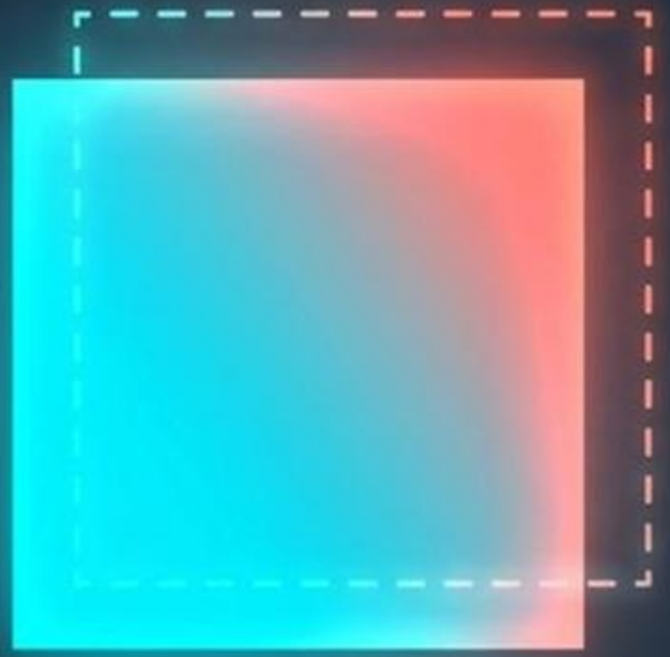

Illuminating the Unknown: An Innovative Model for Mastering Equations in Primary Mathematics

A Longitudinal Pedagogical Study (2022–2025)

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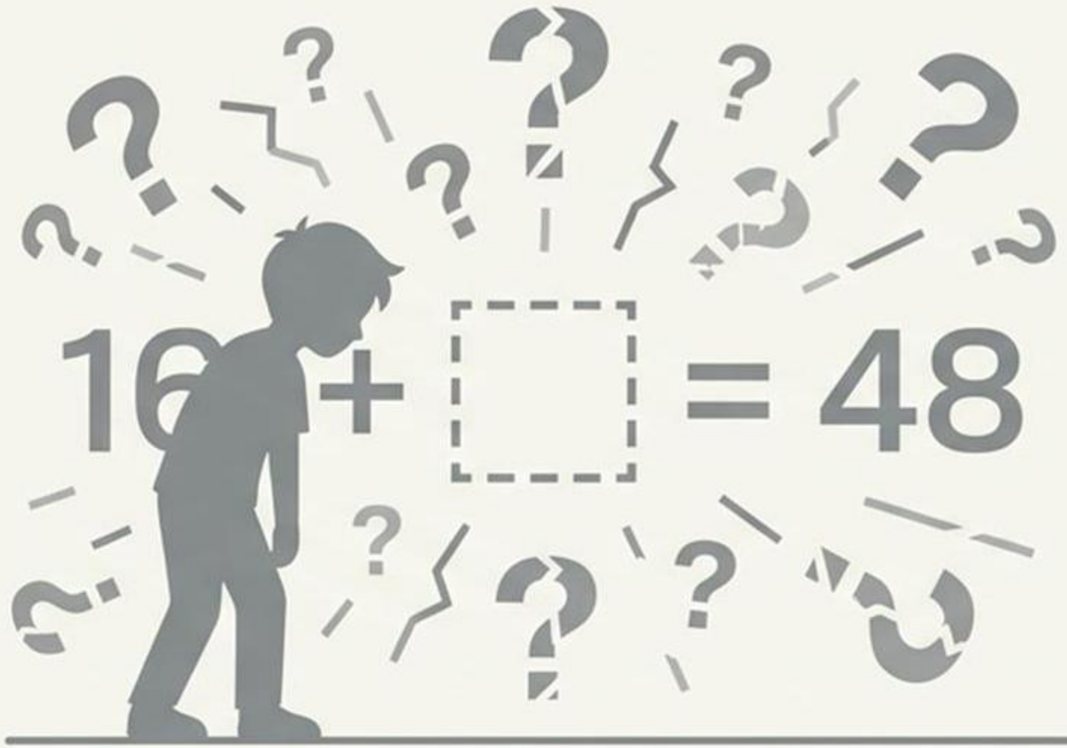
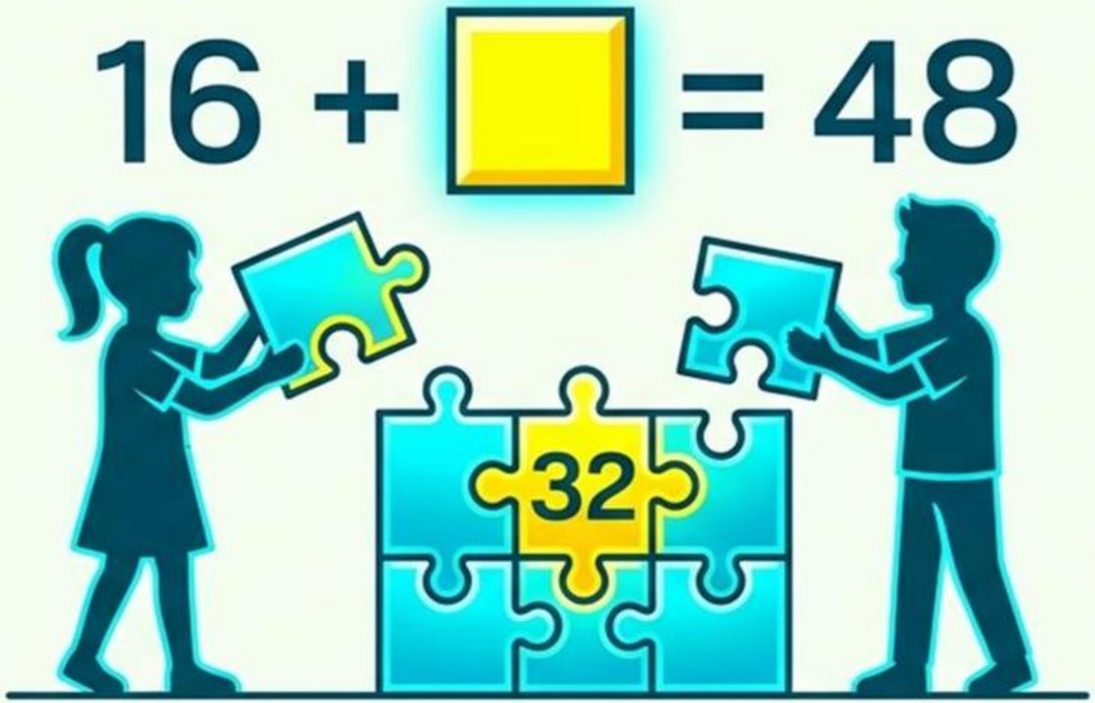
The Fragility of “Guess and Check”



Currently, 8-10-year-olds often find unknown components in equations by intuitively ‘guessing’ the number and checking if it works.



This intuitive method is fragile and breaks down as multi-digit complexity scales. True mastery requires understanding the relationships between arithmetic operations, not just memorizing facts.

Acknowledgements: This study is financed by the European Union-NextGenerationEU, through the National Recovery and Resilience Plan of the Republic of Bulgaria, project № BG-RRP-2.004-0001-C01.

Moving from Intuition to Mathematical Logic

	Traditional Approach	Innovative Teaching Model
The Method	Guess & Check and rote memorization.	**Visual and contextual triggers** that naturally reveal mathematical rules.
The Focus	Isolated, disconnected equations.	**Relational deduction** (e.g., understanding how addition proves subtraction).
The Application	Abstract number-crunching.	**Integrated word problems** that give mathematical models real-world context.

Summary: The innovative model provides a **structured, multi-year scaffolding** that replaces guesswork with rule-based deduction.

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A 3-Year Scaffold of Cognitive Complexity

The model builds upon itself like architectural **scaffolding**, scaling progressively for students **aged 8 to 10**. Each year leverages the foundational rules mastered in the last.

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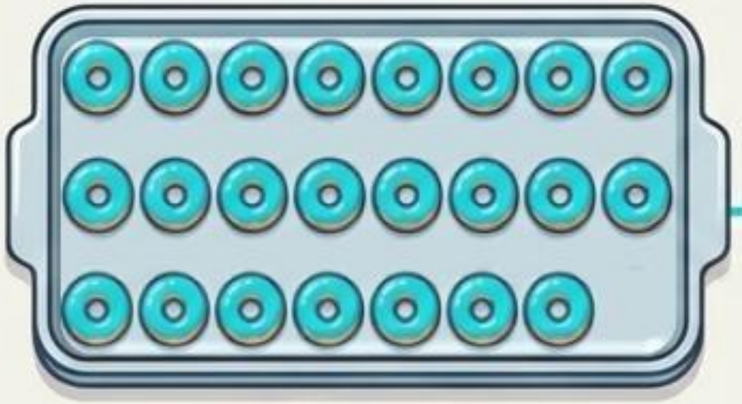
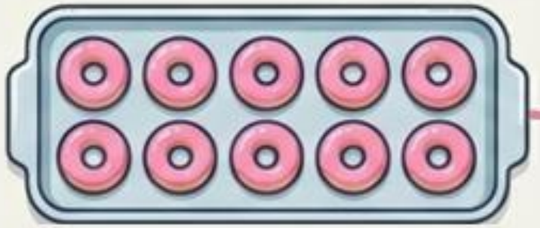
Grade 2: Anchoring Rules in Visual Context



Blue donuts



Pink donuts

23 + 10 = 33

Total

33 - 10 = 23

Pink

33 - 23 = 10

Blue

16 + □ = 48

48 - 16 = 32

The model starts with tangible word problems. Instead of forcing memorization, students naturally derive the core rule through visual observation.

The Rule: The unknown addend is found by subtracting the known addend from the sum.

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Grade 2: Anchoring Rules in Visual Context

$\square \times 4 = 20$

$20 \div 4 = 5$

Check: 5 x 4 = 20

The Rule: The unknown multiplier is found by dividing the product by the known multiplier.

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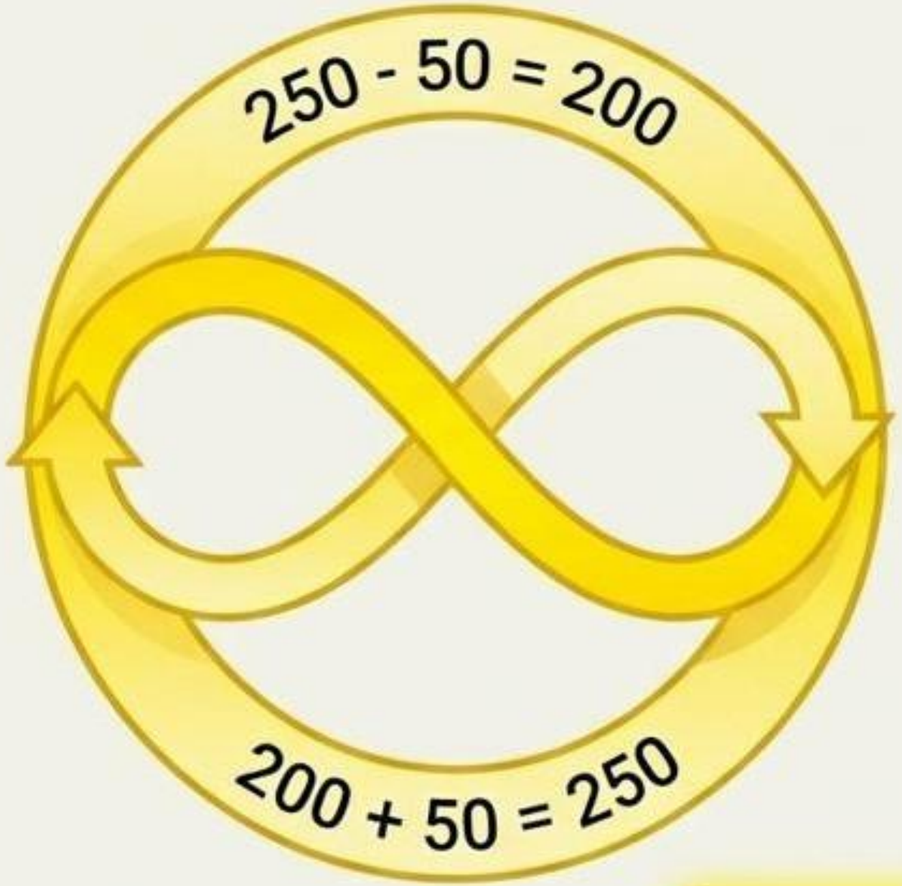
Grade 3: Leveraging Inverse Relationships

As students progress to minuends and dividends, they leverage their existing knowledge of addition and multiplication.

They use inverse operations to verify their work, solidifying the interconnectedness of mathematics.



□ − 30 = 100

↓

100 + 30 = 130

The Rule: The unknown minuend is found by adding the difference and the subtrahend.

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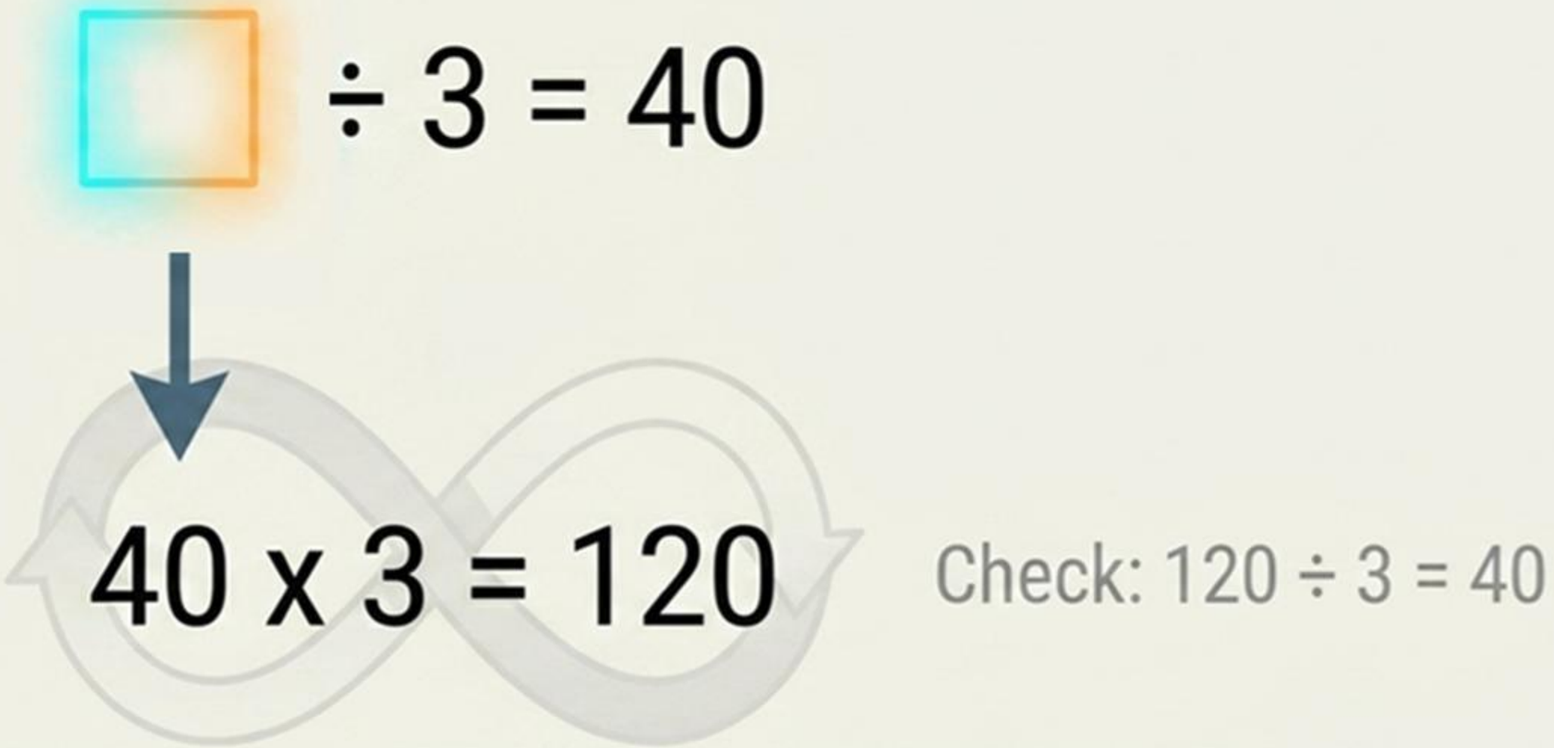
Grade 3: Leveraging Inverse Relationships



The Rule: The unknown dividend is found by multiplying the quotient by the divisor.

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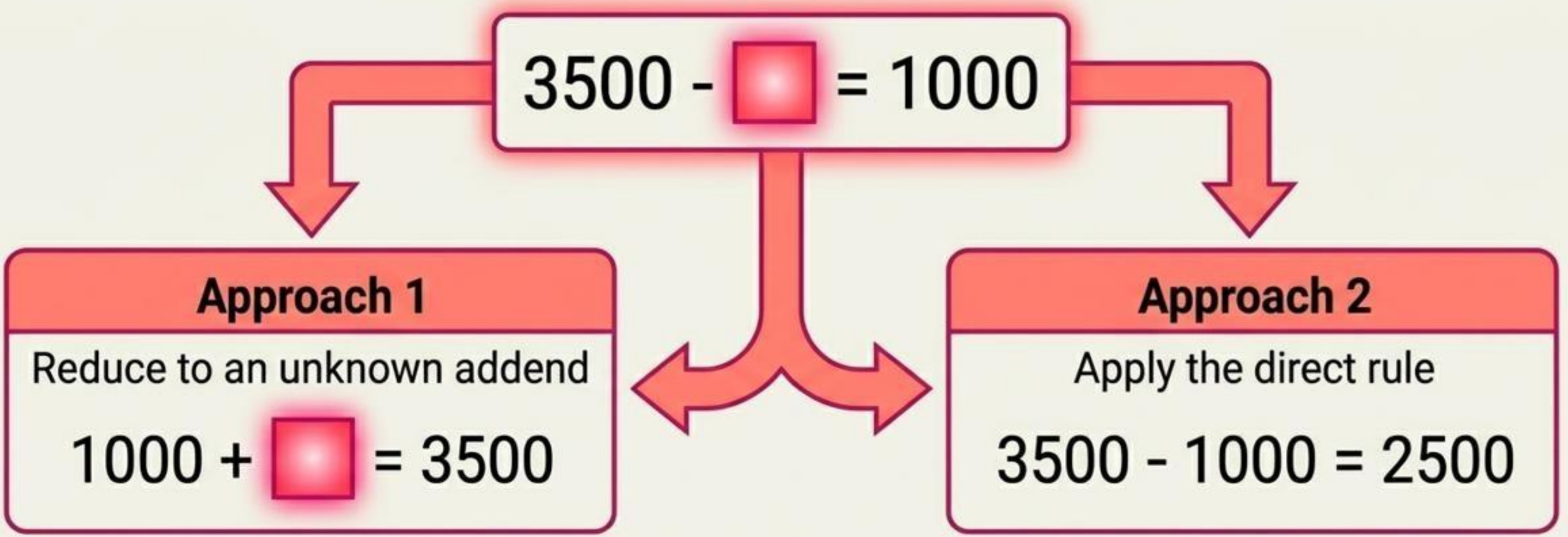
Grade 4: Building Mathematical Fluency and Choice



By fourth grade (finding subtrahends and divisors), the model **shifts from single-path rules** to **flexible, dual-approach problem-solving** for **multi-digit** equations. The ultimate goal is **independent mathematical fluency.**

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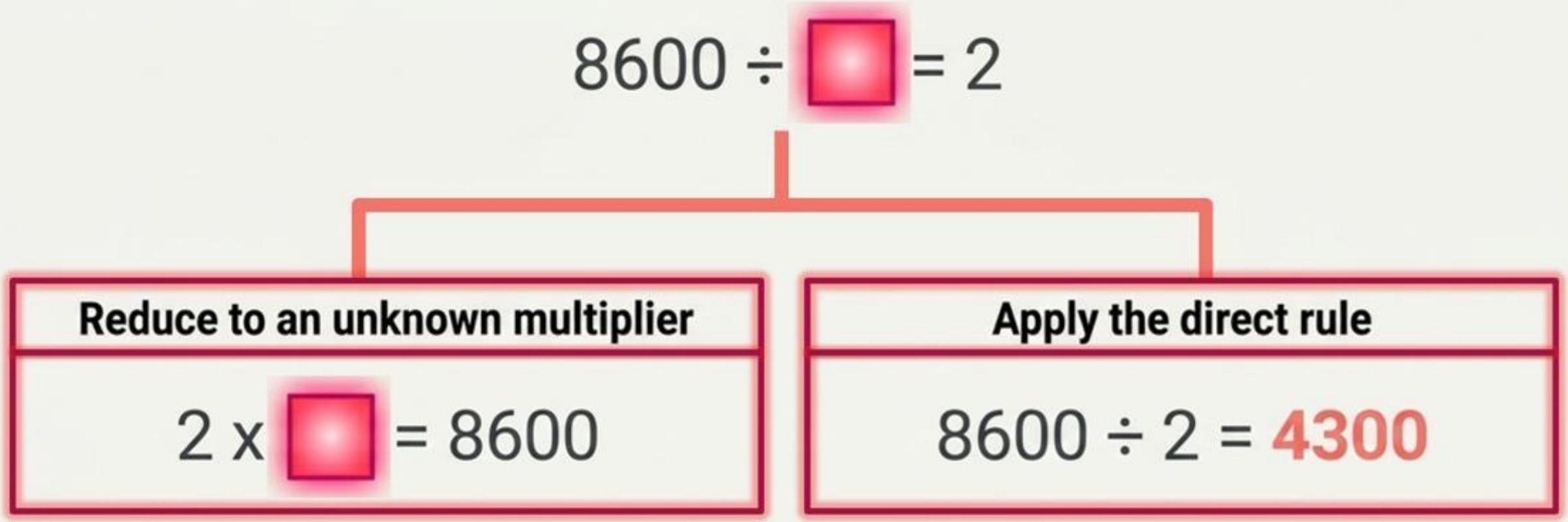
Grade 4: Building Mathematical Fluency and Choice



The Rule: The unknown divisor is found by dividing the dividend by the quotient.

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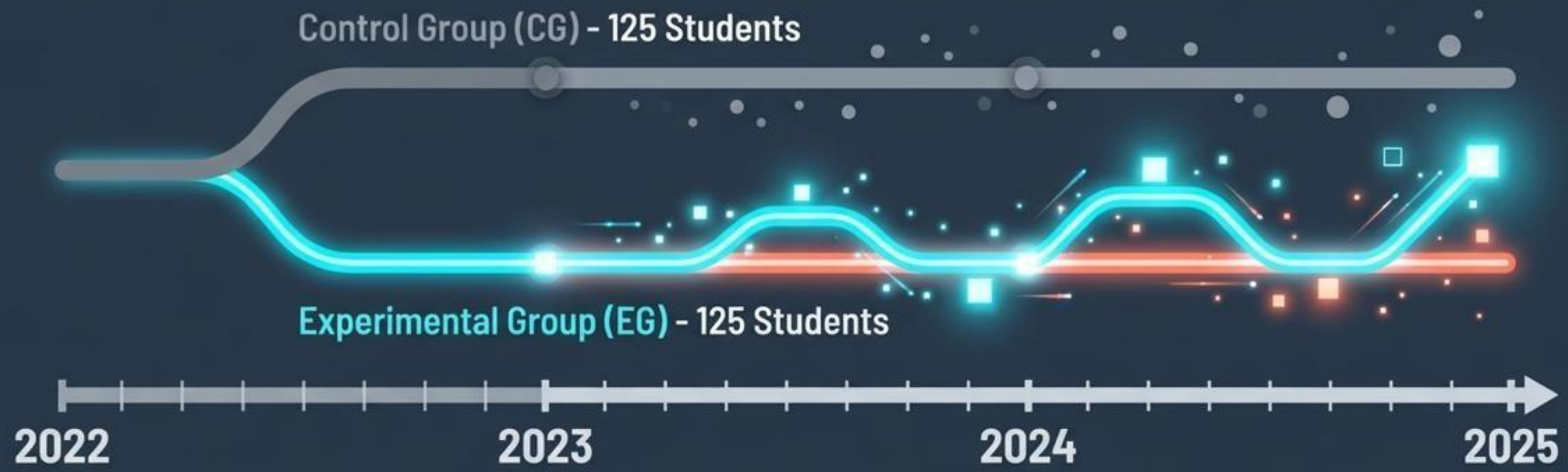
The Crucible: A 3-Year Longitudinal Study



To test the model's effectiveness, a long-term pedagogical experiment was conducted in Plovdiv, Bulgaria. By tracking the same 250 students from second through fourth grade, the study captured the true sustainability and accumulation of mathematical competencies over time.

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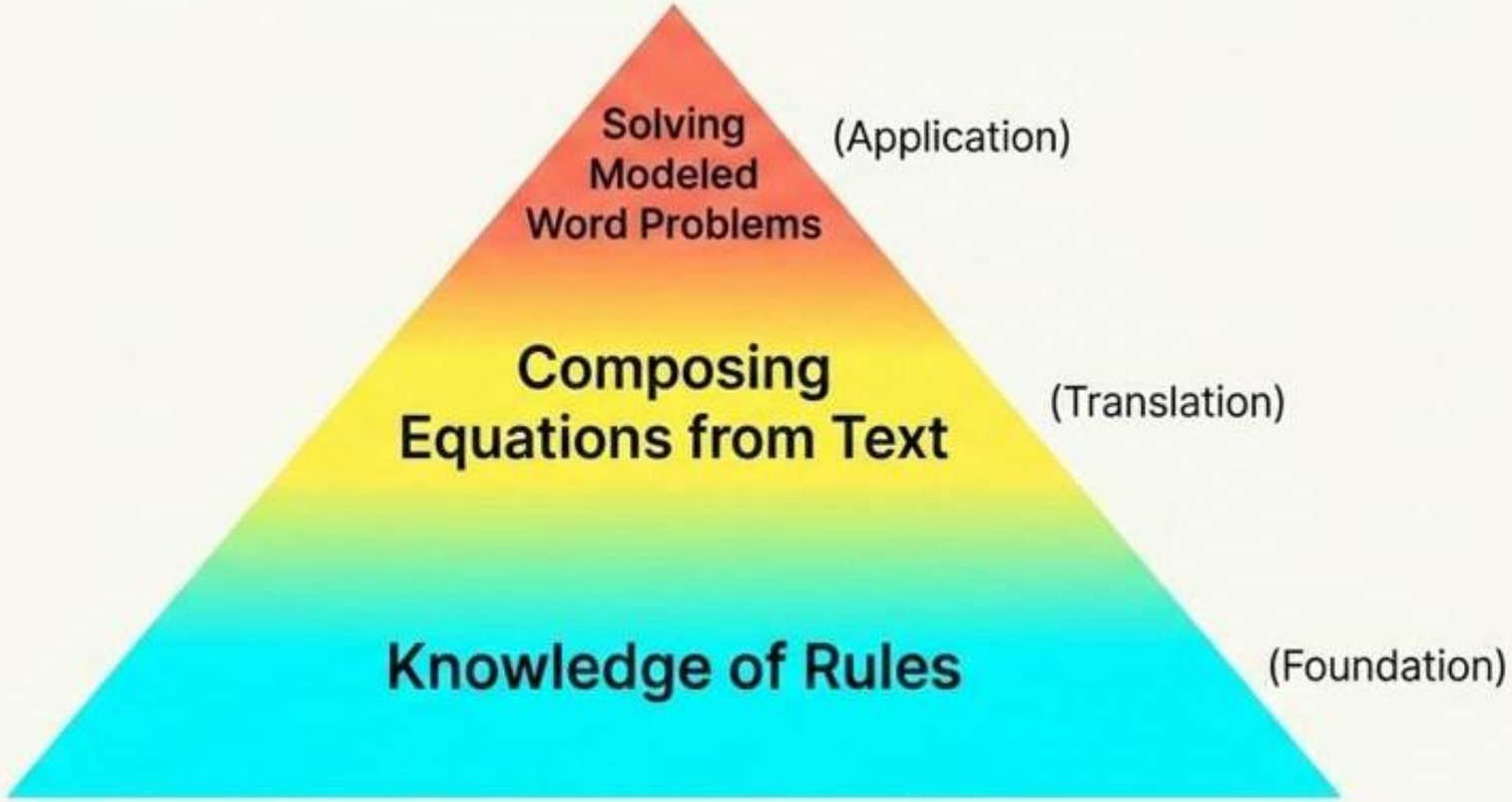
Defining Mastery: The Three Tiers of Assessment



How do we **measure true mastery?** The final diagnostic assessment in Grade 4 measured students against three escalating indicators of cognitive load—moving from simple recall to complex, practical application.

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Tier 1 Results: Near-Universal Rule Mastery

The base layer of the pyramid. The experimental group demonstrated near-universal mastery of the foundational rules for finding unknown components. However, knowing the rules is only the prerequisite for actual problem-solving.

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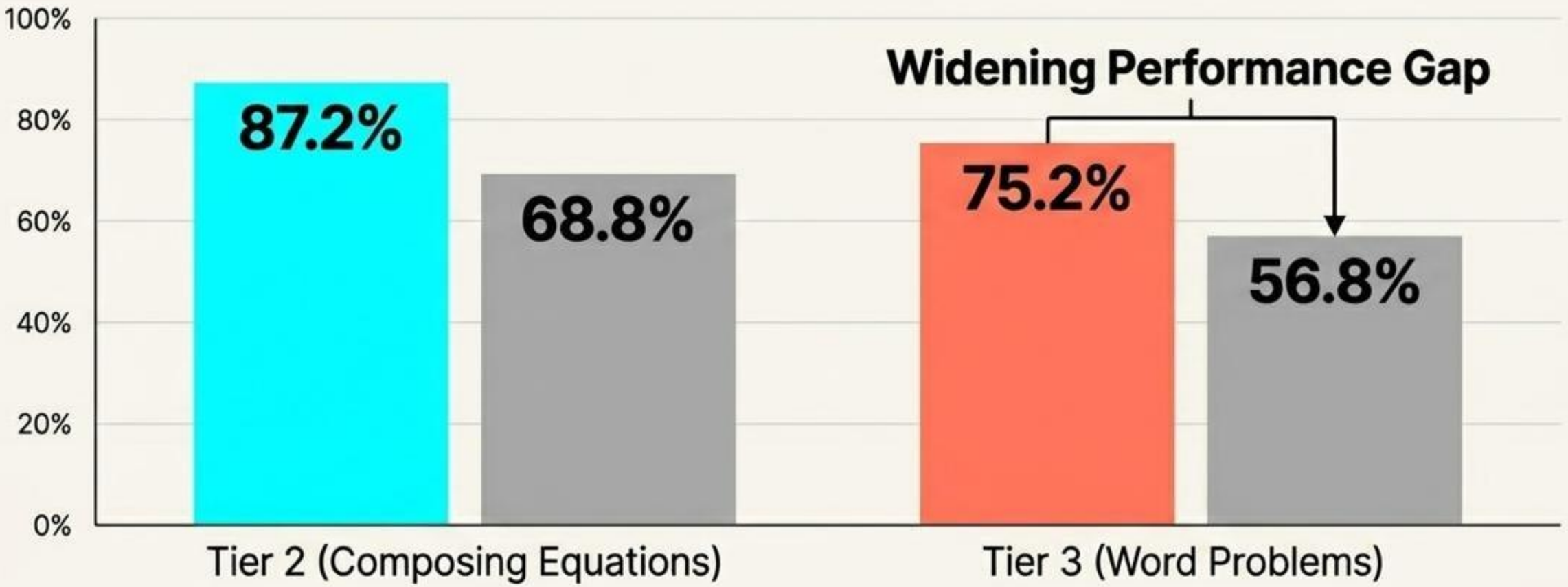
Tiers 2 & 3 Results: The Application Gap



As cognitive load increases, the performance gap widens. The innovative model proves highly resilient under the stress of translating mathematical texts and solving complex word problems, whereas traditional instruction degrades.

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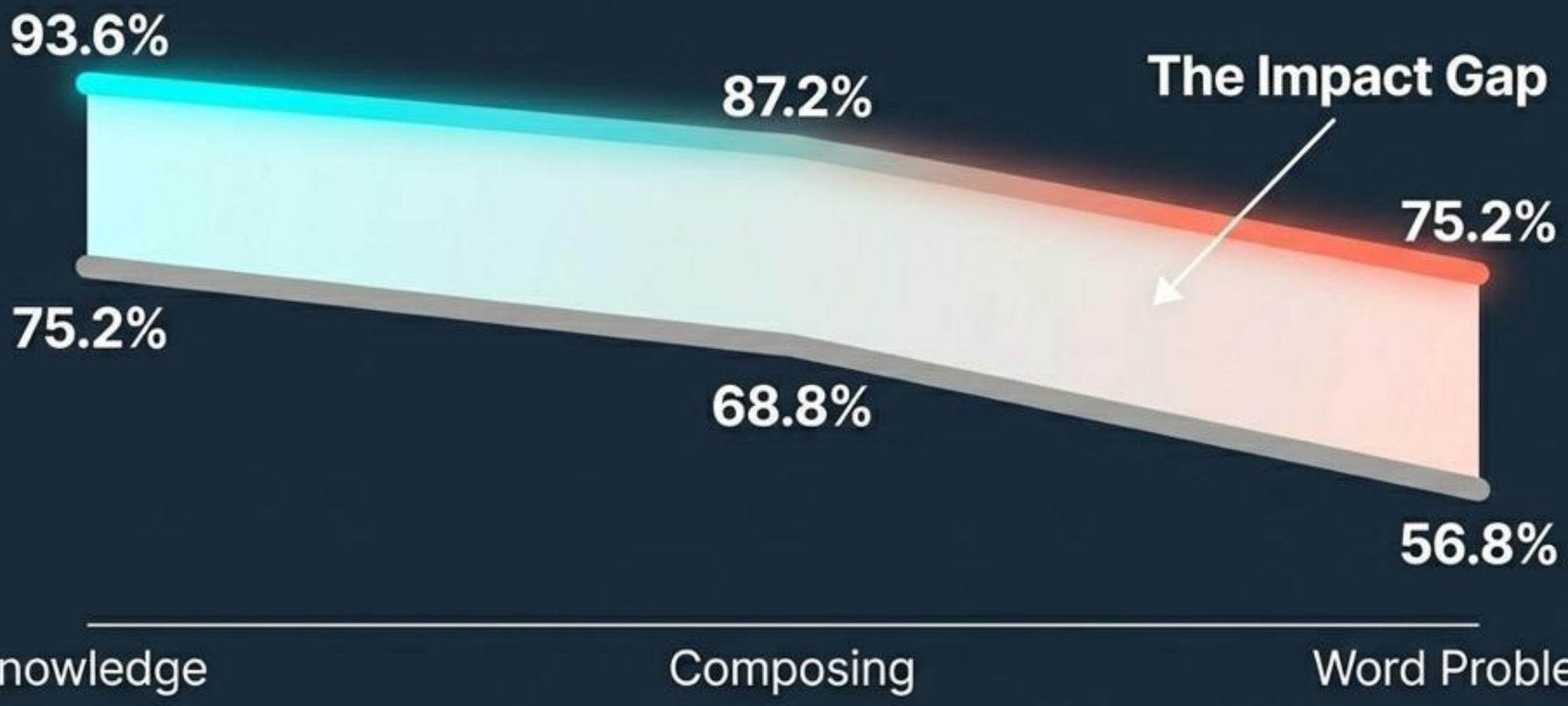
The Resilience of Structural Understanding



The true power of the model is sustainability. Traditional instruction degrades quickly when context gets complicated. The innovative model builds robust logical thinking that survives complexity.

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Elevating Logic, Elevating Confidence

Systematic training in primary mathematics does more than improve test scores. By transitioning students from guesswork to conscious, rule-based deduction, we build deep logical reasoning and permanently elevate student confidence in their mathematical abilities.

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