What Math Are You Teaching? ISAT: A Tool for Individual and Organizational Analysis of Math Activities

Yossi Elran[1], Naama Bar-On[2], Lior Ben-Gai[3], Michal Elran[4], Tamar Levy[5], Tal Levy[6], Michael Gorodin[7], Aurellie Zalait-Lachish[8] and Liat Ben-David[8]

Davidson Institute of Science Education, Israel[1,2,3,4,5,6,7,8]

Abstract

When we teach or learn math, whether in a pure, mathematical context, i.e., a math class, or supplementary to a general context, for example, a discussion about symmetry in a course on the history of art, the nature of the math can be significantly different [1]. Yet, we are not always aware of this. Understanding what, how, and why we teach math in different learning situations is important for individual educators and whole organizations [2]. Individual educators can use this information to better design their lesson plans and improve their instruction [3]. On the management level, organizations can benefit from this analysis to make better fact-based decisions based on the difference between the desired policies and the existing practices [4]. This paper presents a tool for educators, organization leaders, and stakeholders to analyze the nature of the math taught in many learning situations. These can be short-term, such as classroom lessons and extra-curricular activities, or long-term, such as learning units, semester-long courses, or even learning that spans several years. We will show how this tool has been successfully implemented in different learning situations in a large science education organization [5] and discuss how to further use this tool in other disciplines.

Keywords: *Mathematics Education, Analysis Tool, Curriculum Assessment*

1. Introduction

Thinking about the content matter being taught, why it is being taught, for what purpose, and how it is being taught is crucial for effective and meaningful instruction in education. By examining the content matter, educators can ensure that it aligns with the goals and standards of the organization and that it is relevant and meaningful to students' lives [6]. Understanding the purpose of teaching the content helps to guide instruction and ensures that it is in line with the educational goals and objectives [2]. Reflecting on how the content is being taught is critical for effective instruction, as different teaching methods can impact students' engagement, motivation, and learning outcomes [7]. For example, a student-centered approach that encourages interaction, exploration, and hands-on learning may be more effective than a lecture-based approach that relies on passive listening [8]. By considering what content matter is being taught, why it is being taught, for what purpose, and how it is being taught, educators can make informed decisions that support student learning and promote academic success [9]. Hence, analyzing the “What?,” “Why?” and “How?” of instruction is beneficial at many levels of any educational institution: the educator who is looking to improve his or her instruction, project managers, and even the organization's management to ensure that programs align with the organization's goals and standards. There are many ways to assess programs [10]. However, online tools capable of processing data from diverse programs and activities that can then graph the data to create compact visualizations of both individual programs and all the programs, are scarce. In the following sections, we describe ISAT (Instruction Self-Analysis Tool), a tool we developed for analyzing instruction and its application to math activities and programs in a science education organization.

2. Description of the Tool

ISAT is a visual, online analysis tool designed for educators to self-reflect on their instruction and for organizations to get a broad understanding of the nature of a subject that is being taught in different programs and whether it is aligned with organizational goals. It is very easy to implement and produces a smart, fast visual analysis of multiple activities, providing information spanning six methodological themes grouped by pairs into three sections. The tool comprises a table where the rows are distinctive features regarding the instruction, and the columns are the educator's scores for each question (Figure 1). The number of features and their definition can be defined in advance by the organization. The table can be used to analyze a single activity or a whole program of any length. The rows are divided into three sections: “What?”, “How?” and “For what purpose (why)?” We prefer to use “For what purpose?” instead of “Why?” to emphasize that we are interested in educational goals rather than causes. Each section is subdivided into a list of features according to the question, one row per feature, where each feature can be characterized along a scale that has two extreme points, written as the right and left row header. For example, in the “How?” section, a feature might be a scale going from “passive” to “active”. The scale itself is defined along the six score columns in order from left to right: “main feature”, “somewhat”, “slightly leans towards” (the feature on the left), “slightly leans towards”, “somewhat”, and “main feature” (of the feature on the right). We purposely chose an even number of columns (six) so that the educator is forced to make a choice and cannot stay neutral by choosing a score in “the middle”. The educator reflects on the different features of the activity or program and clicks on the appropriate score for each feature. For example, in the “active” versus “passive” feature, the educator chooses the score after reflecting on the interaction between the instructor and the students. If the activity is a lecture, then the students are passive, so passive instruction is the “main feature” of the activity, and the cell on the extreme right would be clicked. On the other hand, if there is some questioning and answering during the class, the score should be: “slightly leans towards” passive instruction. Of course, an activity with greater student-teacher interaction would call for a score on the row's left (“active”) side. Once the instructor finishes filling in the table, line segments are drawn, connecting the marks from each feature, using different colors for each section. The resulting image is a graphic representation that gives the individual instructor a birds-eye view of the features of the instruction being analyzed. When ISAT is applied to a group of activities, courses, or programs, the collective graphs can be examined and compared to find trends. This can be very useful for decision-makers in educational organizations.

3. Applying ISAT to Individual and Organizational Math Activities

In this section, we describe the use of ISAT to examine math education in a large science education organization in Israel. We decided to focus on programs that teach math, at least to a small degree, since math is one of the most difficult subjects to teach [11] and has always been a challenge for educational organizations. The expectations were that the insights provided by ISAT would help us understand what math is being taught in the different programs, how and why it is taught, and whether what is actually going on is aligned with the organization's philosophy and policies. The organization’s aim is to promote scientific literacy for all and encourage students of all ages to pursue scientific careers [5]. It offers a wide variety of programs, including workshops, after-school clubs, curricular and extra-curricular courses, and teacher training. These programs are diverse in many ways. Some are focused on science, so the math curriculum within the program is scarce, while others are math programs per se. The length of the programs ranges from one-time activities to programs that span several years. The target audience of each program is very specific. Some programs target teachers, others cater to average K-12 students, and there are programs for underachievers, overachievers, adults, and the general public [3]. The organization purports to use state-of-the-art science education pedagogy.

The first step in using ISAT is choosing the features that we wish to analyze – the “what”, “how”, and “for what purpose” questions. We chose two features for each section, totaling six questions. The first “What” feature we were interested in was how much the content taught in each program was aligned with the curriculum and how much extra-curricular math was being taught. This is important since in K-12 education, curricular and extra-curricular mathematics serve different purposes and offer distinct learning experiences. Curricular mathematics refers to the formal math curriculum taught within the school day as part of the required coursework, typically focusing on building foundational mathematical knowledge and skills [6]. Extra-curricular mathematics encompasses activities and programs that are outside of the regular school day, such as math clubs, contests and enrichment programs, that focus on recreational math [12], and the interconnection between math and the arts [13,14]. The second “What” feature we were interested in was whether the content was mathematics or meta-mathematics. The subject matter of mathematics and meta-mathematics differ in their focus and objectives. Mathematics is the study of numbers, quantities, and shapes and the relationships between them, including arithmetic, algebra, geometry, and calculus [6]. Meta-mathematics is the study of the foundations, methodology, and philosophy of mathematics [1]. It encompasses the history, epistemology, and sociology of mathematics and its role in society [15]. By engaging in meta-mathematics, students can develop a deeper understanding of mathematical concepts from a historical perspective and broaden their perspective on the field of mathematics and its place in the world.

The “How” questions that we chose were focused on two aspects: how “rigorous” is the math instruction and how active are students in the learning. Regarding the former, there has been an ongoing debate over the best approach to teaching mathematics: a rigorous, traditional approach or a more narrativistic approach. The traditional approach emphasizes the development of mathematical skills through repetitive practice and drills, with a focus on formulas and algorithms [6]. In contrast, the narrativistic approach [12] prioritizes broad mathematical understanding by emphasizing the story behind the math, connecting the math to real-world situations and to other disciplines, and encouraging students to make sense of mathematical concepts. When teaching proofs or theorems, for example, this usually means understanding the "big picture" of the theorem or the proof, sometimes with the cost of leaving some of the technical details unexplained or undeveloped.

The second feature in the “How?” category was “active” versus “passive” learning. Active, hands-on learning refers to educational methods where students actively participate in their own learning process through engaging in practical activities and experiences [16]. These activities can include using manipulatives, playing math games, solving real-world problems, and engaging in projects. In contrast, passive learning occurs when students passively receive information through lectures, reading, or watching presentations. Studies have shown that active, hands-on learning can lead to better long-term retention of information and a deeper understanding of the subject matter compared to passive learning methods [17].

The features that we chose to analyze within the “Why” section are perhaps more philosophical, yet they are indicative of the organization’s instructional design. Mathematics can be studied for both intrinsic and extrinsic purposes. On the one hand, mathematics can be viewed as a tool to solve practical problems in various fields such as engineering, science, and finance [18]. In this perspective, mathematics is learned for an instrumental purpose and its value lies in its ability to help mankind achieve specific goals. On the other hand, mathematics can also be seen as a subject in its own right, with intrinsic value that comes from the beauty and intellectual challenge it offers [19]. In this view, mathematics is learned for the sake of mathematics itself and the enjoyment of exploring abstract concepts and solving complex problems. This was the first feature we asked instructors to assess within this section.

The second feature asked whether instructors designed their courses in order to learn one math topic in depth or whether they scanned a broad range of mathematical topics. Learning and teaching one topic in depth can lead to a greater understanding and mastery of the subject [20], so students gain a thorough understanding of the topic. Learning and teaching a broad range of mathematical topics can provide students with a wider perspective of the subject and its connections to other areas [4]. However, this approach can lead to a lack of depth in understanding and a shallow grasp of the topics covered. Teaching one topic in depth requires a significant amount of time and resources, whereas a broad-ranging approach can be more manageable within limited time constraints. The former approach may also lead to student boredom, whereas the latter may not provide enough time for students to fully understand the material.

Figure 1 shows the main screen of ISAT after being calibrated to the features described in the previous paragraphs and after the educator has recorded his or her assessment of these features for their math program. Each section is described by a unique, colored line segment connecting the educator’s choices for each of the two features described in the section. The red line segment describes the “What?” features. In the specific example in Figure 1, the educator reports that the program is highly non-curricular and focuses solely on mathematics (as opposed to meta-mathematics). The green line segment addresses the “How?” features. In the example, the educator’s program leans slightly towards being rigorous (as opposed to narrativistic) with somewhat active learning of the students. The green line segment, “For what purpose?” tells us that the program is somewhat instrumental; that is, the math is learned in order to pursue some specific aim and involves learning quite deeply about the topic. The shape of the three line segments is the self-analyzed characteristic feature of the program. They can be overlayed to present a concise image that can be quickly interpreted. Figure 2 shows an ensemble of ten such images obtained by applying ISAT to ten different programs of the organization. Interestingly, although there was no official directive from the organization regarding what, how, and why math should be instructed in the programs, and even though the programs were diverse both in the target audience and length of time, there are some distinct features that are similar in all the programs. Note that the red line segments in all the programs tend to be upper-right to lower-left diagonals. This means that the organization leans (heavily) towards non-curricular mathematics, dealing with mathematical content (rather than meta-mathematical). All but one of the green line segments lean the other way, signaling that most programs take a rigorous approach to mathematics, i.e. precise and structured content, but do so with much interaction between the instructors and the students. The features in the “for what purpose?” section, depicted as the blue lines, seem to be unique for each program with no global trends.

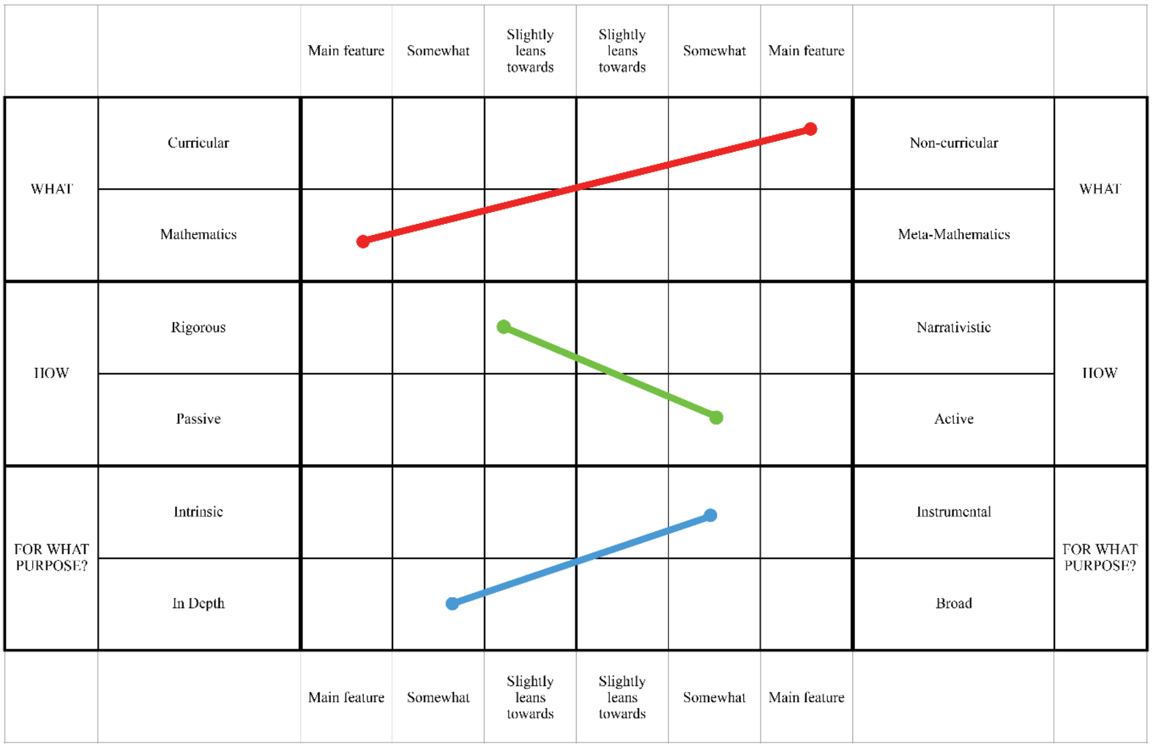

Figure 1. The main screen of ISAT applied to one math activity

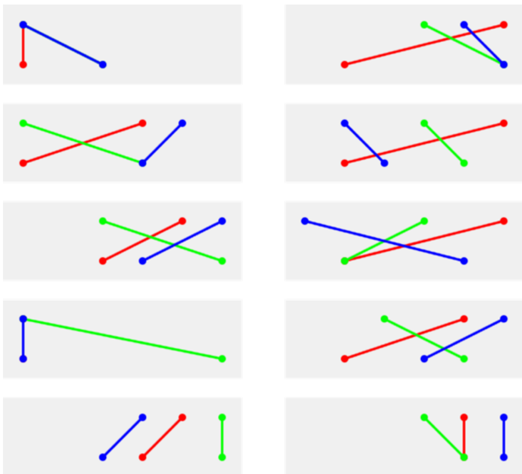

Figure 2*.* ISAT graphs after applying ISAT to 10 diverse programs in the organization

4. Discussion and Summary

In this paper, we described ISAT, an online self-analysis tool that educators use to gain insight into the “what”, “how” and “why” of their own personal instruction in any given activity. ISAT also produces a map of the overall characteristics of a subject taught in programs and activities *throughout* an organization by creating a concise, distinctive visual image for each activity. Using ISAT to examine math activities in a large science education organization gave program leaders the opportunity and information needed to reflect on their instruction, one of the most important steps to significant improvement of students’ learning. Moreover, the leaders could see if their instruction is aligned with what they intended to teach within their activities. On the organization management level, the organization learned that their math curriculum is very much focused on non-curricular, mathematical content, taught rigorously but in an active way. Whether or not this is the intended direction the organization wants to go is up to the managers, but the information gained provides valuable insight.

The organization might also want to rethink whether it is acceptable that different programs have different purposes regarding their contents, a fact that also arose from the ISAT analysis.
Like many assessment tools, educators have to be aware of some “pitfalls”. First, the tool is administered by educators or program managers on an individual base. Since this involves personal impression, it is influenced by positions, perceptions, and mainly preferences and biases of the person conducting the reflection. Therefore, the gaps between the evaluation that one person will give and the one that another person will give on the same program can be significant. Care must be taken to properly understand the features and the sometimes-subtle differences between categories. On the organizational level, one must be aware that the map compares different programs on the same footing, and this might lead to misconceptions. Among other things, the visualization in its present form does not reflect the difference in the number of participants in each program or how long the program runs, or how important the program is for achieving organizational goals. The organization might mistakenly decide to change course based on a trend that is apparent only in short-running programs or those with a small number of participants. We have rectified this to some extent by adding supplementary information fields to the tool that users fill in. The fields can be defined by the user prior to the initial use of the program. For our organization, we asked users to fill in the length of the activity and the number of participants. These properties were then displayed beneath each graph. However, one can think of better ways to do this visually. Instead of just displaying the information as text underneath the graph, we could assign a score that reflects the weight we want to give a program based on the information on these fields and then display this visually, for example, by adjusting the opacity of the graph. This way, for example, a short-term program with a small number of participants could be given a low score, and its graph would be more opaque and draw less attention.
ISAT is a very versatile tool. Different versions of the program can be easily duplicated from the original to accommodate different scenarios. For example, ISAT is easily modified to input any reasonable number of features in the “What?”, “How?” and “Why?” sections, and what these features are, are also adjustable in advance. In its current form, ISAT provides the backbone for a collection of assessment tools. Versions of ISAT for analyzing features of science education instruction and assessing innovation in programs and activities are currently being administered for use in the organization.

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