Prospective Teachers' Knowledge of Multiplication

Stein A. Berggren[1], Paal Jom[2]

[1]Østfold University College, Norway
[2]Nord University, Norway

Abstract

According to the Norwegian curriculum ([1]), students are expected to learn the basics of multiplication in third grade. It is therefore important that prospective primary school teachers possess solid knowledge of how to teach multiplication ([2]). While there is a substantial body of research on prospective teachers' understanding of multi-digit multiplication algorithms ([3]), comparatively little attention has been given to how prospective teachers solve multiplication problems that do not require the use of standard algorithms. Inspired by the study of Fischbein et al. (1985) ([4]), which investigated how Italian students solved word problems involving multiplication and division, we conducted a study examining how 46 prospective primary school teachers, in their first semester of teacher education, solved eight arithmetic problems. Four of these problems focused on multiplication and proportionality. The prospective teachers' solutions to these four problems constitute the data material for the present study. The prospective teachers' solutions were analyzed using a thematic qualitative analysis approach ([5]). Since the tasks solved by the prospective teachers fall between what the literature describes as single-digit arithmetic and multi-digit arithmetic ([6]), we developed an analytical framework tailored to this intermediate level. This framework was inspired by Hickendorff et al.'s (2019) ([6]) classification system for multi-digit multiplication strategies, as well as by Cramer and Post's (1993) ([7]) categorization of solution strategies for proportionality tasks. The results indicate that prospective teachers predominantly rely on procedural knowledge when solving problems related to multiplication and proportionality. This suggests a need for changes in teacher education, with greater emphasis on fostering prospective teachers' conceptual understanding of these topics.

Keywords: *Prospective teachers, multiplication, proportionality, solution strategies, procedural knowledge*

1. Introduction

The four basic arithmetic operations are fundamental to further learning in mathematics. It is therefore crucial that students acquire solid knowledge and skills in these operations during primary school. It is difficult to argue that one of the four operations is more important than the others; however, multiplication is sometimes described as the core of basic arithmetic instruction, as an operation, and it is a key component of several other mathematical topics, such as division, fractions, and proportionality ([8]). In the Norwegian school system, multiplication is introduced in Grade 3 (ages 7–8) ([1]). At this grade level, four out of eleven competence aims relate to multiplication. In Grade 4 (ages 8–9), three out of ten competence aims include content relevant to multiplication, such as exploring and explaining relationships between the four arithmetic operations. In addition, three of the competence aims focus on division, which is the inverse operation of multiplication. This highlights multiplication as a central topic for students in Grades 1–4 in the Norwegian school system.

Teachers' competence is crucial in the challenging and complex process of teaching mathematics. More specifically, specialized content knowledge in mathematics is essential for teachers to teach mathematics effectively ([9]). Teachers who have developed advanced levels of specialized content knowledge are better equipped to understand students' thinking and to interpret the solutions students produce ([2]). For this reason, teacher education programs aim to educate teachers who develop and acquire deep specialized content knowledge. Furthermore, although this may seem self-evident, it is worth emphasizing that teachers who teach multiplication should themselves have a solid understanding of the operation of multiplication ([10]).

It has long been recognized that students' arithmetic is characterized by strategic variation, meaning that students use a variety of different strategies to solve arithmetic problems ([11]; [6]). This variation includes both interindividual variation, which refers to different individuals relying on different strategies to solve a given arithmetic task, and intraindividual variation, which refers to a single individual using different strategies to solve different tasks, or even the same task at different points in time and/or in different contexts ([4]; [12]).

Several studies have examined how prospective teachers apply multi-digit multiplication algorithms ([3]) and have concluded that they experience difficulties in understanding these algorithms. However, there is limited research on how prospective teachers solve multiplication problems that do not require the use of algorithms. Therefore, in the present study, we investigate how 46 prospective teachers enrolled in the primary teacher education program (Grades 1–7) solve multiplication tasks that do not require the use of a multiplication algorithm.

The data material in this study is based on eight arithmetic tasks solved by 46 prospective teachers enrolled in the primary teacher education program (Grades 1–7). The first four tasks required the use of addition and subtraction and are not included in the present study. The remaining four tasks address multiplication and proportionality, and it is the students' solutions to these tasks that are analyzed in this study. The tasks are inspired by the study by Park and Nunes (2001) ([13]) and are presented in Table 1.

2. Theory

In this section, we highlight different situations and definitions of multiplication that are considered relevant for the present study. Greer (1992) ([14]) describes ten different situations for multiplication. Two of these are identified in this study: repeated addition with equal groups and multiplicative change of quantities (proportionality). An example of repeated addition with equal groups is: *"3 children each have 4 oranges. How many oranges do they have altogether?"* In contrast, the task *"A piece of elastic can be stretched to 3.3 times its original length. What is the length of a piece that is 4.2 meters long when fully stretched?"* exemplifies multiplicative change of quantities ([14]).

Lampert (1986) ([15]), on the other hand, describes multiplication as an operation involving two factors, $a \cdot b$, where the factor a represents the number of groups (the multiplier) and the factor b represents the number of objects in each group (the multiplicand). The product ($a \cdot b$) indicates the total number of objects across all groups. This relationship can be expressed as $a \cdot b = p$ ([16]). This definition is consistent with Fischbein et al. (1985) ([4]), who describe multiplication as $operator \cdot operand$, where the operator represents the number of groups and the operand represents the number of objects in each group.

When considering the commutative property of multiplication ($a \cdot b = b \cdot a$), the factors take on different roles (cf. [15]). According to Beckmann and Izsák (2015) ([17]), the fact that the multiplier and multiplicand assume different roles results in two different perspectives on proportional relationships: the *multiple-batches perspective*, where the multiplier varies and the multiplicand remains constant, and the *variable-parts perspective*, where the multiplicand varies and the multiplier remains constant.

Although multiplication can be defined as $multiplier \cdot multiplicand = product$ ([15]; [16]), research shows that students perform multiplication in multiple ways ([6]; [11]). Sherin and Fuson (2005) ([11]) describe the development of different solution strategies for single-digit multiplication, including counting with drawings, fingers, and rhythm; additive calculation through repeated addition and grouping; pattern-based calculation; recalled products; and multiplicative reasoning. In contrast, Hickendorff et al. (2019) ([6]) identify several solution strategies for multi-digit multiplication, such as sequential strategies, decomposition, variation strategies, column-based methods, and standard algorithms for multiplication.

Our theoretical framework is based on the two situations for multiplication identified by Greer (1992) ([14]) that are present in this study: repeated addition with equal groups and multiplicative change of quantities (proportionality). In the study by Fischbein et al. (1985) ([4]), repeated addition is presented as a primitive model of multiplication. Most of the students in the study by Park and Nunes (2001) ([13]) initially used additive strategies when learning multiplication. However, the goal is for students to adopt multiplicative solution strategies, which represent a more conceptual understanding of multiplication and are based on a correspondence scheme ([13]).

Proportionality refers to a multiplicative relationship between quantities represented in a situation and can be expressed algebraically as $y = m \cdot x$ ([7]). Cramer and Post (1993) ([7]) describe four different solution strategies for proportionality problems, two of which are included in our framework: the *unit-rate strategy* and the *factor-of-change strategy*. The unit-rate strategy involves determining the value of one unit and then scaling it up to find the required quantity. The factor-of-change strategy involves solving the problem through multiplication; for example: *"It takes 20 minutes to drive four miles. Since Mark is driving three times as far, it should take three times as long. Therefore, the answer is 20 minutes times three, or 60 minutes"* ([7]).

Mathematical knowledge can be divided into two types: *procedural* and *conceptual knowledge* ([18]). Procedural knowledge of multiplication involves understanding multiplication as repeated addition, for example interpreting $4 \cdot 5$ as taking the number 5 four times (5 + 5 + 5 + 5) ([19]). Conceptual knowledge of multiplication, by contrast, involves: (1) understanding the concepts and mathematical content associated with multiplication; (2) the ability to solve multiplication problems using a variety of strategies and representations; and (3) the ability to retrieve basic multiplication facts directly from memory ([20]).

One approach to teaching multiplication that supports the development of conceptual understanding includes the use of manipulatives, symbolic representations of these manipulatives, group or pair discussions, and the use of everyday contexts ([21]). Manipulatives can help students develop an understanding of mathematical concepts and operations, and when combined with symbolic representations, they can further strengthen mathematical understanding ([22]). Working in groups or pairs and engaging in discussion allows students to articulate their mathematical thinking to peers, which can help consolidate their understanding ([23]). Using everyday context enables students to connect mathematics to their own experiences ([22]).

The study by Park and Nunes (2001) ([13]) shows that the choice of strategy distinguishes between the development of procedural and conceptual understanding of multiplication. Repeated addition was used to foster procedural understanding, whereas conceptual understanding of multiplication emphasized the relationship between the number of groups and the number of objects in each group. One finding of their study was that students who received instruction emphasizing conceptual understanding developed a deeper understanding of multiplication. This suggests that repeated addition may be useful in introducing multiplication as a calculation procedure, but not as a foundation for understanding the concept of multiplication itself.

Özel et al. (2022) ([10]) also highlights the importance of being aware of the distinction between procedural and conceptual knowledge. They found that most of the prospective teachers in their study correctly evaluated students’ solutions to multiplication problems. However, the evaluations were primarily based on the prospective teachers’ procedural knowledge rather than their conceptual understanding of multiplication.

Repeated addition can be viewed as a primitive model of multiplication (cf. [4]). Based on Greer’s (1992) ([14]) multiplication situations, Bao (2023) ([24]) investigated students in Grades 3–6 and their solutions to four multiplication tasks. Bao found that, for tasks involving equal groups, many students used repeated addition as a solution strategy, suggesting that such tasks may not sufficiently support the development of multiplicative thinking.

Proportionality is a fundamental skill in mathematics education and serves as a bridge between basic arithmetic and more advanced mathematical concepts ([27]). It encompasses a range of mathematical ideas, including fraction equivalence, division, place value, percentages, and unit conversions ([28]). Developing a strong understanding of proportionality is therefore considered essential for students’ overall mathematical learning ([27]). Despite its importance, numerous studies indicate that students often lack a sound understanding of proportionality. Avcu and Avcu (2010) ([28]) found that most sixth-grade students in their study used the cross-product algorithm to solve rate and proportion problems. However, this strategy lacks a meaningful connection to the context of the tasks, and there is no inherent logical reason for students to select this method ([7]).

In the study conducted by Torma and Kosztolányi (2025) ([30]), in which seventh-grade students solved proportion problems, more than 90% of the students answered the problems correctly on both the pre-test and post-test. Most students used the unit-rate strategy. A comparison of strategy used between the pre-test and post-test revealed no significant change in students’ approach to solving proportionality problems.

Modestou and Gagatsis (2009) ([31]) examined how students in primary and secondary school solved proportional and non-proportional tasks. One of their findings was that primary school students primarily used the unit-rate strategy, whereas secondary school students predominantly used cross-multiplication. This result suggests that secondary school students relied on cross-multiplication as a procedural strategy without a corresponding conceptual understanding of proportionality.

3. Methodology

In this study, 46 prospective teachers enrolled in the primary teacher education program (Grades 1–7), during their first semester of teacher education, solved a set of tasks that included four multiplication problems. We analyzed the solutions to the multiplication tasks in order to identify the strategies used by the prospective teachers. The strategies were identified using a qualitative thematic analysis, as described by Braun and Clarke (2013) ([5]). The first step of the analysis involved reviewing the students’ responses to the four multiplication tasks and initially categorizing them according to whether the tasks had been answered or left unanswered. The responses were then further classified into correct and incorrect solutions. The vast majority of the students answered all tasks; one student did not respond to Task 7, and another student did not respond to Task 8. In the next step, the solutions were examined and categorized according to the solution strategies employed. This process resulted in the following categories: proportionality, additive, unit-rate (the “way through 1”), multiplicative, other, and incorrect answers ([11]; [6]; [4]), see Table 2.

The four tasks with a multiplicative structure, which constitute the data material of the study, are presented in Table 1.

Task	Task description	Mathematical content
5	In a forest, there are 4 birdhouses. There are 4 birds living in each birdhouse. How many birds live in the birdhouses?	Multiplication The multiplier and the multiplicand are known
6	Two gingerbread cookies cost 8 kroner. You buy 4 gingerbread cookies. How much do you have to pay?	Proportionality The mulitple-batchs perspective
7	Two caterpillars eat 5 leaves per day. How many leaves are needed to feed 12 caterpillars for one day?	Proportionality The mulitple-batchs perspective
8	There are 6 pieces in a pack of chewing gum. How many pieces are there in 3 packs of chewing gum?	Multiplication The multiplier and the multiplicand are known

Table 1. The four tasks included in the study.

4. Analysis and Results

It has long been recognized that students’ arithmetic is characterized by strategic variation, meaning that students use a range of different strategies to solve arithmetic problems ([4]; [12]). However, there are no corresponding studies focusing on prospective teachers. Therefore, in this study, we investigate how prospective teachers solve multiplication tasks that do not require the use of the standard algorithm, but that demand more advanced strategies than those typically used for single-digit multiplication (cf. [11]).

The tasks solved by the students in our study fall between what the literature refers to as single-digit arithmetic and multi-digit arithmetic ([6]). Consequently, we developed our own analytical tool, inspired by the classification system proposed by Hickendorff et al. (2019) ([6]) for multi-digit multiplication and by the solution strategies for proportionality tasks described by Cramer and Post (1993) ([7]).

The analysis in this study is a descriptive analysis ([32]), in which the researcher examines individual occurrences without the need for multiple events or situations. In our case, the students’ solutions to the four multiplication tasks constitute the units of analysis. We reviewed the students’ solutions and searched for patterns in the data in order to generate categories ([32]). The categories identified through this process were: multiplicative, additive, proportionality, unit-rate (“the way through 1”), other, and a category for incorrect answers.

Category	Content
Multiplicative strategy	In this category, the students arrived at the answer by multiplying the numbers, that is, they used multiplicative reasoning. We did not distinguish between the two cases 3 × 6 = 18 and 6 × 3 = 18 (example from Task 8), nor between the multiple-batches perspective and the variable-parts perspective as described by Beckmann et al. (2015) ([17]).
Additive strategy	In this category, the students add the multiplicand the number of times indicated by the multiplier; that is, they write 4 + 4 + 4 + 4 = 16 instead of 4 × 4 = 16 (example from Task 5).
The factor of change strategy	Here, the students solved the tasks by viewing the situation as two quantities increasing simultaneously; for example, in Task 6, that when the number of gingerbread cookies is doubled, the price is also doubled.
The unit-rate strategy	This is illustrated, for example, in Task 7, by first determining how many leaves one caterpillar eats (5 ÷ 2 = 2.5) and then multiplying this by the number of caterpillars (2.5 × 12 = 30).
Other	This category includes alternative solution methods that are not mentioned above, for example solutions based on illustrations (drawings). In this category, we also included responses that did not contain any calculations.

Incorrect answer	We chose to place responses that resulted in incorrect answers in a separate category. This category also includes incomplete responses, for example those in which a calculation was set up but not carried out.

Table 2. Description of the categories used in the analysis.

We analyzed the prospective teachers' solutions by identifying which solution fit into the different categories, then counting the number of solutions and calculating the percentage distribution across the categories. The results of the analysis are summarized in Table 3.

	Multiplicative strategy	Additive strategy	The factor of change strategy	The unit-rate strategy	Other	Incorrect answer
Task 5	84,8	2,2	0	0	8,7	4,4
Task 6	0	19,6	17,4	30,4	13,0	19,6
Task 7	0	4,4	28,9	26,7	4,4	35,6
Task 8	88,9	2,2	0	0	6,7	2,2

Table 3. Percentage distribution across the different categories for each task.

As shown in Table 3, a large majority of the prospective teachers solved Task 5 (84,8%) and Task 8 (88,9%) using multiplicative reasoning. Only a small proportion answered these tasks incorrectly: 4,4% for Task 5 and 2,2% for Task 8. In both tasks, 2,2% used additive reasoning. Furthermore, 8,7% used other methods in Task 5, while 6,7% did so in Task 8.

The factor-of-change strategy (proportional reasoning) was used by 17,4% of the students in Task 6 and by 28,9% in Task 7. A larger proportion of incorrect solutions was observed for these tasks, with 19,6% answering Task 6 incorrectly and as many as 35,6% answering Task 7 incorrectly. None of the prospective teachers used multiplicative reasoning in either of these tasks. In Task 6, there was considerable variation in strategy choice: 19,6% used additive reasoning and 15,2% used other methods. For Task 7, 4,4% used additive reasoning and 4,4% used other methods.

In summary, the main findings indicate substantial agreement in strategy choice for Tasks 5 and 8, where 84,8% and 88,9%, respectively, used multiplicative reasoning. In contrast, there is greater variation in strategy use for Tasks 6 and 7. Task 6 is also notable in that it has the highest proportion of additive reasoning (19,6%). When considering the factor-of-change strategy and the unit-rate strategy together as approaches to solving proportionality tasks, we find that 45,7% of the responses to Task 6 and 55,6% of the responses to Task 7 were solved using one of these two strategies. These findings form the basis for the discussion in the following section.

5. Discussion and Didactic Implications

We now discuss our findings. In Task 5, multiplicative reasoning dominates, with 84,8% of the prospective teachers using this approach ([7]). This is not surprising given the numbers in the task and the fact that the multiplier and multiplicand are equal (symmetric) ([26]). In Task 8, multiplicative reasoning is also dominant, with 88,9% of the prospective teachers multiplying the numbers given in the task. However, it should be noted that in the analysis we did not distinguish between the order of the multiplier and multiplicand ([25]); we considered $3 \cdot 6 = 18$ and $6 \cdot 3 = 18$ as equivalent solutions. Had we instead emphasized the context ([23]), represented by packages of chewing gum, 95% of those who used multiplicative reasoning would have applied an order of multiplier and multiplicand that does not correspond to the context. They simply multiplied the factors in the order presented in the problem statement, without recognizing that the factors in the commutative property of multiplication ($a \cdot b = b \cdot a$) take on different roles (cf. [15]). This tendency to multiply the numbers as given in the text indicates that the prospective teachers do not understand the significance of context and therefore demonstrate procedural rather than conceptual knowledge of multiplication ([18]).

An additive solution strategy in multiplication involves solving tasks through repeated addition, reflecting procedural knowledge of multiplication. In Tasks 5 and 8, 2,2% of the prospective teachers used repeated addition, while in Tasks 6 and 7 the corresponding percentages were 19,6% and 4,4%, respectively. This suggests that these prospective teachers rely on a primitive model of multiplication ([4]).

Prospective teachers who rely on a primitive model of multiplication ([4]) are likely to teach multiplication as procedural knowledge, using repeated addition as their primary strategy ([13]). These teachers may also apply procedural knowledge when evaluating students' solutions to multiplication tasks ([10]). Bao (2023) ([24]) emphasizes that providing students with equal-group multiplication tasks may encourage continued use of repeated addition and does not necessarily promote the development of multiplicative thinking.

In this study, the prospective teachers solved two proportionality tasks, both of which involved the multiple-batches perspective ([17]). The analysis shows that 47,8% used proportional reasoning in Task 6, while 55,6% did so in Task 7. The strategies applied within proportional reasoning were the factor-of-change strategy and the unit-rate strategy.

In Tasks 6 and 7, 30,4% and 26,7% of the prospective teachers, respectively, used the unit-rate strategy, which is the same strategy that Modestou and Gagatsis (2009) ([31]) found to be commonly used by students at the primary school level. Based on the competence aims in LK20 ([1]), it is reasonable to assume that the prospective teachers have been taught other methods; nevertheless, they continue to use the same strategy, a finding consistent with the study by Torma and Kosztolányi (2025) ([30]). This suggests that strategy choice is not grounded in an understanding of proportionality, a pattern also noted by Avcu and Avcu (2010) ([29]) regarding the use of the cross-product algorithm. In such cases, students rely on a learned method without a meaningful reference to the context of the problem (cf. [7]). In our study, this implies that the prospective teachers apply a learned method even when it is not the most efficient.

Proportionality serves as a bridge between arithmetic and more advanced mathematical concepts ([27]), such as fraction equivalence, division, place value, percentage calculations, and unit conversions ([28]). Our analysis shows that 19,6% of responses to Task 6 and 35,6% of responses to Task 7 were incorrect. The most common incorrect answer in Task 7 was that 60 leaves are needed to feed 12 caterpillars; among those who answered incorrectly, 68,8% provided this response. This indicates that the prospective teachers demonstrate limited or insufficient procedural knowledge of proportionality ([18]).

Our findings indicate that the prospective teachers primarily possess procedural knowledge of multiplication and proportionality. This suggests that teacher education must be changed to place greater emphasis on the development of conceptual knowledge in these areas. One approach is to problematize the commutative property of multiplication ($a \cdot b = b \cdot a$) by demonstrating how the factors assume different roles when situated in context (cf. [15]; [17]). In addition, instruction for prospective teachers should foreground concepts and mathematical content in multiplication, promote the use of varied strategies and representations, and support the development of a broader foundation of multiplication facts ([20]).

With regard to proportionality, instruction should likewise foreground conceptual understanding. One possible approach is to use examples that demonstrate that although the unit-rate strategy can lead to correct answers, a strong understanding of proportionality supports the selection of more efficient strategies that preserve relationships. This can be seen as analogous to solving multiplication problems through repeated addition. To foster this understanding, instruction should provide opportunities for prospective teachers to engage deeply with proportional functions and the role of the proportionality constant. In this way, teacher education can support the development of prospective teachers' conceptual knowledge of proportionality.

REFERENCES

[1] Utdanningsdirektoratet (2020). Læreplan I Matematikk 1.-10. Trinn (Mat01-05). Fastsatt Som Forskrift. Læreplanverket For Kunnskapsløftet 2020. Https://Www.Udir.No/Lk20/Mat01-05?Lang=Nob

[2] Ball, D.L., Thames, M.H. & Phelps, G. (2008). Content Knowledge For Teaching. What Makes It Special? Journal Of Teacher Education, 59 (5), 389-407. Https://Doi.Org/10.1177/0022487108324554

[3] Apsari, R. U., Wulandari, N. P., Novitasari, D. (2021). Prospective Mathematics Teachers Making Sense Of Multiplication Algorithm. *Journal Of Physics: Conference Series, 1778,* 1-9. Https://Doi.Org/10.1088/1742-6596/1778/1/012023

[4] Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The Role Of Implicit Models In Solving Verbal Problems In Multiplication And Division. Journal For Research In Mathematics Education, 16, 3-17. Https://Doi.Org/10.2307/748969

[5] Braun, V., & Clarke, V. (2013). Successful Qualitative Research: A Practical Guide For Beginners. Sage.

[6] Hickendorff, M., Torbeyns J. & Verschaffel L. (2019), Multi-Digit Addition, Subtraction,Multiplication, And Division Strategies. In: Fritz A., Haase V.G., Räsänen P. (Eds.)International Handbook Of Mathematical Learning Difficulties. Switzerland: Springer. 543-560. Https://Doi.Org/10.1007/978-3-319-97148-3_32

[7] Cramer, K. & Post, T. (1993). Proportinal Reasoning. The Mathematics Teacher. Https://Www.Jstor.Org/Stable/27968390?Seq=1

[8] Larsson, K. (2016). Students´Understandings Of Multiplication. [Doktorgradsavhandling, Stockholm University]. Diva Portal. Https://Www.Divaportal.Org/Smash/Record.Jsf?Pid=Diva2%3a1038458&Dswid=-1497

[9] Ball, D. L. (1990). Prospective Elementary And Secondary Teachers' Understanding Of Division. Journal For Research In Mathematics Education, 21(2), 132-144. Https://Www.Jstor.Org/Stable/749140?Seq=1

[10] Özel, Z., Yilmaz, A., Işiksal-Bostan, M., & Özkan, B. (2022). Investigation Of The Spesicalized Content Knowledge Of The Pre-Service Primary School Teachers About Multiplication. International Journal For Mathematics Teaching And Learning, 23(2), 115-143. Https://Doi.Org/10.4256/Ijmtl.V23i2.464

[11] Sherin, B. & Fuson, K. (2005). Multiplication Strategies And The Appropriation Of Computational Resoursces. Journal For Research In Mathematics Education, 2005, Vol. 36, No. 4, 347-395. Https://Www.Jstor.Org/Stable/30035044?Seq=1

[12] Giberti, C. & Maffia, A. (2022). Primitive Model Of Partitive Division: A Replication Of The Fischbein Et Al. Study. Implementation And Replication Studies In Mathematics Education, 2(2), 149-173. Https://Doi.Org/10.1163/26670127-Bja10007

[13] Park, J.-H. & Nunes, T. (2001). The Development Of The Concept Of Multiplication. Cognitive Development 16(3), 763-773. Https://Doi.Org/10.1016/S0885-2014(01)00058-2

[14] Greer, B. (1992). Multiplication And Division As Models Of Situations. In D. A. Grouws (Ed.), Handbook Of Research On Mathematics Teaching And Learning (Pp. 276-295). New York Macmillan Publishing Company. - References - Scientific Research Publishing

[15] Lampert, M. (1986) .Knowing, Doing, And Teaching Multiplication. Cognition And Instruction, 1986, Vol. 3, No. 4, Pp. 305-342. Https://Www.Jstor.Org/Stable/Pdf/3233589.Pdf

[16] Ma, L. & Kessel, C (2018). The Theory Of School Arithmetic: Whole Numbers In M.G. Bartolini Bussi, X.H. Sun (Eds.), Building The Foundation: Whole Numbers In The Primary Grades, New Icmi Study Series. Https://Doi.Org/10.1007/978-3-319-63555-2_18

[17] Beckmann, S. & Izsák, A. (2015). Two Perspectives On Proportional Relationships: Extending Complementary Origins Of Multiplication In Terms Of Quantities. Journal For Research In Mathematics Education 2015, Vol. 46, No. 1, 17–38. Https://Www.Jstor.Org/Stable/10.5951/Jresematheduc.46.1.0017

[18] Rittel-Johnson, B. & Alibali, M. W. (1999). Conceptual And Procedural Knowledge Of Mathematics: Does One Lead To The Other? Journal Of Educational Psychology 1999, Vol. 91, No. 1,175-189. Https://Psycnet.Apa.Org/Fulltext/1999-10190-015.Pdf

[19] Haylock,D.(2010). Mathematics Explained For Primary Teachers (4th Ed.).L Ondon, Unitedkingdom:Sage.

[20] Kling, G. & Bay-Williams, J.M. (2015). Three Steps To Mastering Multiplication Facts. Teaching Children Mathematics , Vol. 21, No. 9, Pp. 548-559. Https://Www.Jstor.Org/Stable/10.5951/Teacchilmath.21.9.0548?Seq=1

[21] Mills, J. (2019). Making Multiplication Meaningful: Teaching For Conceptual Understanding. Teachers And Curriculum, 19(1), 17-25. Http://Dx.Doi.Org/10.15663/Tandc.V19i1.334

[22] Ma, L. (2010). Knowing And Teaching Elementary Mathematics: Teachers' Understanding Of Fundamental Mathematics In China And The United States (Anniversary Edition). New York, Ny: Routledge.

[23] Kazemi, E., & Stipek, D. (2001). Promoting Conceptual Thinking In Four Upper-Elementary Mathematics Classrooms. The Elementary School Journal, 102(1) 59-80. Https://Journals.Sagepub.Com/Doi/Pdf/10.1177/0022057409189001-209

[24] Bao, L. (2023). Identifying And Assessing Students' Transition Barriers Between Additive And Multiplicative Thinking. In B. Reid-O'connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), Weaving Mathematics Education Research From All Perspectives. Proceedings Of The 45th Annual Conference Of The Mathematics Education Research Group Of Australasia (Pp. 91–98). Newcastle: Merga. Https://Files.Eric.Ed.Gov/Fulltext/Ed631601.Pdf

[25] Brickwedde, J. (2024). Multiplication And Division Strategies. An Overview Of The Progression Of Children's Strategies In Multiplication And Division. Https://Docs.Google.Com/Document/D/163j9vavo7nuzwzzuci-Aaaidykyhlwxj/Edit
[26] Baroody, A.J., Wilkins, J.L.M., & Tiilikainen, S. (2003). The Development Of Children's Understanding Of Additive Commutativity: From Protoquantitative Concept To General Concept? In A.J. Baroody & A. Dowker (Eds.), The Development Of Arithmetic Concepts And Skills: Constructing Adaptive Expertise. Mahwah, Nj: Lawrence Erlbaum Associates. Https://Www.Taylorfrancis.Com/Chapters/Edit/10.4324/9781410607218-4/Development-Children-Understanding-Additive-Commutativity-Arthur-Baroody-Jesse-Wilkins-Sirpa-Tiilikainen
[27] Lutfi, A., Juandi, D. & Martadiputra, B.A.P. (2023). Development Proportional Reasoning In Mathematics Learning Environment: Systematic Literature Review. Pedagogy: Journal Pendidikan Matematika, 8(1), 107-122. Https://Doi.Org/10.30605/Pedagogy.V8i1.2469
[28] Supply, A-S., Dooren, W.V. & Onghena, P. (2022). Can We Count On Early Numerical Abilities For Early Probabilistic Reasoning Abilities?, Mathematical Thinking And Learning, 24:1, 19-37, Https://Doi.Org/10.1080/10986065.2020.1805551
[29] Avcu, R. & Avcu, S. (2010). 6th Grade Students' Use Of Different Strategies In Solving Ratio And Proportion Problems. Procedia: Social And Behavioral Sciences, Volume 9, 1277-1281. Https://Doi.Org/10.1016/J.Sbspro.2010.12.320
[30] Torma, G. & Kosztolányi, J. (2025). Strategies Used In Solving Proportion Problems Among 7th-Grade Students. Teaching Mathematics And Computer Science, 23(2), 101–127. Https://Doi.Org/10.5485/Tmcs.2025.14608
[31] Modestou, M. & Gagatsis, A. (2009). Proportional Reasoning: The Strategies Behind The Percentages. Acta Didactica Universitatis Comenianae Mathematics, Issue 9, 2009, Pp. 25−40. Https://Www.Researchgate.Net/Profile/Athanasios-Gagatsis/Publication/259654527_Proportional_Reasoning_The_Strategies_Behind_The_Percentages/Links/561f57e608aecade1acd41f0/Proportional-Reasoning-The-Strategies-Behind-The-Percentages.Pdf
[32] Creswell, J.W. & Creswell, J.D. (2023). Research Design - International Student Edition. Qualitative, Quantitative, And Mixed Methods Approaches. Sixth Edition. Sage Publications, Inc.