



## Usage FlexPDE Package in the Courses of Mathematical Modeling and Mechanics

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### Abstract

*The paper describes some features of finite element package FlexPDE that makes it a very suitable tool for teaching such courses as continuum mechanics, strength of materials, elasticity theory, mathematical modelling etc. Finite element program FlexPDE stays apart of the typical modern FEM packages being oriented not on the engineering design but serving as a tool for numerical analysis of equations with partial derivatives arising in many fields of natural sciences. Flexibility of FlexPDE scripting language allows user easily describe the system of differential equations, boundary conditions, and shape of the region and set up the required quantities to be calculated. The proven effectiveness of this package in the elasticity theory makes it very useful for designing and implementation courses on strength of materials, continuum mechanics, the theory of elasticity and plasticity etc.*

*Some examples which can be of use in student labs are presented. They demonstrate not only the nice way from the physical object to numerical result through mathematical modelling but also the problems that are intended to inspire students for independent work, self-study and research activities. Nowadays different types of engineering software accompany the teaching process. Using the finite element package FlexPDE when teaching mechanics and mathematical modelling related courses allows students to better absorb the course material, to understand the essence of the problem, to propose new ideas and methods of solution and to attract additional math for solving complex problems.*

### 1. Introduction

Specialists engaged in field of engineering education recognize the need to compose two fundamentals such as theory and practice to improve student's qualification in general. Natural experiments are not always possible to make in education processes because as a rule they require many resources, time and equipment. In these cases, especially if we deal with structural analysis, fracture mechanics, or chemical process, numerical simulation is a real connection between theory and practice.

Over the past few decades, finite element (FEM) packages became the primary means of computer modeling in engineering. Such FEM complexes as ANSYS, Abaqus, Nastran, Fluent and some others de facto turned into a standard for engineering analysis, design, and production decisions [1-4]. That is why these packages are introducing widely in the engineering education [5, 6]. However, in this paper we will focus on the use of less common and not so huge FEM package FlexPDE [7].

Unlike the majority of modern FEM software, it is concentrated mostly on equations describing the physical process, thus allowing deeper understanding underlying mathematics, easy switching to new models, and conducting numerical experiments to test the material and functional characteristics of these models. The second reason why we recommend to use FlexPDE in education is very wide variety of its application areas. The problems of physics and chemistry, biology and economy are among them. Wide variety of cases described by initial value problem or boundary value problem can be analyse and resolve with this package. And the last, but not least reason to use FlexPDE in teaching process is availability of student's version that has some limitations but is distributed freely.

The usage of FlexPDE package provides good training opportunities because of the transparency of the differential equations and boundary conditions [8–9]. They are not hidden from the user; moreover, the boundary value problem is the main object of investigation. The scripting language is quite simple so student can start working rather quickly. Nice help system substantially increases the speed of this start. The user creates the program script containing the differential equation and the boundary conditions defining the problem. We think that this is very important for teaching and learning.

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The script program contains sections some of which are obligatory, such as VARIABLES, EQUATIONS, BOUNDARIES. The sample script of the stationary heat conductivity problem is presented on the Fig.1. Solving this beginner level problem student can learned to build 3d shape, to recall the heat conductivity theory and to compare numerical solution with the theoretical one.

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TITLE 'Stationary Heat Conductivity'
COORDINATES
  cartesian3
SELECT
  painted
VARIABLES u
DEFINITIONS
  K = 0.1 { conductivity }
  R0 = 1 { radius }
  H0 = 100 { total heat }
  heat =H0*exp(-x^2-y^2-z^2) { volume heat source }
EQUATIONS
  div(K*grad(u)) + heat = 0
extrusion
  surface z = -sqrt(R0^2 - (x^2+y^2)) { the bottom hemisphere }
  surface z=0
  surface z = sqrt(R0^2 - (x^2+y^2)) { the top hemisphere }
BOUNDARIES
  surface 1 value(u) = 0
  surface 3 value(u) = 0
  Region 1
  layer 1 K=0.1
  layer 2 K=0.5
  start (R0,0) arc(center=0,0) angle=360
PLOTS
  grid(x,y,z)
  contour(u) on x=0
END
  
```

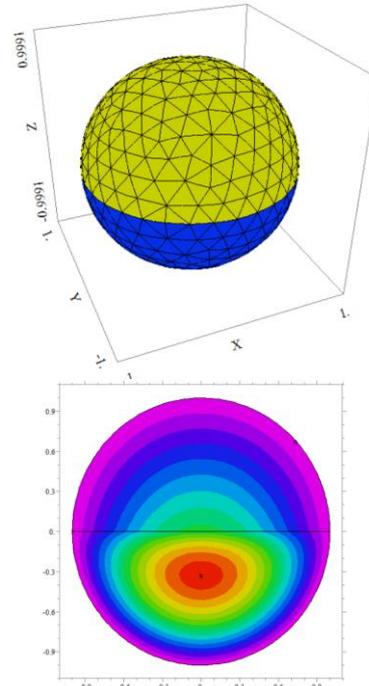


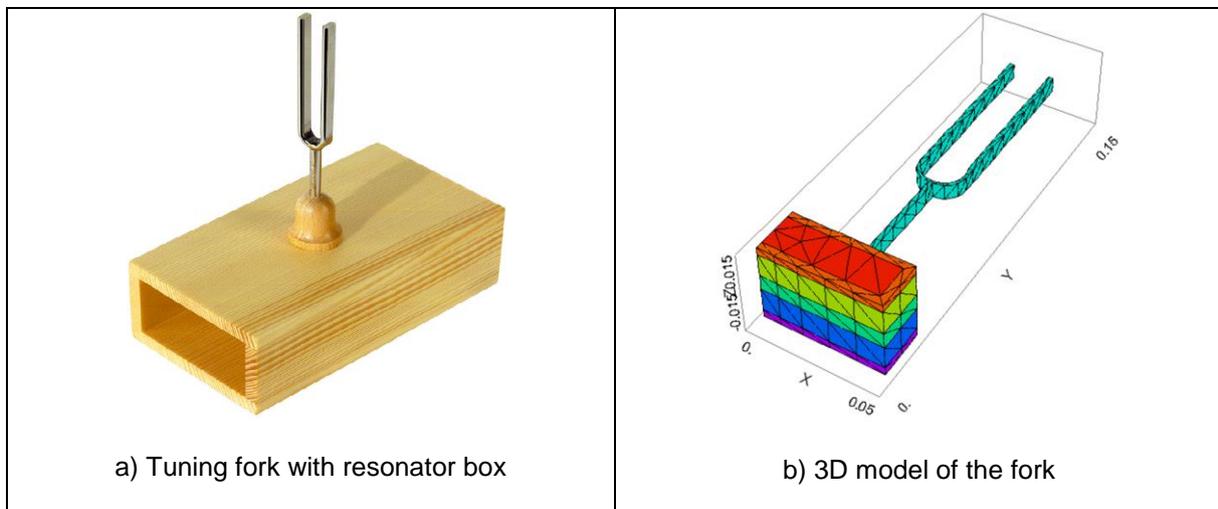
Fig.1. The example of FlexPDE script for the stationary heat conductivity problem

## 2. Examples

### 2.1. Frequency analysis of tuning fork

An interesting educational task for students is a problem to determine the natural frequencies of the tuning fork. Students are required to build tuning fork geometry and calculate the natural frequency (that coincides with the 440 Hz for classical materials and dimensions).

The cells in the Table 1 below correspond to the main steps in creating a numerical model for examining and analysis of the problem: (a) studying the real object, (b) building its 3D model (or triangulated shape) in FlexPDE package, (c) setting up mathematical model based upon elasticity theory equations of motions, and (d) writing program script corresponding the formulated equations and boundary conditions.





<p>Equations of motion</p> $\text{div } \boldsymbol{\sigma} + \rho \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2},$ <p>Oscillations</p> $\mathbf{u}(x, y, z, t) = \mathbf{u}(x, y, z) \cos(\omega t),$ <p>Cauchy relation</p> $\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),$ <p>Hooke's law</p> $\sigma_{ij} = \lambda \theta + 2\mu \delta_{ij} \varepsilon_{ij},$ <p>Equation for frequency analysis</p> $\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} = -\omega^2 \rho \frac{\partial^2 u_1}{\partial t^2},$ $\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{32}}{\partial x_3} = -\omega^2 \rho \frac{\partial^2 u_2}{\partial t^2},$ $\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} = -\omega^2 \rho \frac{\partial^2 u_3}{\partial t^2},$ <p>Boundary conditions</p> $\mathbf{n} \cdot \boldsymbol{\sigma} = 0,$ $\mathbf{u} = 0.$ <p>c) Mathematical model</p>	<p>COORDINATES cartesian3 SELECT { numbers of frequency } modes = 3 VARIABLES U1 U2 U3 { displacements } DEFINITIONS nu=0.3 E=200e9 lambda=E/(1+nu)/(1-2*nu) mu=E/(2*(1+nu)) ... { Cauchy relation } E11 = dx(U1) E12 =(dy(U1) + dx(U2))/2 ... { Hooke's law } S11=lambda*((1-nu)*E11+ nu*E22) S12=mu*E12 ... EQUATIONS U1: dx(S11) + dy(S12) + dz(S31) + Omega*rho*U = 0 U2: dx(S12) + dy(S22) + dz(S32) + Omega*rho*V = 0 U3: dx(S13) + dy(S23) + dz(S33) + Omega*rho*W = 0  EXTRUSION z = 0, h BOUNDARIES region 1 start(0,0) { Boundary conditions } load(U1)=0 load(U2)=0 load(U3)=0 line to... PLOTS grid(x+U1, y+U2, z+U3) as "Shape" ... summary report lambda END</p> <p>d) FlexPDE realisations (fragment)</p>
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Table 1. Main steps of the modelling process

This problem is not complicated for modelling. At the same time obtained results can be easily compared with known data, or even with a full-scale experiment. Students also can study the dependence of frequencies on various parameters of the construction. It is necessary to explore and to plot various dependencies between the parameters of the problem as shown at Fig. 2.

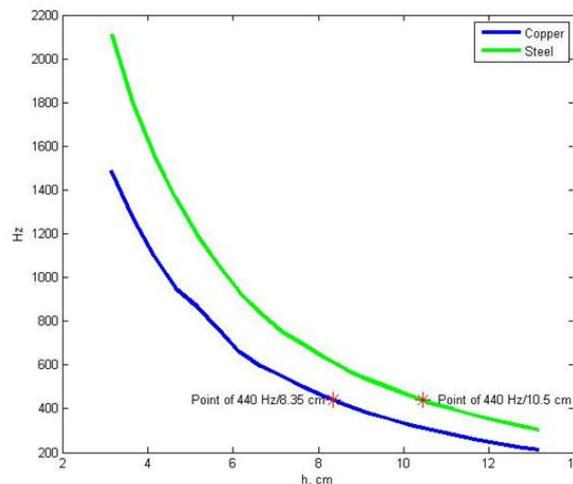


Fig.2. The dependence of frequency versus the material properties and the size of the tuning fork



Typically, every student calculates only small portion of such graph – three or four points. So, to obtain the whole picture students should work together in groups, checking and adjusting their calculations. The problem above can be considered for different levels of difficulty – two dimensional, three dimensional, fork with a resonator box, etc. The option of difficulty of this task depends upon the level of students.

## 2.2. The Kirsch problem

At the initial stage of training it is important to confirm the theoretical principles by showing good agreement between theoretical predictions and numerical results. To demonstrate it the well-known Kirsch problem could be of great use because it has an analytical solution which may be compared with a numerical one. Among others it is the classical example presenting the concept of stress concentration. The formulation of the problem is following: the uniform strip with a central circular hole is exposed to uniaxial tension. The main task is to define stress-strain state and to calculate the coefficient of stress concentration.

Theoretical solution the Kirsch problem [10] in the two-dimensional case is described by following expression for the components of Cauchy stress tensor in polar coordinates  $\sigma_{rr}$ ,  $\sigma_{\varphi\varphi}$ ,  $\sigma_{r\varphi}$

$$\begin{aligned}\sigma_{rr} &= \frac{p}{2} \left( 1 - \frac{R^2}{r^2} \right) + \frac{p}{2} \left( 1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4} \right) \cos 2\varphi \\ \sigma_{\varphi\varphi} &= \frac{p}{2} \left( 1 + \frac{R^2}{r^2} \right) - \frac{p}{2} \left( 1 - \frac{3R^4}{r^4} \right) \cos 2\varphi \\ \sigma_{r\varphi} &= -\frac{p}{2} \left( 1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4} \right) \sin 2\varphi\end{aligned}\quad (1)$$

where  $p$  is the tension load,  $R$  is the radius of the hole.

Fig.3. shows typical stress distributions. Students should use such pictures to analyse and discuss the agreement between the numerical and the analytical solution based on (1).

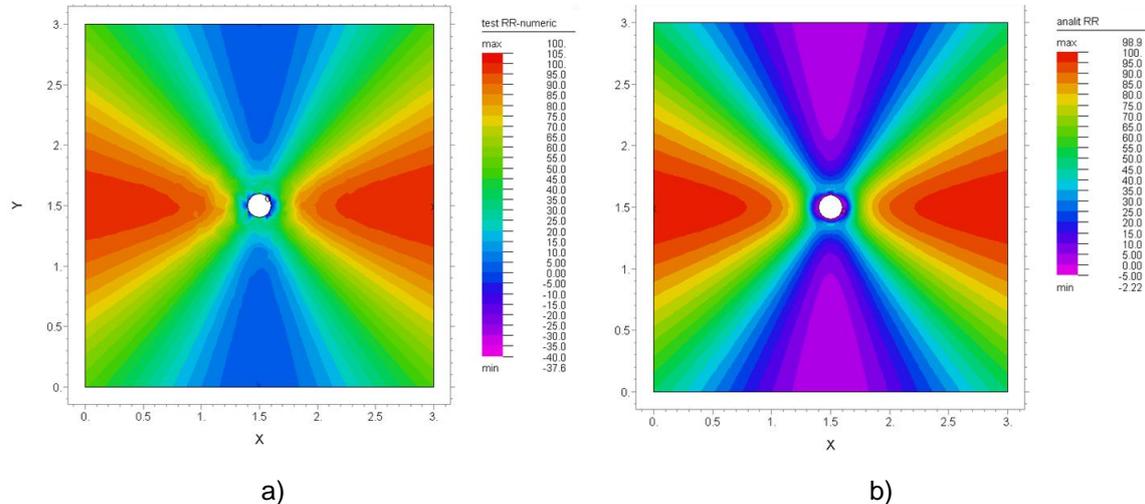


Fig.3. Numerically (a) and analytically (b) determined distributions of  $\sigma_{rr}$

Students are invited to solve similar problems as educational tasks, but with some variation of parameters (holes form, the method of loading, different materials of the plate and others).

## 3. Conclusions

Using the finite element package FlexPDE in the learning process has, in our opinion, a serious of advantages. These include on the one hand the transparency in the definition of equations and boundary conditions, and on the other hand the simplicity of scripting language. Free availability of the student version is also a definite advantage. Moreover, usage of FlexPDE in conjunction with other programs, such as computer algebra package, significantly expands the range of possibilities and scope of tasks. This allows students to take the initiative, to propose new solutions, to attract



additional mathematical methods, to better absorb the material. This may be one of the first steps that will allow the younger generation to conduct their own research and increase scientific activity.

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