



The Future of Mathematics Pre-University Education (With an Emphasis on Iranian High School Educational System)

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1. Introduction

Since 1967 Iranian pre-university educational system has been divided into three separate periods of education: "Elementary School" (5 years), "Middle School" -or "Guidance School"- (3 years), and "High School" (4 years). Although in early 1990's, by the change of educational system, the high school period has been reduced to 3 years and a two-semester (one-year) course called "Pishdanehshgahi" –means pre-university– was added instead of high school's fourth grade, but in this paper by the word "Pre-university" we specially mean the whole 4 years of high school period in both two educational systems since 1967.

The textbooks in the new high school educational system -since 1990's- and the older one –1967 until early 1990's- are totally different, so we examine them separately in the next two sections of the research. From now on in this paper we call the older high school educational system, "the previous system" and the newer high school educational system, "the new system". In the last section of the research we express the difficulties and deficiencies and suggest some practical changes to get the textbooks more practical and useful for nowadays life and science.

2. Previous system

When we talk about the textbooks in the previous system, the main theme would be that they're foul of mathematical methods, and they're very attentive to give the exact mathematical definition of any subject. We take a look at each grade's textbooks.

2.1 First grade

There are three textbooks: "Algebra", "New mathematics" and "Geometry". The most basic subject of this grade is algebra, which contains the basic properties of four principal operations on real numbers, first order equations with one or two indeterminate(s), and the properties of inequalities and inequations. "New mathematics" includes an elementary introduction to sentential logic and set theory.

The last, but not least is "geometry". First comes a review of the geometric information, students have gained in the past, but in a more mathematical manner. The geometric objects, such as line, line segment, triangle, rectangle, quadrangle, parallelogram, trapezoid and circle, inner and outer polygons are precisely defined and their properties are studied deeper. At the end of the book, there's a short biography of a few mathematicians, who had had a fundamental role in developing geometry, such as Archimedes, Pythagoras, Thales, and specially biographies of Riemann and Hilbert, which motivates interested students to do more studies in modern geometry and mathematics.

2.2 Second grade

"Trigonometry" is the textbook added to the educational content. It starts from the definition of an angle and its measurement, trigonometric circle and the trigonometric ratios, followed by their calculations, trigonometric identities and equations, and at the end of the book there's a trigonometric table. But there's an obvious lack of the variation graphs for trigonometric functions, when we look at the textbooks of all four grades, we don't see any graph of trigonometric functions either. Although, relations and functions are simply introduced in "new mathematics" this year, but the exact definition of a function and its graph is postponed to the third grade. Also matrices, vectors, and a chapter on "Group Theory" are included in this textbook. As we see "Group Theory" is more complicated for common students than the graphs of trigonometric functions.

"Calculus and algebra" is the next text which is mainly about second order equations and logarithms. There's a brief history note on logarithm invention, but it says nothing about how logarithm was invented and why; which makes the note totally unhelpful.

"Geometry": Thales' theorem, similarity, Pythagorean Theorem, regular polygons (inner and outer), equations for lines and planes in three dimensional space, crossed and noncrossed lines and planes. One of the best statements in this text is a definition for "radian", but 3 space equations and problems is too difficult for students who are not expert enough in 2 space analytic geometry, yet.

2.3 Third grade

"Algebra": division of polynomials, intervals, functions, limit and continuity, derivation and its applications. There's an obvious fault in chapter 6, where the sample images are always curves, while curve of variations of a function introduced in chapter 7, and students only know the linear graphs.



"New mathematics", more than always includes a large variety of contents: vector spaces, Boolean algebra, combinational analysis, probability, descriptive statistics and linear programming. However linear programming is an applicable subject, which is eliminated from the new system's textbooks.

"Trigonometry": contains a review of last year with more complicated equations, and there are three interesting and useful chapters about map reading, monitoring and calculating trigonometric statements by using logarithms.

2.4 Forth grade

"New mathematics": sentential calculus and logical deductions are studied deeper than the first grade.

"Analytic geometry": point and line in 2 and 3 dimensional spaces, circles and other conic sections.

"Algebra and analysis": real functions, limits, derivable functions, graphical representation of a function, number of roots for a polynomial, approximation of the roots for third order equations, and integration. Considering the student's knowledge so far, the text is so easy at the beginning, but becomes more and more complicated as the time passes. By summarizing the first five chapters, which is repeated from the third grade, and elimination of chapter 6, only differentiation and integration remain, and it saves more time for students to learn the subject and become ready for university.

3. New system

Maybe the most essential crisis about the textbooks in the new system is repeating subjects, such as methods of reasoning, probability, line and plane in 3 space, derivation and its applications. In comparison with the previous system, the texts have become more applicable, but the quality of the subjects and exercises decreases.

3.1 First grade

"Mathematics 1": real numbers, sets and set operations, polynomials, algebraic identities, first and second order equations, line equations, trigonometric ratios, rational statements and first order inequations.

3.2 Second grade

"Mathematics 2": rational and irrational equations, sign of polynomials, relations and functions, translation of axes, matrices and linear systems with two indeterminates, curve of variations for linear, logarithm and trigonometric functions, sequences, geometrical and arithmetical progression, trigonometric identities, vectors, combinational analysis, probability.

"Geometry": geometry and reasoning methods, congruent triangles, similarity, lines and planes in 3 space, area and volume formulas, cavalieri's principle.

3.3 Third grade

"Calculus": functions (review), limit, derivation and its applications, definite integral and the area of the region under the graph of functions.

"Geometry 2": reasoning methods in geometry (induction, deduction, contrary hypothesis and contrapositive), circle and the relevant angles and lines. Homogeneity, translation, rotation and reflection operators, three dimensional analytic geometry of lines and planes.

"Algebra and probability": again reasoning methods, Cartesian product, relations; and two chapters on probability.

3.4 Forth grade

"Discrete mathematics": graphs and their applications, number theory (divisibility, prime numbers, modular arithmetic).

"Analytic geometry": lines, planes, vectors and vector products in 3 space, conic sections, matrix and determinant, linear systems.

"Calculus": real number system, sequences and series, limit and continuity, derivation and its applications, definite and indefinite integrals, fundamental theorem of calculus.

4. Practical suggestions

For the first grade, number theory and modular arithmetic, set theory and operations on sets, are simple and interesting subjects, that motivates the students much more than solid algebra, which included in new and previous systems' first grade. Also the properties of inequalities and first order equations and inequations, linear systems with two indeterminates, are simple subjects that the students are familiar with from the middle school. At this grade, the most difficult but important matter should be algebraic identities, which is to be practiced as much as possible.

The second grade's textbooks must contain an elementary introduction to "Logic", but not sentential logic like the previous system or methods of reasoning in the new system, which are both unhelpful for the students. By this introduction to "Logic", we mean an initial step in "Computability theory" and "Group theory" -which based upon the number-theoretic information, the students had gained last year- and "Axiomatic set theory" accompanied with a brief history of set theory's development in the last century (Russell's paradox and Frege's answer to that paradox...), which is simple, exciting and very motivating for students at this grade, who have already chosen to get their diploma in mathematics.



Also a short history of "Euclidean geometry" (Euclid's five postulates and the parallel axiom's crisis, which led to invention of other geometries), will be a more instructive lesson for students than reviewing methods of reasoning in new system, to learn how the reasoning methods actually work. Geometry at this year must continue with similarity, Pythagorean Theorem, line and circle equations in 2 space.

Besides, some algebraic subjects such second order equations, the definition of a function and graph of the linear functions must be taught, as well as trigonometric ratios/functions, some simple trigonometric identities and the curve of variations for $\sin(x)$ and $\cos(x)$.

This would be continued at the third grade with more trigonometric identities and exercises, logarithm and exponential functions and their graphs; Followed by derivation and some of its initial applications like determining the sign of a polynomial over specific intervals, extremums of functions and drawing their graphs.

Learning about matrix, determinant, vectors in 2 and 3 spaces and the vector products, will increase the special sense of the students at this grade.

As for the fourth grade, lines and planes in 3 space, conic sections, more applications for derivation, more complicated functions, differentiation and integration must be included in the learning material in order to get the students ready for their first semester in college. Likewise, some combinational analysis and elementary probability theory is needed to learn, but probability distributions or other complicated subjects in probability theory are not necessary and must be postponed to college education.

References

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