Education for a Discussion of Analytical Geometry in High School Based Vectors

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Abstract
Due to the great problem of learning in universities that has been growing regarding to the teaching of analytic geometry, it is of great interest to teach within an approach that makes it closer to reality, which is able to attract the interest and leads to the understanding by the students since they arrive at High School. We are also concerned with the teaching methodology, as the university is the repository of knowledge acquired by human effort over the centuries, and these skills can help, even within a topic with an established designation, relax and discuss the various interpretations of its possible content and its various ways for presentations has your importance. In this sense, we proposed here suggestions for the teaching of geometry in High School, trying to answer and discuss the differences between the analytic geometry studied in Higher Education in the exact areas as well as in High School.
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1. Introduction
Unsurprisingly affirmed that education in basic education in Brazil faces serious problems. When trying to identify some of the factors that contribute to this situation is common to find a lack of physical and technological structure of schools and teachers poorly qualified and poorly paid, unmotivated by indiscipline and disbelief reigned among students, especially those in public school.
Another factor that seems to have the potential to maximize the problems encountered with the teaching of mathematics in basic education, strengthening the belief in the almost impossibility of learning the content of this course is on the structuring of the high school curriculum, which can also respond by disinterest on the part of students. The high school curriculum is composed of a large program, fragmented, which makes it impossible to work the teacher so investigative content, otherwise you cannot complete the planned for that series.
In principle, such a proposal seems legitimate, even if one takes into account that, when starting school, the student is too young to be able to objectively decide on the career that will follow. However, the results observed in the early years of the top courses in the areas of sciences reveal that the result achieved with this broad curriculum is far from achieving its goals. The vast majority of public school students are leaving school without this level of minimal knowledge of mathematics needed to continue their studies in this area.
In the specific case of analytic geometry, the focus of this article, the possible applications arising from this study may, at first sight, justify the need to stay on the list of contents, since the teaching of geometry is of great interest nowadays. But, although it is unquestionable the importance and use of geometry in the contemporary world, we must emphasize that these applications come from the study of geometry on the upper level of analytic geometry and not listed as content in high school.
Given these arguments, we put the research question that guided the entire literature search done for this article:
What kind of analytical geometry has figured in high school curriculum?
Seeking to answer, we propose a new way of teaching of analytic geometry in high school, with the goal of improving student learning and consequently that in higher difficulties encountered today are minimized due to better organization of the contents worked.
We think that in this being done, we are contributing to a reflection in teaching analytic geometry, seeking to improve the quality of teaching and learning this content, consistent with its usefulness in the contemporary world, making it therefore more motivating and interesting to students.

2. The Analytic Geometry in High School Based on Vectors

Teacher Márcio Antônio de Faria Rosa of IMECC (Institute of Mathematics, Statistics and Computer Science), State University of Campinas, provided a summary of the discussions held in your study group on teaching Analytic Geometry site called Web of Knowledge Analytic Geometry Actuality and Vectors and Simplicity in Teaching Geometry.
These discussions led to the conclusion that one should teach analytic geometry with vectors instead of analytic geometry Renaissance. According to [3], analytic geometry makes use of two orthogonal axes to connect the concrete geometry to two somewhat abstract branches of mathematics, arithmetic and algebra.
We will discuss in this paper the proposed Rosa for teaching of analytic geometry in high school from the vectors.
To start the study of content, which in our view should be worked in the first year of high school, before the contents of functions, presents the Cartesian plane, name associated with Descartes for his having been responsible for the sign convention for coordinates of the points below the negative x-axis or left y-axis. From the use of the system orthogonal axes, each point to associate a pair of real numbers called coordinates of the coordinate system fixed. Thus, we must draw attention to the fact that the point, which is the object of study of geometry, is associated with a pair of numbers, which are objects of arithmetic and algebraic manipulation. A simple arithmetic operation performed to the point represented by the ordered pair (x, y) will change this point of position in space. For example, if we add a unit to the ordinate of the point (1, 2) have vertical movement of a unit of measure for the point, which will now represented by a pair (1, 3). For a good fixing of initial content of analytical geometry can propose to the students a play known as Battleship, which is a board game for two players, in which players have to guess where coordinates are the ships of the opponent and so destroy them. After this work with the Cartesian plane, one can introduce the concept of vector, which according to the Concise Oxford Dictionary [4], "vector comes from the Latin, one who carries or takes something". The dengue mosquito, for example, is the vector of dengue viruses carrying the disease. Already in mathematics, the best way to define vector is as a transport operation points. The vectors act in the space of points, carrying them straight, this is, the vectors are actions that cause displacements of points. A vector will be denoted by a pair of numbers (vector components) that correspond to the variations that cause the vector to the coordinates of the point that carries. For example, the vector v = < 3, 2 > displaces the point (1, 2) causing variation of 3 and 2 units in the coordinates of the point in the turning point (4, 4). Geometrically we have:

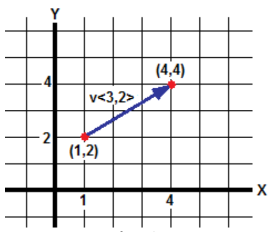

Fig. 01

At this point, one must draw attention to the fact that the line segments oriented in the plane are not drawn vectors, but each is a representation of the action of the vector v. Each vector has infinite oriented segments that can represent it. It would be interesting math teacher make a comparison with the vectors worked in physics classrooms. It is known that the discipline of physics, at the first year of high school, starting with the contents of kinematics, which deals with the study of bodies in motion, first without analyzing the causes a body to move. Then the dynamic content are worked and at this time, the study of forces acting on a body is performed. For this study, it is necessary to use vectors that represent the various forces, and a knowledge of some elementary operations with vectors is vital for the learner to determine the resulting forces on the body. It appears a great difficulty for the students to solve the problems of dynamics for lack of more knowledge about vectors. It seems appropriate that mathematics can work the relevant content, broadly, that will be used in physics, helping to interdisciplinary as desired.
Before starting the elementary operations with vectors, it is essential that the student understands the concept of vector. The previous example on the mosquito vector of dengue, helps the understanding, however, a formalization for vectors is necessary. It is extremely important that students know that a vector is only defined when the following quantities are defined: direction, meaning and module. The direction of a vector in the Cartesian plane is given by the position of the line that contains it in relation

to the coordinate axes. The angle formed by this line and the x axis is sufficient to determine the direction. The sense to indicate which "part" vector will be oriented because, given a starting point on any line that contains the direction, there are two oppositely below. Finally, the module provides information about the size of the vector in question. As already discussed, there are infinite vectors with these characteristics, and use one of them as a representative.
The vector will also be set to inform the coordinates of its end, beginning with the source. In the example illustrated above, the vectors can be defined as v = <2, 4>, even though none of them are designed to start at the origin. This concept must to be working very well, so that the student can understand that such vectors can be positioned elsewhere in the plan, without losing their integrity and thus allow proper repositioning the elementary operations, which we will discuss.

2.1 Elementary operations with vectors

The first operation is the sum of vectors. Initially, the procedure for such operation is through the graphical method, with the main objective to verify that the concept previously mentioned about the change vector in the plane was assimilated. We start then a simple problem, suggesting a shift in two successive steps. From a point P = (-1, -2) we will first displacement vector according v1 = <3, 1> and soon after, the second offset corresponding to the vector v2 = <2, 4>. Note that the vectors are understood as agents that displace the point. The figure below shows the results obtained and displacements corresponding to the arrival point A = (4, 3):

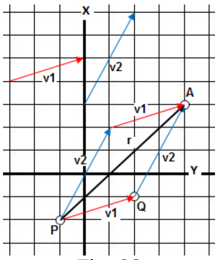

Fig. 02

We emphasize that the vectors v1 and v2 can be anywhere in the plan. Write a representative of v1 with its beginning at the point P. After undergoing the first displacement, the coordinates are those of the arrival point Q = (2, -1). The new displacement leads to the arrival point, A = (4, 3). We can verify that corresponds to the displacement effected caused by the same vector r = <5, 5>. Thus, it appears that such vector r is the sum of the previous two, v1 and v2. This same method can be generalized to *n* vectors. Taking advantage of the previous figure, we can see that the total displacement can be obtained by performing first displacement caused by v2 and then caused by v1, thus confirming that the vector sum enjoys the commutative property. The teacher can illustrate this idea to the student using the example of a bank account that undergoes successive deposits and withdrawals, since within the limits available for this account, and regardless of the order in which deposits and withdrawals are made, the final result will be the same.
From the realization of the geometric model, the students will understand how to get the coordinates of point A algebraically, by doing the following:

$$v_1 + v_2 = <3, 1> + <2, 4> = <3+2\ ,\ 1+4> = <5, 5> = r$$

We define the vector sum $\boldsymbol{v} = <v_1, v_2>$ with $\boldsymbol{w} = <w_1, w_2>$ by:

$$\mathbf{v} + \mathbf{w} = <v_1 + w_1, v_2 + w_2>.$$

Finally, just use the vector r obtained to move the point P to point A:

$$P + r = (-1, -2) + <5, 5> = (-1 + 5, -2 + 5) = (4, 3) = A$$

The addition operation, we can study the subtraction of vectors. For this, we define the vector opposite. Such a vector can be understood as the vector that exactly undoes the displacement caused by another. Thus, given a vector v = <a, b>, its opposite vector is commonly denoted by -v = <- a, - b>. Thus, subtracting a vector sum is the opposite of this vector. Note that the direction and length are the same between the vector v its opposite -v, but occurs in reverse order. Thus we have:

$$v + (-v) = 0 = <0, 0>$$

Another basic operation is the multiplication by a scalar, a vector, or by a real number. We can think of defining twice the vector v = <a, b>, which would cause the effect of the displacement v performed twice repeatedly. It would be natural to denote 2v, or

$$2v = v + v = \langle 2a, 2b \rangle$$

Geometrically, the vector 2v has a representative below:

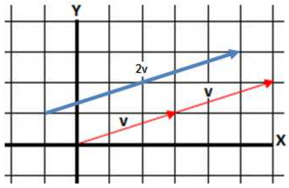

Fig. 03

Similarly, γv = <γa, γb> would cause the effect of this vector when applied γ times. And this is how we define the product of a vector by a scalar γ E R.
It is important to conduct a study on the effects caused by various scalar, positive or negative, with modules or larger than 1 in the range between 0 and 1. Clearly a geometric software can be used in the consolidation of such operations, expanding our understanding, the ability to view, and above all, serving as enabler of interest to the subject worked.
These are the most commonly performed operations with vectors, and of such operations is essential to work with dynamics problems in high school and will also be of great value to the work of this discipline at the top level.
In many situations, especially in problems of physics at the first year of high school, the vectors are not expressed by its coordinates, but by their length (also commonly called intensity, clear reference to the dynamic forces used). Thus, the student should be able to use the information available to solve a problem, and this time, we believe we will study another important aspect: the orthogonal decomposition.
So far not discussed the module vector, this is, on its length. How to determine the length of the resulting vector, or in other words as calculating the distance between the point of departure and point of arrival? The vector sum and scalar multiplication we allow the following operations:

$$v = \langle 4, 3 \rangle = \langle 4, 0 \rangle + \langle 0, 3 \rangle = 4 \langle 1, 0 \rangle + 3 \langle 0, 1 \rangle \qquad (1)$$

On the first pass (manipulation done in (1)) we write the displacement v = <4, 3> as the vector sum of two displacements. The first is the offset <4, 0>, parallel to the x axis, which just adds to the 4 x coordinate of the point, without changing the value of the y coordinate. The second is the offset <0, 3>, parallel to the y axis, which only adds 3 to the y coordinate of the point, without changing the x coordinate. This first manipulation is called orthogonal decomposition of the vector v.
Note that regardless of the starting point, the breakdown between the vectors obtained by orthogonal decomposition will be the same. Now, at this moment, the student himself is able to establish a way for determining the modulus of the vector. Suffice it to check, and for this nothing better than the graphical method, the vector <3, 4> is the hypotenuse of the triangle whose other two sides are <3, 0> and <0, 4>. Thus, the length or norm, a concept that can be introduced at this point to describe the size of the vector will be:

$$||v|| = (3^2 + 4^2)^{1/2} = 5$$

In the second manipulation performed in (1) wrote:

$$\langle 4, 0 \rangle = 4 \langle 1, 0 \rangle \text{ e } \langle 0, 3 \rangle = 3 \langle 0, 1 \rangle.$$

The vectors <1, 0> and <0, 1> cause unit displacement parallel to the x and y axes respectively. For historical reasons these vectors that carry unit displacement given denomination *i* = <1, 0> e *j* = <0, 1>. For whatever the vector v, we can make the manipulation (1) and get:

$$v = \langle a, b \rangle = \langle a, 0 \rangle + \langle 0, b \rangle = a \langle 1, 0 \rangle + b \langle 0, 1 \rangle = a_i + b_j.$$

Multiple combinations of *i* and *j* originate from any vector. We say that the set {i, j} forms a basis for the two-dimensional vectors. From these two vectors, with the use of the product by a scalar and vector sum, all other vectors are constructed. We can observe that

$$||\gamma v|| = ((\gamma a)^2 + (\gamma b)^2)^{1/2} = |\gamma|.((a)^2 + (b)^2)^{1/2}$$

Thus, the magnitude of a scalar product of a vector is the scalar product of the module by the module of the vector. It is worth to use different scalars for the study of standards obtained using, again, a geometric software. It is hoped that the students realize that when using scalar whose magnitude is greater than 1, there is an expansion in the vector and the scalar case has modulus less than 1, there is a contraction of the same.
We have seen that a vector is fully defined when we know three quantities, one of which direction. We can establish the direction determining the angle formed by the vector and the coordinate axes. The following figure shows the relationship between the orthogonal decomposition and the angle θ.

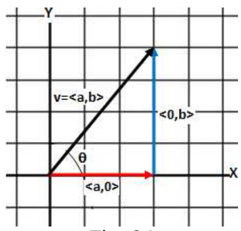

Fig. 04

Using the trigonometric ratios in a right triangle, we believe that students are able to relate the angle θ with the vector v orthogonal vectors obtained from the decomposition of v, since such relationships have been studied in the last grade of primary school.

$$\text{sen}\theta = b/\mathbf{||v||} \rightarrow b = ||v||.\text{sen}\theta \quad \text{and} \quad \cos\theta = a/\mathbf{||v||} \rightarrow a = ||v||.\cos\theta$$

If we consider the action of the vector v = <a, b> the origin of the coordinate system, taking the point (0, 0) to the point (a, b), the direction θ can be interpreted as the angle between the x axis and arrow representative action vector, measured in a counterclockwise direction.
However, we can always use the angle between the vector v and its horizontal projection <a, 0> and adopt the sign convention of the Cartesian axis.
So be adopting θ from the first quadrant or as angle between the vectors (where v is in the second quadrant) will always be possible to determine the direction of the vector from its coordinates, and indeed, we can decompose v in their orthogonal components always know that their standard, direction and sense, which, in the case of physics will be widely used in solving problems. We believe that this introductory notion about vector may impact on the study of the dynamics and in the future, facilitate the introduction of analytic geometry in higher education, for those who embark for the exact sciences.

3. Final Thought

Whenever we act on a more didactic teaching, we suggest methodological changes that influence and interest to the student. If our goal is to teach students to analytic geometry concepts considering the vector, then the proposal is that we start at school. If we want to offer more to students, there is no doubt that it is necessary to seek changes to our teaching, whether, primary, secondary or higher. Although many concepts of analytical geometry being abstract, we apply techniques that involve some simple math to solve some practical problems, the application of the study of vectors.
This curriculum is still in the implementation phase, however, is of utmost importance to be heard and seen by the teaching community so that it can be improved and secure a significant space in the current curriculum in mathematics education, because the seek benefits for there is the process of teaching and learning, we correlated subjects equivalent of analytic geometry in high school. Proposed activities should give margins of many observations, providing discussions between teacher and students. This helps students to draw their own conclusions regarding the mathematical content of each activity, helping to develop in them a logical and critical spirit, beyond the vision that offers dynamic geometry, making a link with math facts that, for the student, is a factor of discomfort when this process does not occur.

Regarding the willingness and purpose of the activities, we are absolutely sure we do not cover all situations relating to vectors, however, the proposals offer the creation of other activities that complement these initials.

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