representations**; and
3) the ability **to retrieve basic multiplication facts** directly from memory ([20]).

Analysis and results

The tasks solved by the students in our study fall between what the literature refers to **as single-digit arithmetic and multi-digit arithmetic** ([6]). Consequently, we developed our own analytical tool, inspired by the classification system proposed by **Hickendorff et al. (2019)** ([6]) for multi-digit multiplication and by the solution strategies for proportionality tasks described by **Cramer and Post (1993)** ([7]).

The analysis in this study is a **descriptive analysis** ([32]). In our case, the students' solutions to the four multiplication tasks constitute **the units of analysis**. We reviewed the students' solutions and searched for patterns in the data in order to generate categories ([32]). The categories identified through this process were: **multiplicative, additive, proportionality, unit-rate ("the way through 1"), other, and a category for incorrect answers.**

Category	Content
Multiplicative strategy	In this category, the students arrived at the answer by multiplying the numbers, that is, they used multiplicative reasoning. We did not distinguish between the two cases 3 × 6 = 18 and 6 × 3 = 18 (example from Task 8), nor between the multiple-batches perspective and the variable-parts perspective as described by Beckmann et al. (2015) ([17]).
Additive strategy	In this category, the students add the multiplicand the number of times indicated by the multiplier; that is, they write 4 + 4 + 4 + 4 = 16 instead of 4 × 4 = 16 (example from Task 5).
The factor of change strategy	Here, the students solved the tasks by viewing the situation as two quantities increasing simultaneously; for example, in Task 6, that when the number of gingerbread cookies is doubled, the price is also doubled.
The unit-rate strategy	This is illustrated, for example, in Task 7, by first determining how many leaves one caterpillar eats (5 ÷ 2 = 2.5) and then multiplying this by the number of caterpillars (2.5 × 12 = 30).
Other	This category includes alternative solution methods that are not mentioned above, for example solutions based on illustrations (drawings). In this category, we also included responses that did not contain any calculations.
Incorrect answer	We chose to place responses that resulted in incorrect answers in a separate category. This category also includes incomplete responses, for example those in which a calculation was set up but not carried out.

Table 2. Description of the categories used in the analysis.

The analys gave us following results

	Multiplicative strategy	Additive strategy	The factor of change strategy	The unit-rate strategy	Other	Incorrect answer
Task 5	84,8	2,2	0	0	8,7	4,4
Task 6	0	19,6	17,4	30,4	13,0	19,6
Task 7	0	4,4	28,9	26,7	4,4	35,6
Task 8	88,9	2,2	0	0	6,7	2,2

Table 3. Percentage distribution across the different categories for each task.

Task	Task description	Mathematical content
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6	Two gingerbread cookies cost 8 kroner. You buy 4 gingerbread cookies. How much do you have to pay?	Proportionality The mulitple-batchs perspective
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Discussion and Didactic implications

In Task 5 and 8

- **multiplicative strategy** dominates, with over 80 % of the prospective teachers using this approach ([7]).
- An **additive solution strategy** in multiplication involves solving tasks through repeated addition, reflecting procedural knowledge of multiplication. In Tasks 5 and 8, 2,2% of the prospective teachers used repeated addition, while in Tasks 6 and 7 the corresponding percentages were 19,6% and 4,4%, respectively. This suggests that these prospective teachers rely on **a primitive model of multiplication** ([4]).

In Tasks 6 and 7,

- 30,4% and 26,7% of the prospective teachers, respectively, **used the unit-rate strategy**, which is the same strategy that Modestou and Gagatsis (2009) ([31]) found to be commonly used by students at the primary school level.

Based on the competence aims in LK20 ([1]), it is reasonable to assume that the prospective teachers have been taught other methods; nevertheless, **they continue to use the same strategy**, a finding consistent with the study by Torma and Kosztolányi (2025) ([30]).

This suggests that strategy choice **is not grounded in an understanding** of proportionality, a pattern also noted by Avcu and Avcu (2010) ([29]) regarding the use of the cross-product algorithm. In such cases, **students rely on a learned method without a meaningful reference to the context of the problem** (cf. [7]).

In our study, this implies that the prospective teachers **apply a learned method** even when it is not the most efficient.

Our findings indicate that the prospective teachers primarily possess a **procedural knowledge of multiplication and proportionality.** This suggests that teacher education must be changed to place greater emphasis **on the development of conceptual knowledge** in of multiplication and proportionality.

One approach is to problematize **the commutative property of multiplication** ($a \cdot b = b \cdot a$) by demonstrating how the factors assume different roles when situated in context (cf. [15]; [17]). In addition, instruction for prospective teachers should foreground concepts and mathematical content in multiplication, promote the use of varied strategies and representations, and support the development of a broader foundation of multiplication facts ([20]).

With regard to **proportionality**, instruction should likewise foreground conceptual understanding. One possible approach is to use examples that demonstrate that although the unit-rate strategy can lead to correct answers, a strong understanding of proportionality supports the selection of more efficient strategies that preserve relationships.

This can be seen as analogous to **solving multiplication problems through repeated addition**. To foster this understanding, instruction should provide opportunities for prospective teachers to engage deeply with proportional functions and the role of the proportionality constant. In this way, teacher education can support the development of prospective teachers' conceptual knowledge of proportionality.

Thank you!

Question?

REFERENCES

[1] UTDANNINGSDIREKTORATET (2020). LÆREPLAN I MATEMATIKK 1.-10. TRINN (MAT01-05). FASTSATT SOM FORSKRIFT. LÆREPLANVERKET FOR KUNNSKAPSLØFTET 2020. HTTPS://WWW.UDIR.NO/LK20/MAT01-05?LANG=NOB

[2] BALL, D.L., THAMES, M.H. & PHELPS, G. (2008). CONTENT KNOWLEDGE FOR TEACHING. WHAT MAKES IT SPECIAL? JOURNAL OF TEACHER EDUCATION, 59 (5), 389-407. HTTPS://DOI.ORG/10.1177/0022487108324554

[3] APSARI, R. U., WULANDARI, N. P., NOVITASARI, D. (2021). PROSPECTIVE MATHEMATICS TEACHERS MAKING SENSE OF MULTIPLICATION ALGORITHM. *JOURNAL OF PHYSICS: CONFERENCE SERIES, 1778,* 1-9. HTTPS://DOI.ORG/10.1088/1742-6596/1778/1/012023

[4] FISCHBEIN, E., DERI, M., NELLO, M. S., & MARINO, M. S. (1985). THE ROLE OF IMPLICIT MODELS IN SOLVING VERBAL PROBLEMS IN MULTIPLICATION AND DIVISION. JOURNAL FOR RESEARCH IN MATHEMATICS EDUCATION, 16, 3-17. HTTPS://DOI.ORG/10.2307/748969

[5] BRAUN, V., & CLARKE, V. (2013). SUCCESSFUL QUALITATIVE RESEARCH: A PRACTICAL GUIDE FOR BEGINNERS. SAGE.

[6] HICKENDORFF, M., TORBEYNS J. & VERSCHAFFEL L. (2019), MULTI-DIGIT ADDITION, SUBTRACTION,MULTIPLICATION, AND DIVISION STRATEGIES. IN: FRITZ A., HAASE V.G., RÄSÄNEN P. (EDS.)INTERNATIONAL HANDBOOK OF MATHEMATICAL LEARNING DIFFICULTIES. SWITZERLAND: SPRINGER. 543-560. HTTPS://DOI.ORG/10.1007/978-3-319-97148-3_32

[7] CRAMER, K. & POST, T. (1993). PROPORTINAL REASONING. THE MATHEMATICS TEACHER. HTTPS://WWW.JSTOR.ORG/STABLE/27968390?SEQ=1

[8] LARSSON, K. (2016). STUDENTS´UNDERSTANDINGS OF MULTIPLICATION. [DOKTORGRADSAVHANDLING, STOCKHOLM UNIVERSITY]. DIVA PORTAL. HTTPS://WWW.DIVAPORTAL.ORG/SMASH/RECORD.JSF?PID=DIVA2%3A1038458&DSWID=-1497

[9] BALL, D. L. (1990). PROSPECTIVE ELEMENTARY AND SECONDARY TEACHERS' UNDERSTANDING OF DIVISION. JOURNAL FOR RESEARCH IN MATHEMATICS EDUCATION, 21(2), 132-144. HTTPS://WWW.JSTOR.ORG/STABLE/749140?SEQ=1

[10] ÖZEL, Z., YILMAZ, A., IŞIKSAL-BOSTAN, M., & ÖZKAN, B. (2022). INVESTIGATION OF THE SPESICALIZED CONTENT KNOWLEDGE OF THE PRE-SERVICE PRIMARY SCHOOL TEACHERS ABOUT MULTIPLICATION. INTERNATIONAL JOURNAL FOR MATHEMATICS TEACHING AND LEARNING, 23(2), 115-143. HTTPS://DOI.ORG/10.4256/IJMTL.V23I2.464

[11] SHERIN, B. & FUSON, K. (2005). MULTIPLICATION STRATEGIES AND THE APPROPRIATION OF COMPUTATIONAL RESOURSCES. JOURNAL FOR RESEARCH IN MATHEMATICS EDUCATION, 2005, VOL. 36, NO. 4, 347-395. HTTPS://WWW.JSTOR.ORG/STABLE/30035044?SEQ=1

[12] GIBERTI, C. & MAFFIA, A. (2022). PRIMITIVE MODEL OF PARTITIVE DIVISION: A REPLICATION OF THE FISCHBEIN ET AL. STUDY. IMPLEMENTATION AND REPLICATION STUDIES IN MATHEMATICS EDUCATION, 2(2), 149-173. HTTPS://DOI.ORG/10.1163/26670127-BJA10007

[13] PARK, J.-H. & NUNES, T. (2001). THE DEVELOPMENT OF THE CONCEPT OF MULTIPLICATION. COGNITIVE DEVELOPMENT 16(3), 763-773. HTTPS://DOI.ORG/10.1016/S0885-2014(01)00058-2

[14] GREER, B. (1992). MULTIPLICATION AND DIVISION AS MODELS OF SITUATIONS. IN D. A. GROUWS (ED.), HANDBOOK OF RESEARCH ON MATHEMATICS TEACHING AND LEARNING (PP. 276-295). NEW YORK MACMILLAN PUBLISHING COMPANY. - REFERENCES - SCIENTIFIC RESEARCH PUBLISHING

[15] LAMPERT, M. (1986) .KNOWING, DOING, AND TEACHING MULTIPLICATION. COGNITION AND INSTRUCTION, 1986, VOL. 3, NO. 4, PP. 305-342. HTTPS://WWW.JSTOR.ORG/STABLE/PDF/3233589.PDF

[16] MA, L. & KESSEL, C (2018). THE THEORY OF SCHOOL ARITHMETIC: WHOLE NUMBER. IN M.G. BARTOLINI BUSSI, X.H. SUN (EDS.), BUILDING THE FOUNDATION: WHOLE NUMBERS IN THE PRIMARY GRADES, NEW ICMI STUDY SERIES. HTTPS://DOI.ORG/10.1007/978-3-319-63555-2_18

[17] BECKMANN, S. & IZSÁK, A. (2015). TWO PERSPECTIVES ON PROPORTIONAL RELATIONSHIPS: EXTENDING COMPLEMENTARY ORIGINS OF MULTIPLICATION IN TERMS OF QUANTITIES. JOURNAL FOR RESEARCH IN MATHEMATICS EDUCATION 2015, VOL. 46, NO. 1, 17–38. HTTPS://WWW.JSTOR.ORG/STABLE/10.5951/JRESEMATHEDUC.46.1.0017

[18] RITTEL-JOHNSON, B. & ALIBALI, M. W. (1999). CONCEPTUAL AND PROCEDURAL KNOWLEDGE OF MATHEMATICS: DOES ONE LEAD TO THE OTHER? JOURNAL OF EDUCATIONAL PSYCHOLOGY 1999, VOL. 91, NO. 1,175-189. HTTPS://PSYCNET.APA.ORG/FULLTEXT/1999-10190-015.PDF

[19] HAYLOCK,D.(2010). MATHEMATICS EXPLAINED FOR PRIMARY TEACHERS (4TH ED.).L ONDON, UNITEDKINGDOM:SAGE.

[20] KLING, G. & BAY-WILLIAMS, J.M. (2015). THREE STEPS TO MASTERING MULTIPLICATION FACTS. TEACHING CHILDREN MATHEMATICS , VOL. 21, NO. 9, PP. 548-559. HTTPS://WWW.JSTOR.ORG/STABLE/10.5951/TEACCHILMATH.21.9.0548?SEQ=1

[21] MILLS, J. (2019). MAKING MULTIPLICATION MEANINGFUL: TEACHING FOR CONCEPTUAL UNDERSTANDING. TEACHERS AND CURRICULUM, 19(1), 17-25. HTTP://DX.DOI.ORG/10.15663/TANDC.V19I1.334

[22] MA, L. (2010). KNOWING AND TEACHING ELEMENTARY MATHEMATICS: TEACHERS' UNDERSTANDING OF FUNDAMENTAL MATHEMATICS IN CHINA AND THE UNITED STATES (ANNIVERSARY EDITION). NEW YORK, NY: ROUTLEDGE.

[23] KAZEMI, E., & STIPEK, D. (2001). PROMOTING CONCEPTUAL THINKING IN FOUR UPPER-ELEMENTARY MATHEMATICS CLASSROOMS. THE ELEMENTARY SCHOOL JOURNAL, 102(1) 59-80. HTTPS://JOURNALS.SAGEPUB.COM/DOI/PDF/10.1177/002205740918900l-209

[24] BAO, L. (2023). IDENTIFYING AND ASSESSING STUDENTS' TRANSITION BARRIERS BETWEEN ADDITIVE AND MULTIPLICATIVE THINKING. IN B. REID-O'CONNOR, E. PRIETO-RODRIGUEZ, K. HOLMES, & A. HUGHES (EDS.), WEAVING MATHEMATICS EDUCATION RESEARCH FROM ALL PERSPECTIVES. PROCEEDINGS OF THE 45TH ANNUAL CONFERENCE OF THE MATHEMATICS EDUCATION RESEARCH GROUP OF AUSTRALASIA (PP. 91–98). NEWCASTLE: MERGA. HTTPS://FILES.ERIC.ED.GOV/FULLTEXT/ED631601.PDF

[25] BRICKWEDDE, J. (2024). MULTIPLICATION AND DIVISION STRATEGIES. AN OVERVIEW OF THE PROGRESSION OF CHILDREN'S STRATEGIES IN MULTIPLICATION AND DIVISION. HTTPS://DOCS.GOOGLE.COM/DOCUMENT/D/163J9VAVO7NUZWZZUCI-AAAIDYKYHLWXJ/EDIT