New Perspectives in Science Education
International Conference
NEW PERSPECTIVES in SCIENCE EDUCATION

Prospective primary school teachers' difficulties dealing with multiplying fraction word problems

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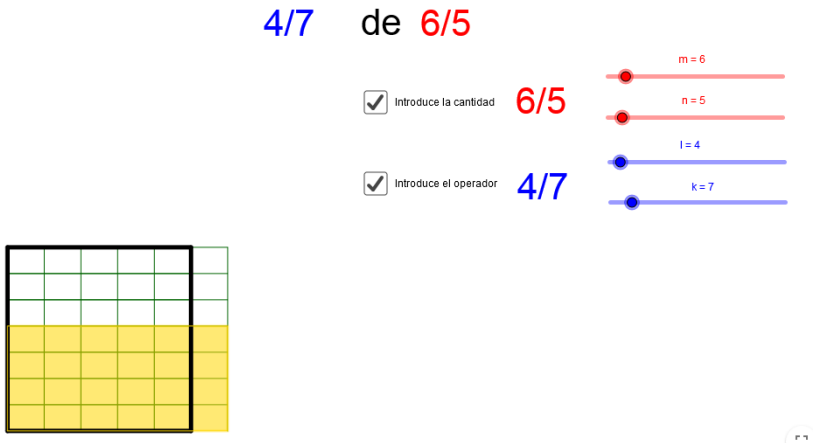
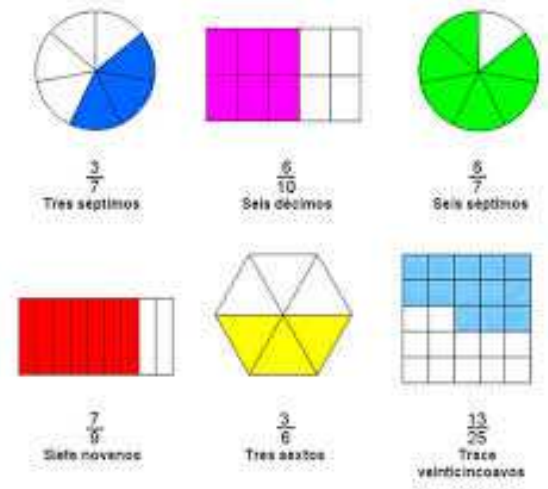
Introduction

- Technological instruments rarely become an essential part of the learning/teaching processes with Prospective Primary School Teachers (PPST).
- Traditional methods include very limited interpretations of the rational number.



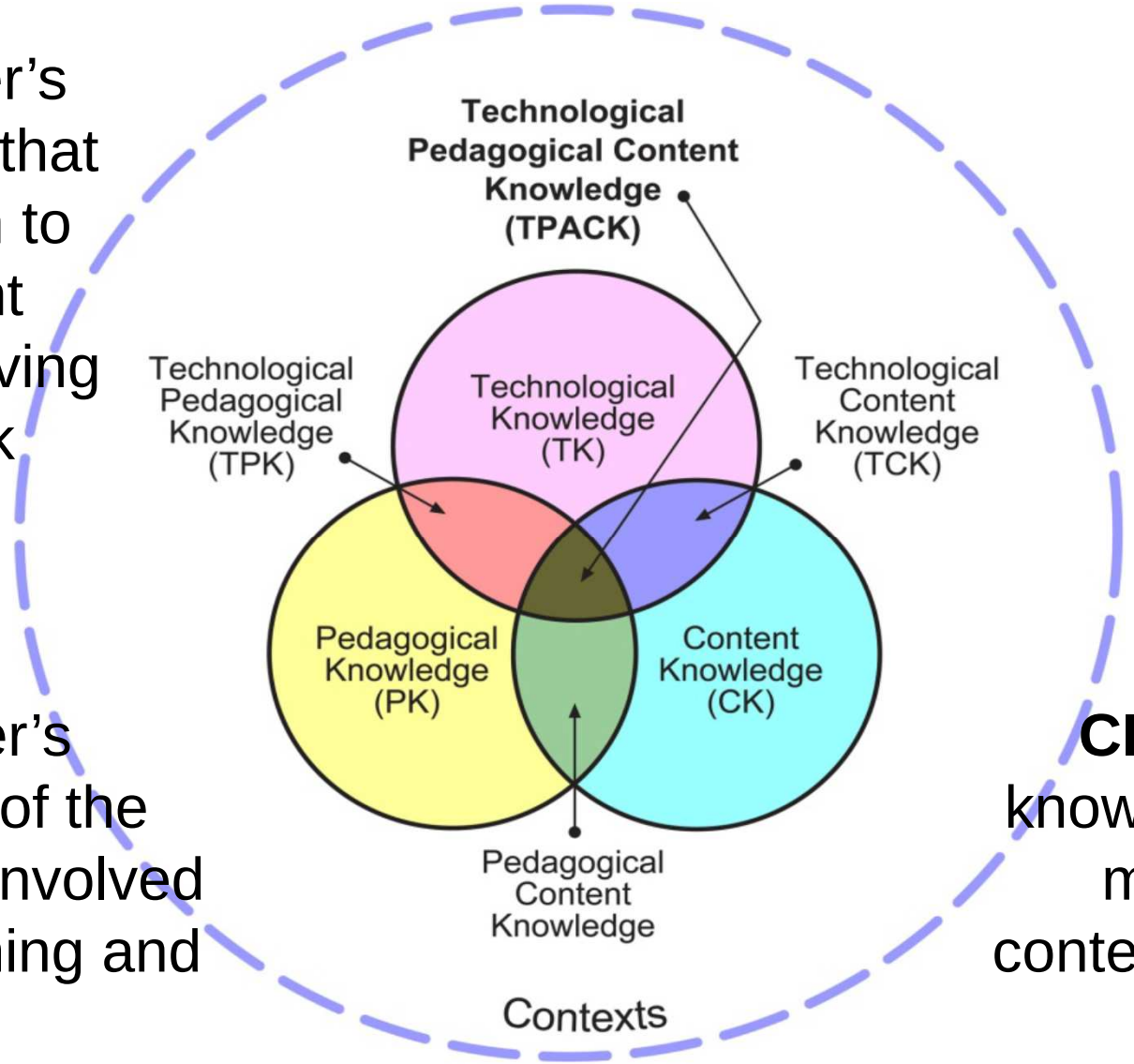


Introduction

- Goals:
  i. To analyze if our PPST are willing to include technology in their teaching activities.
  ii. To study if our PPST are prepared to link different interpretations of the rational number.

Theoretical Framework - TPACK

TK: Teacher's knowledge that permits him to find different ways of solving a given task using IT .

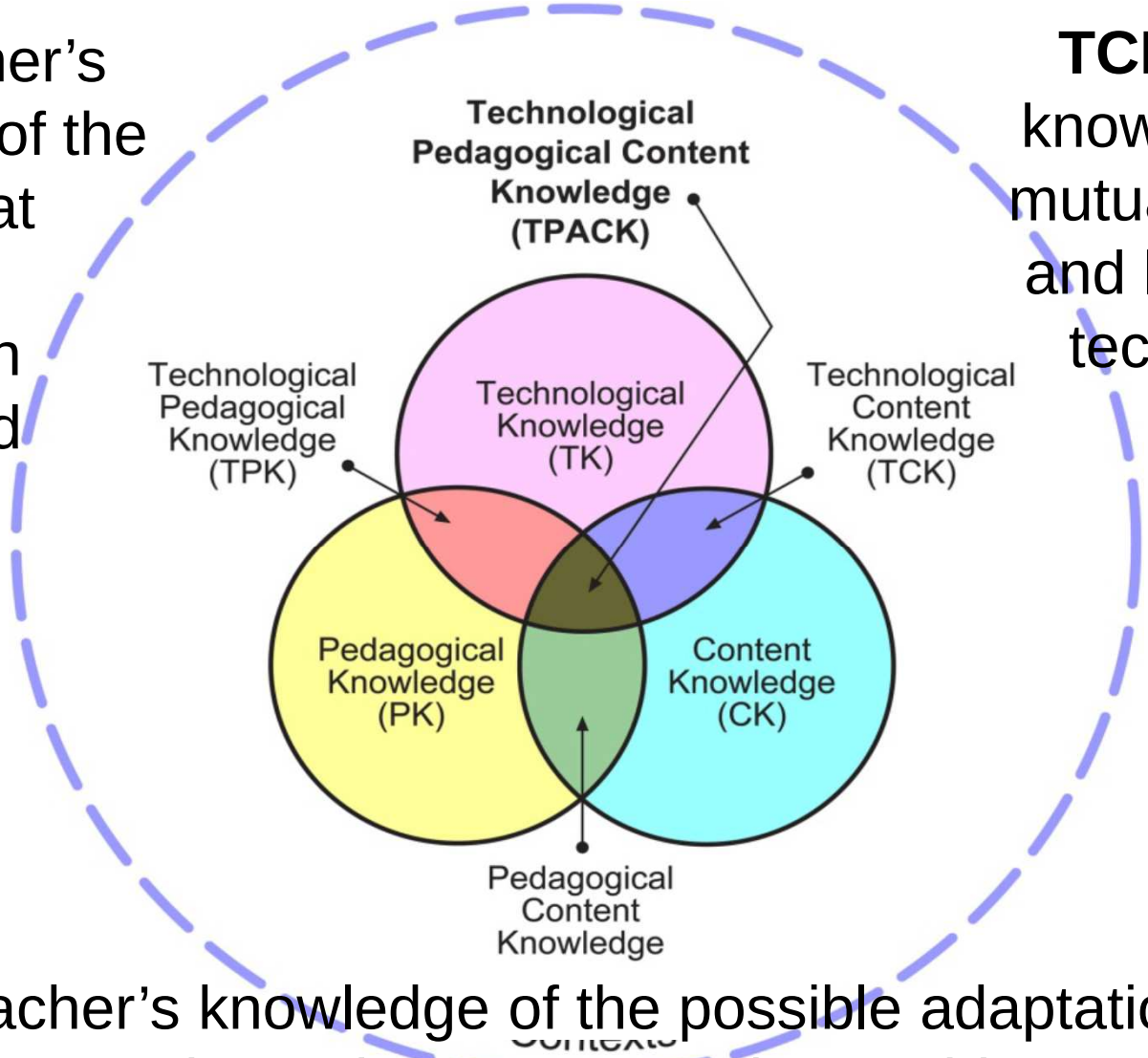


PK: Teacher's knowledge of the processes involved in the teaching and learning of mathematics.

CK: Teacher's knowledge of the mathematical content, including concepts, theories, etc.

Theoretical Framework - TPACK

TPK: Teacher's knowledge of the changes that technology generates in learning and teaching.



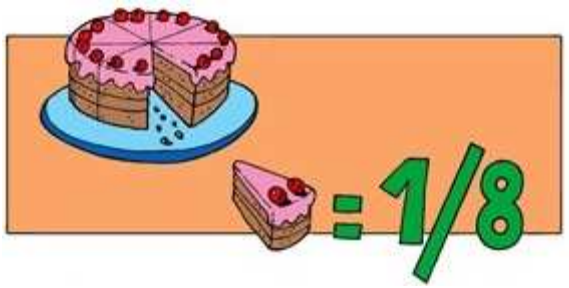
TCK: Teacher's knowledge of the mutual influences and limitations of technology and content.

PCK: Teacher's knowledge of the possible adaptations of the mathematical content to its teaching.

Theoretical Framework

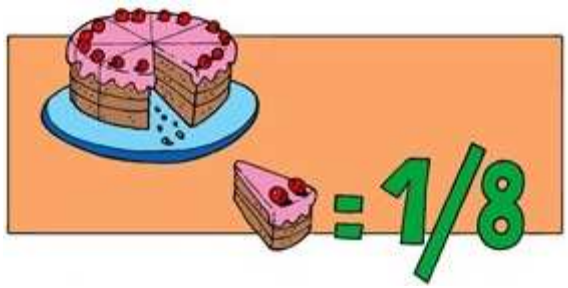
Interpretations of the Rational Number

- Part-whole
- Measure
- Quotient or division
- Operator
- Ratio



Theoretical Framework
Interpretations of the Rational Number

- Part-whole
  - The fraction is understood as a pair of natural numbers.
  - The magnitude becomes less important.
  - Difficulties dealing with improper fractions.
  - The unit is not defined.
  - Passive learning.



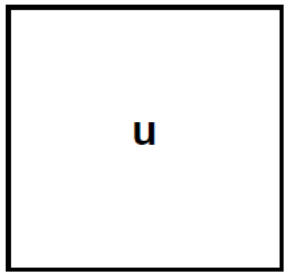
Theoretical Framework
Interpretations of the Rational Number

- Quotient or division

$$\frac{5}{6}$$

Theoretical Framework
Interpretations of the Rational Number

- Measure



Area Unit

Object to measure

The Word problem

Antonio had pizza for lunch with his friends on Monday and Thursday. On Monday they were 5 friends and shared 3 pizzas. On Thursday, they were 8 friends and shared 5 pizzas. On Monday, he gave one fourth of his food to his sister Sara, eating the rest of his lunch. On Thursday Antonio decided to eat all the food he received, but he dropped one fifth of it on the ground. Which day did Antonio eat the most?

(Note: all the pizzas are alike.)

The Word problem

Antonio had pizza for lunch with his friends on Monday and Thursday. On Monday they were 5 friends and shared 3 pizzas. On Thursday, they were 8 friends and shared 5 pizzas. On Monday, he gave one fourth of his food to his sister Sara, eating the rest of his lunch. On Thursday Antonio decided to eat all the food he received, but he dropped one fifth of it on the ground. Which day did Antonio eat the most? (Note: all the pizzas are alike.)

- Solve the problem without using arithmetic operations, but using the graphic support of the given applet (available at https://www.geogebra.org/m/b3XaeVVV). Justify your answer. (You can use as many screenshots as you want to clarify the resolution.)
- Considering your previous justifications, what could you say about the graphics used?
- Solve the problem without using any graphic strategy, just by using arithmetic operations.
- Imagine that you are preparing a mathematics class for your primary school pupils to teach them how to solve problems about comparing quantities coming from the application of operators. Describe step by step the mathematical instructions you would give to your students to teach them how to solve the given problem.

The applet

Multiplicación de fracciones gráfica

Autor: Alberto Arnal Bailera

4/7 de 6/5

Introduce la cantidad 6/5

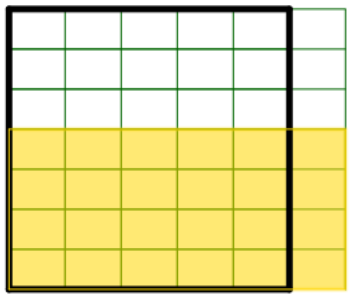
Introduce el operador 4/7

m = 6

n = 5

l = 4

k = 7



The Word problem

a) Solve the problem without using arithmetic operations, but using the graphic support of the given applet (available at https://www.geogebra.org/m/b3XaeVVV). Justify your answer. (You can use as many screenshots as you want to clarify the resolution.) **(TCK)**

b) What could you say about the graphics used? **(TCK)**

c) Solve the problem without using any graphic strategy, just by using arithmetic operations. **(CK)**

d) Imagine that you are preparing a mathematics class for your primary school pupils to teach them how to solve problems about comparing quantities coming from the application of operators. Describe step by step the mathematical instructions you would give to your students to teach them how to solve the given problem. **(TPACK)**

The Word problem

d) Imagine that you are preparing a mathematics class for your primary school pupils to teach them how to solve problems about comparing quantities coming from the application of operators. Describe step by step the mathematical instructions you would give to your students to teach them how to solve the given problem. **(TPACK)**

Results

Task d tries to integrate the mathematical parts of the problem (K) with a hypothetical explanation (P) and the GeoGebra (T). We have classified the given explanations in four categories:

- **Theoretical** - no references to the problem conditions
- **Abstract** - references to the elements of the problem but no to numbers
- **Concrete** - references to the elements of the problem including numbers
- **Complete** - comprehensive solution of the problem

Results

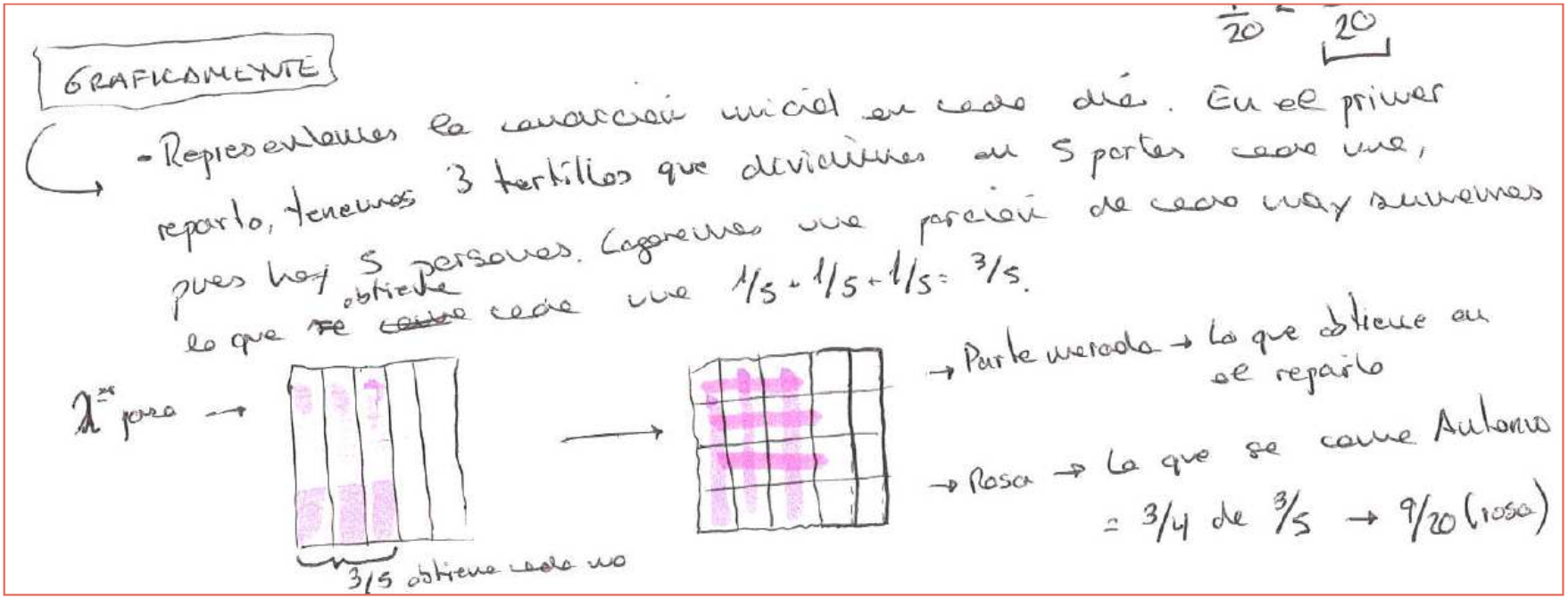
	T	A	Cn	Cm	
No reference in the instructions to the operator interpretation nor the meaning of the comparison	1 (0)	1 (1)	1 (1)		7%
References only to the operator interpretation	2 (2)	1 (0)	**9 (6)**		**28%**
References only to the meaning of the comparison	**5 (5)**	0 (0)	2 (0)		**17%**
References to both concepts	**7 (3)**	2 (0)	**5 (3)**	**6 (3)**	**48%**
	36%	10%	**40%**	14%	

Results

Math. WRONG answers (18)	T	A	Cn	Cm	
No reference to any concept	1	0	0		6%
Ref. only to the operator	0	1	3		22%
Ref. only to the comparison	0	0	2		11%
References to both concepts	**4**	**2**	**2**	**3**	**61%**
	28%	17%	39%	17%	

Math. CORRECT answers (24)	T	A	Cn	Cm	
No reference to any concept	0	1	1		8%
Ref. only to the operator	2	0	**6**		33%
Ref. only to the comparison	**5**	0	0		21%
References to both concepts	3	0	3	3	**38%**
	42%	4%	42%	12%	

8 Concrete – correct – references only to operator



3 Concrete – correct – references only to operator

- Primero, representaríamos la cantidad inicial. Luego, les explicaríamos que el operador es una cantidad que actúa dentro de la inicial (la modifica), y entonces, representaríamos el operador dentro de esa representación inicial.

$9/20 < 10/20$ → Come más en el 2º reparto, 10/20.

Una vez representadas las cantidades, atendiendo a los tamaños de las subunidades, tienen que colorear o marcar la parte correspondiente al producto de los numeradores. Por ejemplo, representar las subunidades:

Es decir, $3/5 \times 3/4 = 9/20$

Luego, haríamos el mismo proceso para calcular lo que come el jueves y finalmente, compararíamos ambas fracciones.

4 Theoretical – correct –references only to comparison

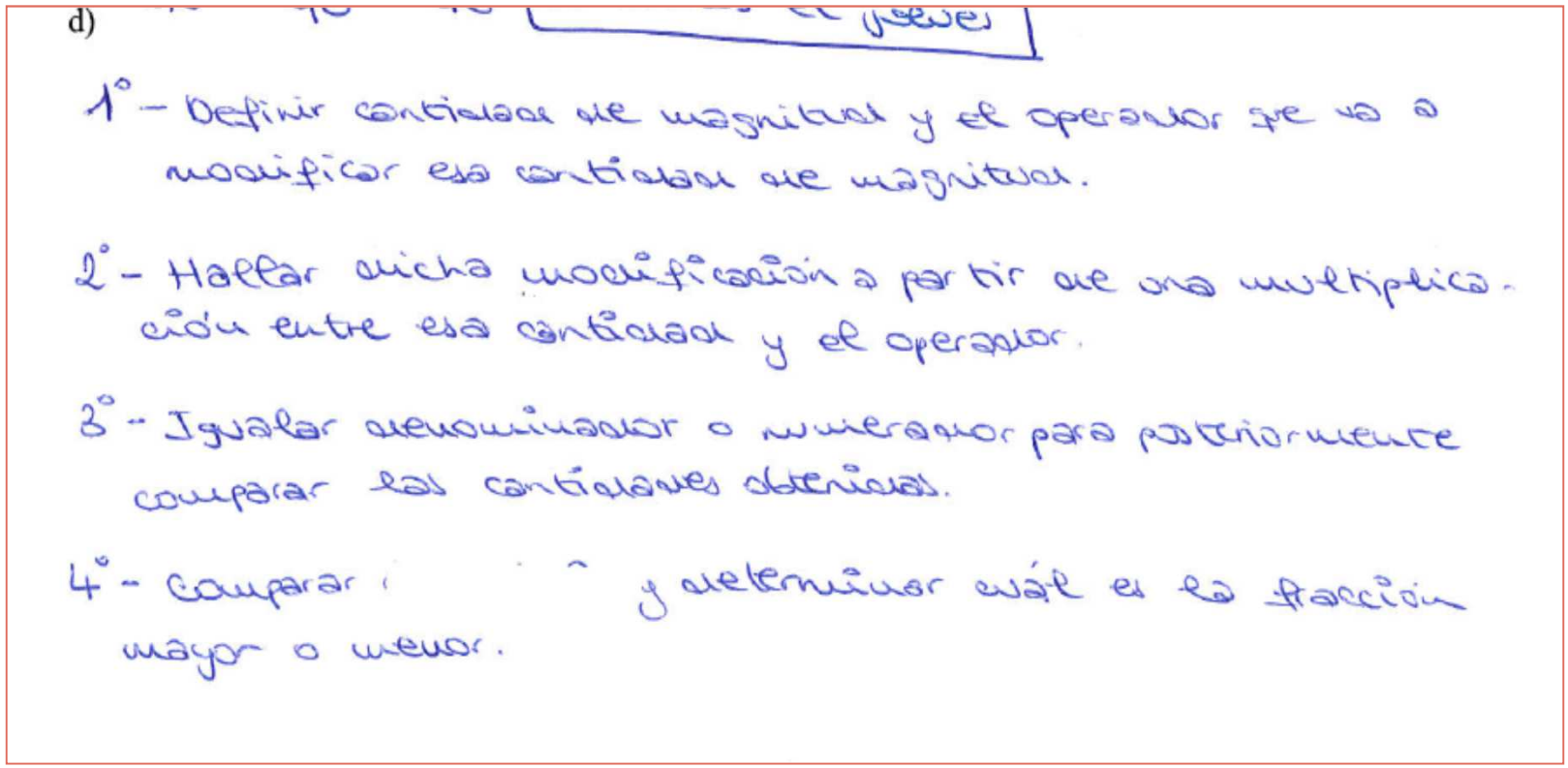
Para comparar fracciones resultado de operaciones de reparto, tenemos que hacerles ver a los niños que hay que igualar o numerador o denominador en ambas fracciones y lo podemos hacer multiplicando tanto el numerador como el denominador por la misma cantidad, ya que el reparto será el mismo pero repetido más veces.

5 Theoretical – correct – references only to comparison

1. Factorización de los denominadores
2. Realizar el mínimo común múltiplo, es decir, comunes y no comunes al mayor exponente.
3. Adecuar cada numerador al denominador común.
4. Comprobar que numerador es mayor para saber que fracción es mayor que la otra.

Lo que se hace es igualar el tamaño de las subunidades (denominador) para saber en que fracción hay más cantidad de subunidades (numerador).

46 Theoretical – correct – References to both concepts



d)

1° - Definir cantidad de magnitud y el operador que va a modificar esa cantidad de magnitud.

2° - Hallar dicha modificación a partir de una multiplicación entre esa cantidad y el operador.

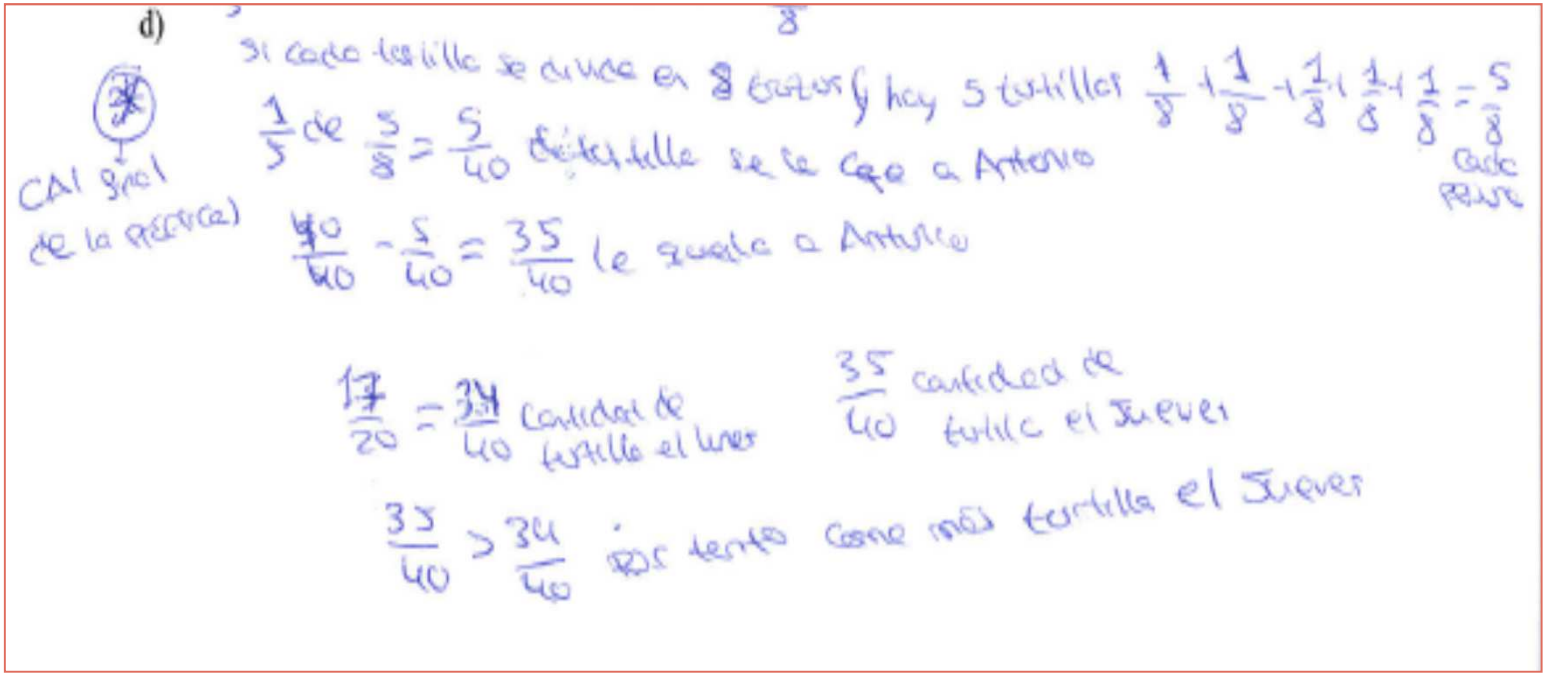
3° - Igualar denominador o numerador para posteriormente comparar las cantidades obtenidas.

4° - Comparar y determinar cuál es la fracción mayor o menor.

31 Concrete – correct – references to both concepts

d) Para representar las fracciones usando la aplicación, deberán poner la fracción que representa el primer reparto donde pone "introduce la cantidad" que en este caso será 3/5, que indica que son 3 tortillas para 5 personas. Y donde pone "introduce el operador" deberan poner la fracción que indica la cantidad de tortilla que se come, en este caso 3/4, que indica las tres cuartas partes del reparto. Así en la gráfica apareceria representado 3/4 de 3/5. Se realizaría lo mismo con el 2º reparto y así podrían compararlo gráficamente.

30 Concrete – wrong – references to both concepts

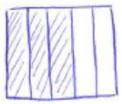
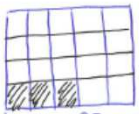


30 Complete – wrong – references to both concepts

d) Primero se les leería el enunciado. Analizado parte por parte se corregiría explicando la expresión "3 tortillas para 5 personas" con el siguiente dibujo.

= ← lo que recibe 1

Les mostramos las 3 tortillas del reparto y especificamos con las 5 participantes, después mostramos lo que recibe una persona para llegar así al apoyo gráfico que nos muestra 'Geogebra' y poder utilizarlo.

Ahora introducimos la segunda parte del problema "el operador" una cantidad que modificará la que ya tenemos. Entonces con el gráfico de $\frac{3}{5}$  y ahora cogemos $\frac{1}{4}$ de esa cantidad, por lo que fraccionamos en 4 

De esta manera nuestra tortilla ha sido dividida en 20 partes de las cuales cogeremos 3 partes de la misma adquiriendo dos soluciones una que será lo que come Sara $(\frac{3}{20})$ y otra lo que cave Antonio $(\frac{17}{20})$.

Con el reparto realizado el jueves repetiríamos el mismo proceso, pero con otras cantidades.

↳

1. Deben representar la cantidad de tortilla que recibe la persona que participa en el reparto a ambos días.
2. Deben calcular la cantidad de tortilla que come Antonio de la cantidad de tortilla inicial de cada uno de los días considerando las variables que se generan (lo que se come Sara y lo que se cae al suelo).
3. Comparar la cantidad de tortilla que come Antonio cada día.

Results

- Students with more complete instructions had worse mathematical answers.
- Students with a better domain of the mathematical content had Pedagogical difficulties.

Results

- Very few couples decided to use technology.
- Only two couples considered solving the problem in two different ways
- None proposed to check the answer by solving it in two different ways.
- Ten of the couples missed any interpretation of the rational number other than a formal explanation of the arithmetic operations.

Results

- They emphasized the most difficult mathematical aspects, mainly the operator interpretation and the meaning of the comparison.
- From this point of view they may have thought that comparison is easier to be understood by a primary school kid than the operator interpretation. Moreover, instructions about comparison are shown to be more theoretical than the ones about the operator interpretation.

Consequences for teaching training in mathematics education

- To include tasks covering all the TPACK subdomains.
- To combine different interpretations of the rational number.
- To promote the use of one technique by making more difficult the use of the others. It means, for example, we have to use higher figures in the activities to promote the use of GeoGebra by making more difficult for them the use of other techniques.

Consequences for teaching training in mathematics education

- To include actual answers of primary school kids to analyze errors and give tips to correct them by using different techniques.
- To ask for an analysis of the mathematical content before writing the instructions.
- To include role-playing activities with prospective teachers to make them understand better that, when designing instructions, they should focus in pupils' troubles rather than in their own ones.

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