Tangibles, Construction of Meaning and Math Problem Solving

[1]Amparo Lotero-Botero, [1]Edgar Andrade-Londoño, [1, 2]Luis Alejandro Andrade-Lotero

Alandra- investigación educativa (Colombia), [2]Indiana University (USA)
a.loterobotero@alandradifuciencia.org, edandrade@alandradifuciencia.org,
al.andrade@alandradifuciencia.org

After experiencing during 7months a program designed for development of mathematical thinking in basic primary education, named *Math in Color*, a sample of 75 second –grade and 60 third-grade students took a test intended to measure their progress in word problem solving. These results were contrasted with those of a similar sample of students who had not experienced the program. Statistical analysis showed that experimental groups scored higher than the control groups and that the differences were significant.

Another outcome of these tests, which included a questionnaire to assess reading comprehension, was that there were no significant differences in this respect.

Closer examination of students' work showed:

- Students scoring poorly in reading comprehension could not solve the math problem. However, not all those who scored acceptably or higher could solve the problem. Reading comprehension does not necessarily led to problem solving.

- Characteristically, students of control groups set out to try arithmetical operations with the quantities provided in the problem, whether they made any sense or not (in most cases it did not). In contrast, most of the students in the experimental groups started out to make drawings attempting to represent the variables and their relationships, as derived from the word problem enunciation.

Above outcomes suggest that the focus of traditional math education on symbolic algorithms leads to "meaningless" operativity, since students appear to understand problem-solving as *applying* some operation.

Instead, the focus of the *Math in Color* program is on student´s construction of meaning, both out of everyday-life situations involving quantification/measuring as well as of mathematical symbolism. Algorithms here are not a starting point, but rather the final stage of a process of constructing meaning.

Starting at modeling with tangible objects, every student works on his/her own, following instructions on a work-notebook, and is led to experience by herself and represent in drawings, words and symbols:

- Numbers as representing quantities sharing some attribute. Any quantity may be increased by adding another amount, or decreased by taking out. So, adding/subtracting are seen, firstly, as transforming actions of a subject.

- These actions take place along a timeline of events: What it is at first, the transformation (taking in/taking out), and the resulting amount. The sequence is reversible and flexible, but, at the end, quantities need to be conserved.

- There is a special case in this composing/de-composing of quantities: grouping so that each group contains the same quantity as the others. Here, three different quantities arise: The quantity of things in each group, the amount of groups, and the total quantity. This is also a reversible sequence: adding groups so many times ends up in total amount, which can also be decomposed in so many times of groups containing such amount of things each.

- How many times a group needs to be taken, is indicated by a correspondence relationship between the set of things contained into every group and a group of reference. So, there is a new level implied: while adding/subtracting takes place into the same set of things, multiplication/division requires this set of reference stating how *many times.*

- Finally, problem situations arise in those types of sequences when there is an unknown quantity, whether any of the moments in the timeline of events, or any of the three quantities involved in grouping. However, experiencing with tangibles and representing those sequences as reversible allowed students to: a) identify which one is missing; b) how this unknown is related to those quantities that are known; and, c) how this unknown could be found.

These features of the *Math in Color* program seem to explain above outcomes. The final paper would present in detail the analysis, examples of how the *Math in Color* program treats above aspects, student's work samples and would discuss conclusions for the future of education, as derived from this experience.