Formulation, Resolution and Revision of Mathematical Problems in Cooperation in a 3rd Grade Classroom

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Abstract

Since the last century, problem-solving has been assigned a central role in the mathematics curriculum (1), recently enshrined in the guidding document for Essencial Learning in Portugal (2). Despite this focus, students continue to face and express difficulties when tasked with problem-solving. In this sence, encouraging students to write, review, share and solve their own problems (3), can prove important, in improving their skills in this área. Thus, this study seeks to understand students' perception regarding the formulation, resolution and revision of problems and how performing these tasks in cooperation with their peers can encourage them to create more and better problem statements, as well as solve them more successfully. (4)

This study was conducted with a group of 20 students from a 3rd grade class, in de first cycle of basic education, and had the following specific objetives: (i) charaterize students' perceptions about mathematical problems formulated by themselves and their peers; (ii) charaterize students' perceptions regarding cooperative discussions about mathematical problems formulated by themselves, and their peers; (iii) charaterize students' perceptions of the charateristics of the mathematical problems they formulated and those formulated by their peers; (iv) charaterize students' perceptions of the learning promoted by mathematical problems they and their peers formulated and solved; (v) identify potencial improvements in the formulation, resolution, and revision of problems resulting from the specifics strategies implemented.

This study is qualitative in nature, using action-research methodological procedures. Data collection techniques, included exploratory documents analysis, direct and participative observation, field notes, and semi-structured questionnaires. The data was processed using categorical content analysis. The results allowed us to better understand the processes explicity described by the students, in data treatment, the mobilization of grammar, writing, reading and interpretative correlation in portuguese, as well as the importance of the task of formulating and solving problems in cooperation for acquiring mathematical, social, and linguistic/ comunication skills.

Keywords: *Problem formulation, problem resolution, problem revision, cooperation.*

1. Introduction

This study aims to understand how to improve students' skills in formulating, solving, and reviewing mathematical problems.
The teacher in the class where the study was conducted followed the pedagogical model of the Modern School Movement (MEM). This model is based on principles of cooperation, participation, and communication, where students actively participate in their learning process within the context of a learning community [5].
The study originated from the significant difficulty students experienced in formulating and solving problems in their daily lives. It references the need to emphasize problem-solving in learning contexts, as advocated by the National Council of Teacher Association [1] and the guiding document on Essential Learning in the Portuguese national context [2].

Problem-solving is a challenging and complex task, often due to comprehension issues or difficulty in identifying the best strategy for solving the problem. In line with these challenges, the study had the following specific objectives: (i) to analyze students' perceptions of the mathematical problems they and their peers formulated; (ii) to examine students' perceptions of the discussions held in class about the mathematical problems they and their peers formulated; (iii) to assess students' perceptions of the characteristics of the mathematical problems they and their peers formulated; (iv) to evaluate students' perceptions of their learning through self- and peer-formulated mathematical problems; and (v) to identify improvements in the formulation, solving, and reviewing of problems based on the strategies implemented.
The study is presented in several sections. First, the methodology used is described, followed by the results and progress in students' mathematical skills during the process.

2. Methodology

The methodology used was qualitative, guided by principles of action research. This approach focuses on improving teaching practices and student learning "through change and learning from the consequences of these changes" (p. 65) [6].
Data collection instruments were selected based on a theoretical framework. In line with the study's objectives, data collection techniques included direct observation and, initially, a questionnaire with both open and closed questions. This questionnaire aimed to identify students' prior perceptions of what problems are and whether formulating problems helped them solve them.
Later, after analyzing the data using content analysis [7], the researcher shared this information with students, involving them in an active and reflective process. This led to the identification of a set of indicators (developed with the participation of students and the teacher), which supported the creation of a guide for problem formulation and the implementation of a daily routine of problem formulation during Autonomous Study Time (AST). AST is a one-hour daily session dedicated to pedagogical differentiation, guided by an Individual Work Plan (IWP).
At the end of the process, a second open-ended questionnaire was used to assess students' final perceptions of what a problem is and whether working in partnerships contributed to their learning and skill development. Additionally, it aimed to determine whether problem formulation helped them understand and solve problems more effectively.

3. Results

3.1. Students' Perceptions of Self- and Peer-Formulated Mathematical Problems

Initially, only about half of the class enjoyed formulating problems, and they did so individually, starting with a topic or question. Moreover, many students mistakenly thought exercises were problems. In this context, a previous study noted that "students working in groups almost always formulated problems, whereas individual productions tended to result in exercise statements" (p. 4) [8].
A good problem should meet three key criteria:

1. It should be challenging and make sense, where the solution path is not immediately obvious.
2. It should be engaging and interesting, particularly from a mathematical perspective.
3. It should be appropriate, allowing students to connect their knowledge and skills to complete the task (p. 17) [9].

Thus, a problem involves formulating a question aimed at discovery and finding an answer.
After the strategies were implemented, over two-thirds of the group began enjoying problem formulation, viewing it as a fun and purposeful activity. These problems were solved by their classmates during AST as part of a self-correcting file created by all students.

3.2. Students' Perceptions of Cooperative Discussions on Mathematical Problems

While students regularly received feedback on project presentations, they initially lacked feedback on problem formulation, leaving them unaware of areas for improvement.
By the end of the intervention, the class began discussing and reviewing problem statements as they would a "normal" text, recognizing the need for revisions and improvements. This task enhanced writing skills, fostered the exchange of problem-solving strategies, and improved students' understanding of what constitutes a mathematical problem.

One improvement observed was greater collaboration in this task, as "two heads think better than one" (p. 134) [10].
Overall, most students felt that this activity encouraged idea-sharing, although some initially struggled with it.

3.3. Students' Perceptions of the Characteristics of Mathematical Problems

At first, students associated a good problem with simply knowing how to pose a question, without understanding how to improve or refine it.
Through the intervention, students learned to better understand problem requirements, pay closer attention to problem statements, and improve their writing and reading skills while addressing their difficulties.
Active student involvement led to a deeper understanding of mathematical concepts. Creating their own problems allowed students to engage in "new problem creation or reformulation of an existing problem, formulating a sequence of mathematical problems, or generating problems from a given situation or as a resultant activity when a problem is inviting the generation of other problems" (p. 519) [11]. This process motivated students to explore and deepen their mathematical knowledge.

3.4. Students' Learning Through Self- and Peer-Formulated Mathematical Problems

Initially, most students believed that formulating problems only helped in solving them. However, understanding a problem is essential before solving it. Five key benefits of problem comprehension were highlighted:

1. It develops the type of representation students can construct.
2. It helps students coordinate the selection and execution of procedures.
3. It aids in evaluating the reasonableness of results.
4. It promotes knowledge transfer to related problems.
5. It encourages generalization to other contexts [9].

After the intervention, students recognized additional benefits, such as improved comprehension of problem statements, enhanced writing and reading skills, and better strategies for mathematical operations. They appreciated discovering and sharing diverse strategies for solving problems, which could be applied to similar situations.

3.5. Student Progress in Problem Formulation, Resolution, and Revision Resulting from Implemented Strategies

The literature on problem formulation reveals that this activity is relevant from various perspectives, also noting connections between problem formulation and creativity. Creative activity can be seen in the process of formulating, attempting to solve, reformulating, and solving a problem. This is evident in Pólya's work [12], where he considers problems as non-routine activities that present an intellectual challenge, contributing to improving students' mathematical understanding and development. The author also states that investigative problems, due to their multiple results, have characteristics related to creativity, involving complex thought processes [13]. Therefore, the formulation, resolution, and revision of a problem involve a greater degree of complexity, requiring more steps, searching for patterns, generalizations, justifications, and finding new questions for further exploration. This situation was observed during the research, as students began to want to create more complex problems for their peers, making them unsolvable, or even creating problems without solutions. This freedom of creation triggered a creative desire in students that had previously been limited to reading, understanding, and solving problems presented by the teacher.
Studies by different authors focus on concept acquisition and analyze the role of representations in completing tasks and mathematical processes. They find that despite some difficulties students face in performing this task, they recognize its contribution to promoting reasoning and understanding of concepts [14]. Concepts like area or perimeter, which were traditionally presented with simple geometric shapes, were rethought by students in new ways, discovering that everything around them could be measured, even toys and games. This led to a problem being created about the area of toys drawn by the students.
Problem formulation and resolution activities are seen as valuable learning opportunities, as they equally emphasize reasoning and mathematical communication. In fact, it is "through oral discussion in the classroom that students confront their problem-solving strategies and identify the reasoning

produced by their peers" (p. 9). Furthermore, "through writing texts, students have the opportunity to clarify and elaborate their strategies and arguments in greater depth, developing their sensitivity to the importance of rigor in the use of mathematical language" (p. 9) [15].
On the other hand, "as students make their ideas explicit, the teacher has the opportunity to understand how they are thinking, which allows them to identify misconceptions, 'arbitrate' the use of mathematical language, and plan new challenges to present" (p. 61) [16].
Thus, this capacity to think mathematically can be developed through reflective and cooperative practice. It is through the sharing and diffusion of student productions (both individual and in partnerships) that the statements, when communicated or presented, can be cooperatively evaluated, enabling their improvement and understanding. This results in psychological, curricular, and social development for the student. The fact that their problem statements can be solved by their peers gives the activity an immediate social meaning—a sense of accomplishment.

4. Conclusions

Key conclusions include changes in students' perceptions of mathematical problems, increased creativity in problem formulation, and improved problem-solving strategies. By the end of the study, students demonstrated greater care in structuring problem statements, both in reading and writing. They also emphasized connecting problems to real-life contexts, making them personal and relevant. The fact that they presented their problems to their peers helped develop their critical thinking skills, the appreciation of mistakes as a form of learning, and the exchange/sharing of knowledge.
The teacher played a crucial role in challenging, supporting, and evaluating students, fostering investigation, reflection, and connections among concepts. This process also enhanced the teacher's professional, mathematical, and pedagogical knowledge [17].

4.1. Limitations and Future Proposals

During the research, other questions arose that, for various reasons, could not be developed. The study highlighted the importance of addressing various types of problems, such as open-ended, exploratory, and multi-step problems. Non-routine and diverse problems can foster meaningful learning by connecting concepts and encouraging critical reflection.

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