Considering Density through a Numeracy Lens: Implications for Science Teaching

Shelley Dole, Geoff Hilton, Annette Hilton, Merrilyn Goos
The University of Queensland (Australia)
s.dole@uq.edu.au; g.hilton@uq.edu.au; a.hilton@uq.edu.au; m.goos@uq.edu.au

Abstract

Promoting the concept of density is regarded typically as the domain of science teachers. However, density as the relationship between an object's mass and volume highlights the importance of foundational measurement concepts located in the mathematics curriculum. As an intensive relationship, understanding density also requires a degree of proportional reasoning, but research has repeatedly shown that students' understanding of proportion and proportional thinking is generally poor. An instructional sequence for density was developed and iteratively implemented in several middle school (fifth grade to eighth grade) classrooms using a design research approach. This paper presents an outline of the unit sequence and highlights key points within it where particular mathematics knowledge and understanding was required. The capacity of this unit for promoting proportional reasoning is discussed. In this paper, the unit sequence is further analysed through the lens of numeracy. In this analysis, a rich model of numeracy is used to consider the potential of this unit for promoting numeracy, and to draw implications for teaching of density in science with respect to consideration of the numeracy demands of this difficult topic.

1. Promoting understanding of density

The teaching and learning of density has a long and problematic history [1]. Understanding density requires a measure of proportional reasoning [2], yet research has repeatedly highlighted students' difficulties with tasks, topics and activities that require any measure of ratio application and proportional thinking and reasoning [3, 4]. Density is an intensive quantity that requires simultaneous consideration of two measures: an object's mass and its volume. Nunes, Desli and Bell [5] called for greater attention to promoting students' understanding of intensive quantities (and hence promoting proportional reasoning) in the early years mathematics classroom through hands-on learning experiences exploring the inverse proportional nature of intensive quantities (e.g., taste, temperature, density). Sadly, many primary school mathematics classes are deficient in providing hands-on experiences in promoting students' understanding of extensive quantities (length, mass, volume), hence providing little foundation for meaningful understanding of intensive measures.

2. The project

An instructional sequence for promoting students' understanding of density was developed by the researchers, in consultation with project teachers, and implemented in a variety of classroom and school settings with students from Grades 5 to 9. The unit drew upon the work of Adey, Shayer and Yates [6] that advocated using sets of opaque jars. Two sets were created: (i) five jars of the same volume (same size and shape), but of various mass; and (ii) five jars of varying shapes and sizes (different volumes) but all the same mass. Connecting with students' intuitive knowledge of sinking and floating, exploratory activities included predicting and experimenting with various objects before introducing the jars. Students predicted the mass and volume of each jar before measuring, then recorded the information in an organised table where volume and mass could be directly compared. A key teaching point was to focus students' attention on each jar in turn, consider both its mass and volume and then make a prediction of whether it would sink or float. The outcome was recorded. Greater student accuracy in predictions of whether a jar would sink or float occurred: "if the mass is bigger than the volume, it is going to sink...see I told you!" (comment overheard by a Grade 5 student).

Analysing all classroom iterations, the lesson sequence appeared to support students' capacity to use the language of density, but also highlighted the importance of underpinning concepts in mathematics to support students' understanding of density. First and foremost was students' lack of depth of understanding of the concepts of mass and volume. Students were initially hesitant to estimate the
mass or volume of the jars, and clearly had few reference points for considering the units of measure to make sensible estimations. Digital scales were used to measure the mass of the jars and students reacted to the measures they were reading (especially of the smallest jar when its mass was found to be the same as other jars in the second collection), however, they did not link these measures (100g, 200g, 500g) to known objects or consider them in relation to a kilogram. In relation to volume, students in the primary grades had only been exposed to volume through displacement of liquid and hence had some degree of familiarity with liquid volume units of measure. The secondary students readily recalled the formula for volume (of a rectangular prism) but were unsure of how to find the volume of the cylindrical jars in the collection. Familiar units of measure for the secondary students were in relation to cubic volume, but this did not support their capacity to estimate the volume of the jars in cubic centimetres. Another major issue was students’ lack of comparison of numerical quantities in fractional form, which became apparent when students analysed the mass and volume of each jar. Students readily concluded that when the mass was greater than the volume, the jar would sink, but no students made reference to the relative difference between the two measures or the units of measure. There was visible surprise as students used a calculator to determine the density of each of the jars (mass divided by volume) that the measures were greater or less than one.

Although gaps in students’ knowledge appeared as the unit was implemented, adjustments ensured that all students were continually engaged throughout and teachers felt that the unit was successful. As students’ knowledge of the units of volume varied, a specific demonstration of the equivalence of liquid volume (measured in millilitres/litres) and cubic volume (measured in cubic centimetres/metres) was included in the unit via teacher demonstration. A commonly-known container for holding one litre of liquid (milk carton) was filled and then emptied into a cubic measuring container (dimensions 10cm x 10cm x 10cm) to show the relationship of 1000 millilitres (1 litre) to 1000 cubic centimetres. From a set of Dienes (MAB) blocks from the mathematics classroom, a 10cm x 10cm x 10cm ‘thousand’ block was used as a reference point for estimating the volume of common household objects (tins of food from the pantry). For primary students, determining the volume of the density jars was via displacement. The resulting measure in millilitres was then converted to cubic centimetres to match the units they had been using when estimating volumes of household objects. The secondary students measured the dimensions of the jars and calculated their volume using the formula of area of base multiplied by height using cubic units. For determining the actual density of each object, the data table included a column for this purpose with the formula included (m÷V). All students were required to complete this column in the data table using a calculator. The secondary students reported that they knew of the formula for calculating density (although few could recall it prior to the unit); the primary students who had not been introduced to the formula for density, appeared to be pleased at being introduced to something that they “would learn in secondary school science”.

The culminating activity in this unit provided students with hollow plastic cylindrical container with a hole pierced in its side and a metal weight on its end. When first put into water, the vessel would float, but then gradually fill with water and sink to the bottom. The students were required to take a series of photographs of this vessel (named the Titanic) at various stages on its voyage to the bottom of the ‘ocean’, and create a powerpoint presentation that told the story of why their vessel sank in water. The resulting powerpoint presentations indicated students’ facility with describing changes in mass, constant volume and sensible application of the term density.
3. Numeracy implications

As an integrated mathematics and science unit, the unit outlined here appeared to make connections between mathematics and science. However, project teachers openly admitted that they regarded it as a science rather than a mathematics unit. The secondary mathematics teachers did not regard this as something that should be incorporated into their yearly work program. Yet, foundational mathematics knowledge was clearly required. If this is regarded as a science unit, analysis of it from a numeracy perspective highlights its inherent numeracy demands. Numeracy is generally regarded as the sensible application of mathematics in context [8] and promoting numeracy is the responsibility of all teachers, not just teachers of mathematics [9]. A recent reinterpretation of definitions of numeracy has resulted in a rich framework for considering the multi-dimensional nature of numeracy [see 10], and applied here, can show the numeracy demands of this unit of work. The framework identifies key elements for numeracy, with context as central, together with mathematical knowledge (skills, concepts, estimation, problem solving), tools (representational, digital, physical), and dispositions (confidence, flexibility, initiative, risk), encapsulated in a critical orientation (questioning, challenging, arguing, defending) towards mathematical conclusions. The density unit is situated in an authentic context for numeracy. It clearly both requires and demands mathematical knowledge, particularly of mass and volume, but also decimals and ratio. It requires the use of physical measuring tools and representational tools of a data table. The unit was seen to promote students’ positive dispositions identified through their confidence in engaging with all aspects of the unit. It had the capacity to promote a critical orientation through more careful consideration of the relationship between the two measures of mass and volume and the units of comparison in density calculations. From this analysis, it can be seen that the density unit has considerable potential for promoting numeracy (and proportional reasoning), but also serves to highlight the numeracy demands of this aspect of the science curriculum and the need for science teachers to be responsive to this.

4. Concluding comments

The unit outlined here drew upon mathematics and science literature that emphasizes hands-on approaches for learning, and structured experiences to guide students towards building conceptual understanding of key topics. As an integrated unit of work, it attended equally to the mathematics and science curriculum, but also highlighted the importance of foundational mathematics knowledge and understanding for the scientific concept of density as an intensive quantity. This unit of work shows the potential of a numeracy lens for exploring new science teaching methods.

References