Pre-service Teachers' Solution Strategies in Addition and Subtraction Tasks

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Abstract

Specialized content knowledge (SCK) concerns the mathematics teacher's deeper understanding of the specific mathematical topics to be taught ([1]). Flexibility refers to the ability to use a variety of strategies to solve mathematical problems, while adaptivity refers to the selection and use of strategies that are appropriate for a given task ([2]). In this study, we examine which solution strategies first-year pre-service teachers in the primary school teacher education program use when solving addition and subtraction tasks. We asked 22 pre-service teachers to complete a test consisting of addition and subtraction tasks toward the end of their first semester in the teacher education program. The test was analyzed using a thematic qualitative analysis strategy ([3]). The theoretical framework applied in the study was based on the solution strategies for addition and subtraction with multi-digit numbers described by Hickendorff et al. (2019) ([4]). Our results indicate that most pre-service teachers in the primary school program used algorithm-based solution strategies when solving addition and subtraction tasks, while only a small proportion used the number line as a strategy. The findings highlight the importance of strengthening representational competence and conceptual understanding in the early stages of teacher education ([5]; [2]). In particular, the number line, central in the Norwegian curriculum and foundational for understanding number relations, should be emphasized not only as a pedagogical tool for young students but also as a representational resource for future teachers' own mathematical reasoning.

Keywords: *Solution strategies, Pre-service teachers, Addition and Subtraction Tasks, Specialized content knowledge*

1. Introduction

Ball et al. (2008) ([1]) developed a model describing the knowledge required by mathematics teachers, consisting of two main domains: subject matter knowledge and pedagogical content knowledge. Within the domain of subject matter knowledge, the model distinguishes between common content knowledge and specialized content knowledge. Common content knowledge refers to the mathematical knowledge needed to solve mathematical problems and includes the fundamental mathematical skills that are taught in school ([6]). Specialized content knowledge (SCK), on the other hand, concerns the mathematics teacher's deeper understanding of the specific mathematical topics to be taught. This includes, among other aspects, the ability to provide explanations that are comprehensible and useful for students, to represent mathematical ideas and relationships in appropriate and accessible ways, and to solve mathematical problems using multiple approaches ([6]).

The development of flexibility and adaptivity has become a central goal of arithmetic instruction in primary school ([2]). Flexibility refers to the ability to use a variety of strategies to solve mathematical problems, while adaptivity concerns the selection and use of strategies that are appropriate for a given task ([2]). Lin et al. (2011) ([5]) suggest that specialized content knowledge (SCK) consists of three components: representation, justification, and explanation. Representation involves the accurate and effective selection and use of mathematical representations; justification concerns the description and justification of mathematical ideas; and explanation refers to providing mathematical expressions of common rules and procedures. In this study, we focus specifically on representations, more precisely, on the solution strategies used by pre-service teachers.

According to the Norwegian national curriculum (LK20), students are introduced to addition and subtraction in Grades 1 and 2, multiplication in Grade 3, and division in Grade 4 ([7]). One of the competence aims for Grade 2 states that students should use the number line when solving addition and subtraction tasks. In addition, students are expected to explore the commutative and associative properties of addition. At Grade 3, a competence aim emphasizes that students should use appropriate strategies for subtraction in practical situations.

These curriculum requirements imply that mathematics teachers working in Grades 1–7 need strong knowledge of a variety of solution strategies for addition and subtraction tasks. Accordingly, the aim of this study is to investigate the following research question: **Which solution strategies do pre-service teachers in the Grades 1–7 teacher education programme use when solving addition and subtraction tasks early in their teacher education?** To address this research question, pre-service teachers enrolled in the Grades 1–7 programme were asked to solve 22 addition and subtraction tasks at the end of their first semester in teacher education. The students' solutions were analyzed using the solution strategy framework described by Hickendorff et al. (2019) ([4]), and the results are discussed in relation to previous research on the solving of addition and subtraction problems.

2. Theory

The model of Mathematical Knowledge for Teaching (MKT) developed by Ball et al. (2008) ([1]) conceptualizes the knowledge teachers need into two main dimensions: subject matter knowledge and pedagogical content knowledge. Specialized content knowledge (SCK) constitutes one of the three categories within subject matter knowledge and is defined as mathematical knowledge that is specific to the work of teaching. Delaney et al. (2008) ([8]) describe specialized content knowledge as the mathematical knowledge and skills that teachers use in instructional contexts.

Morris et al. (2009) ([9]) suggest that SCK is a particularly suitable focus area for pre-service teacher education. Moreover, SCK is considered essential for effective classroom instruction. In this sense, teachers and pre-service teachers should be able to accurately represent mathematical ideas, provide explanations for common rules and procedures, and examine, interpret, and understand non-standard or unusual solution methods ([10]). Similarly, Petrou and Goulding (2011) ([11]) define SCK as the type of mathematical knowledge teachers and pre-service teachers require for effective mathematics teaching and for use in classroom practice.

Mitchell et al. (2013) ([12]) examined teachers' classroom practices and emphasized that the use of representations is essential in the teaching of integers. They also developed a typology of tasks involving the use of representations in integer instruction. The use of representations is particularly important when dealing with integer operations, as the properties of these operations are often challenging for students. Stephan and Akyüz (2012) ([13]) point out that integer operations pose specific representational challenges because, unlike natural numbers, students cannot construct meanings for integer operations solely through abstraction from concrete objects.

Addition is defined as an operation in which the sum of two numbers results in a third number that contains as many units as the two numbers combined, and it is symbolized by the plus sign (+) ([14]). The two numbers are referred to as addends, expressed as *addend* + *addend* = *sum* ($a + b = c$). Subtraction, on the other hand, is the operation or process of finding the difference between two numbers using the minus sign (–). When the sum and one addend are known, the operation used to find the unknown addend is subtraction. Subtraction is therefore the inverse operation of addition ([14]), and a subtraction task can be expressed as *minuend* – *subtrahend* = *difference* ($a – b = c$).

Over the past decades, the development of flexibility and adaptivity has become a central goal in the teaching of arithmetic in primary education ([2]). Flexibility refers to the ability to use a variety of strategies to solve mathematical problems, while adaptivity concerns the selection and use of strategies that are appropriate for a given task ([2]). In a study by Körner et al. (2026) ([15]), the authors investigated how 91 pre-service teachers early in their education and 91 pre-service teachers at the end of their education solved addition and subtraction tasks on a test. The results showed that many pre-service teachers at the beginning of their education struggled with flexible and adaptive calculations, whereas pre-service teachers nearing the end of their education demonstrated greater flexibility and adaptivity in their solution strategies.

A key characteristic of solution strategies for multi-digit numbers is whether the calculation is based on place value or not. This distinction results in two main types of strategies: *number-based strategies* and *digit-based strategies* (cf. [16]). In number-based strategies, the place value of digits is respected (for example, the number 83 is decomposed into 80 and 3). In contrast, digit-based strategies ignore place value, treating numbers as sequences of individual digits (for example, decomposing 83 into the digits 8 and 3 without considering that 8 represents eight tens, or 80). The most common digit-based strategies are the written algorithms for addition, subtraction, multiplication, and division, which operate on single digits in a proceduralized manner, typically from right to left.

Hickendorff et al. (2019) ([4]) describe both number-based and digit-based strategies for multi-digit addition and subtraction. One dimension for categorizing strategies concerns the underlying operation of the solution process: addition or subtraction. In multi-digit addition, there is only one operational approach, direct addition, where one addend is added directly to the other. In contrast, multi-digit subtraction can be performed in three different ways: as direct subtraction, where the subtrahend is taken away from the minuend; as indirect addition, where one adds on from the subtrahend until the minuend is reached (also referred to as an addition strategy); and as indirect subtraction, where the difference is determined by calculating how much must be taken away from the minuend to reach the subtrahend.

The number-based strategies include sequential strategies, decomposition strategies, varying strategies, and the column-based strategy. In sequential strategies (also known as jump strategies), numbers are primarily viewed as positions on a (mental) number line, and operations involve forward or backward movements along this number line. In decomposition strategies (also referred to as split strategies;[17]), numbers are conceptualized according to their decimal structure, and operations involve partitioning or splitting numbers into place-value-based components. Varying strategies involve adapting numbers and/or operations in the problem, such as in compensation strategies where one operand is rounded to a more convenient number (e.g., subtracting 70 instead of 69 and then compensating by adding back the extra 1 that was subtracted).

The column-based strategy largely relies on the same numerical approach as decomposition strategies but is explicitly taught as a separate strategy in Dutch Realistic Mathematics Education (RME). It functions as an intermediate or hybrid strategy that bridges number-based strategies and digit-based algorithms. On the one hand, it operates with numbers rather than digits; on the other hand, it follows a standardized step-by-step procedure accompanied by a structured vertical notation.

Column-based strategies therefore represent a specific form of number-based strategies designed to facilitate a smoother and more conceptually meaningful transition between number-based strategies and digit-based algorithms (cf. [18]). Although these strategies share features with digit-based algorithms, such as the use of structured vertical notation, they operate on whole numbers rather than digits and proceed from left to right, which clearly distinguishes them from digit-based algorithms.

In most countries, digit-based algorithms fall under the category of direct addition or subtraction. The key difference from number-based strategies is that numbers are treated as individual digits, thereby ignoring their place value. For example, in a digit-based addition strategy, one begins by adding the units (e.g., 8 + 6 = 14), writes down the 4, carries the 10 as a 1, and then adds the tens digits (e.g., 3 + 4 + 1 = 8). Only after combining the resulting digits does the 8 come to represent eight tens. Digit-based addition and subtraction algorithms proceed from right to left, starting with the units, followed by the tens, and so on.

Arithmetic operation	**Type**	**Number-based strategy**				**Digit-based strategy**
		Sequential	**Dekomposisjon**	**Variert**	**Colum-based**	
Addition Example 38 + 46	Direkt addition	38+40=78; 78+6=84	30+40=70; 8+6=14; 70+14=84	38+50=88; 88-4=84	38 46+ 70 14+ 84	1 38 46+ 84
Subtraction Example 82 - 69	Direkte subtraction	82-60=22; 22-9=13	80-60=20; 2-9=-7; 20-7=13	Eksempel, kompensasjon: 82-70=12; 12+1=13	82 69- 20 -7 13	7 10 82 69- 13
	Indirekt addition	69+3=72; 72+10=82; 3+10=13	9+3=12; 60+10=70: 3+10=13	69+1=70; 70+12=82; 1+12=13		
	Indirekt subtraction	82-10=72; 72-3=69; 10+3=13	80-10=70; 2-3=-1; 10+3=13	82-20=62; 62+7=69; 20-7=13		

Table 1. Different solution strategies for multi-digit addition and subtraction tasks (Hickendorff et al., 2019)

In sequential strategies (also referred to as *jump strategies*), numbers are primarily conceptualized as objects on a number line. Number lines reveal the ordinal meaning of numbers, while counters reveal their cardinal meaning. Davidson (1987) ([19]) therefore emphasizes the importance of understanding both ordinal and cardinal meanings of integers in the conceptualization of number concepts. From this perspective, both teachers and pre-service teachers need a deep conceptual understanding of why and how addition and subtraction with integers are performed.

For example, Griffin et al. (1995) ([20]) identified knowledge of number order, such as that represented on a number line, as a crucial foundation for the development of other number concepts. Their study showed that low-performing students who received explicit instruction using number lines were able to catch up with high-performing students in mathematics. Similarly, participants in the study by Bofferding et al. (2013) ([21]) were able to use movements on a number line to add or subtract positive integers from negative integers. However, they did not successfully use the number line to add or subtract negative integers, suggesting that number line instruction needs to address such situations more explicitly and effectively ([21]).

Ipek (2018) ([22]) investigated pre-service teachers' specialized content knowledge related to addition and subtraction. In this study, 42 fourth-year pre-service teachers solved 16 addition and subtraction problems and subsequently participated in interviews. When the pre-service teachers' representations, justifications, and explanations were analyzed collectively, the findings indicated that they lacked sufficient subject matter knowledge. The number line is an early numerical representation that can be strategically used by teachers to develop students' understanding of number magnitude and arithmetic operations ([23]). Number lines support accurate counting, comparisons of number magnitude, and computational processes, and one of their key advantages is that they help connect mathematical content across grade levels ([23]).

When comparing pre-service teachers' representational abilities, Ipek (2018) ([22]) found that the number line was the representation they struggled with the most, whereas counters were the representation they used most successfully. This finding aligns with earlier research, which also reported counters to be more effective than number lines. Finally, the results showed that pre-service teachers primarily focused on procedural or operational knowledge when explaining integer addition and subtraction. According to Skemp (1976) ([24]), symbolic presentations of integer operations that are not connected to meaningful contexts reflect *instrumental understanding*, which may contribute to rote learning. From this perspective, the appropriate use of number lines and counters, combined with meaningful contexts, is crucial for fostering relational or conceptual understanding of integer operations.

3. Method

This study involved 22 pre-service primary school teachers enrolled in the Grades 1–7 teacher education programme during their first semester. The participants completed a written test consisting of 22 tasks on addition and subtraction involving integers. Examples of the test tasks are presented in Table 2.

The students' written solutions were analyzed in order to identify the solution strategies they used when solving the tasks. The strategies (see Table 1) were identified through a qualitative thematic analysis, following the approach described by Braun and Clarke (2013) ([3]). In the analytical process, the students' responses were categorized according to the solution strategy framework proposed by Hickendorff et al. (2019) ([4]) (see Table 1).

Task	Task description	Answer	Mathematical content
4	-20 + 15 = ____	Svar: -5	Addend + Addend = Sum Unknown sum; two-digit numbers; one addend is negative.
5	– 4 + ____ = 10	Svar: 14	Addend + Addend = Sum Unknown addend; the other addend is negative; the sum is positive.
6	___ + 13 = -5	Svar: -18	Addend + addend = sum. Unknown addend; the other addend is positive; the sum is negative.
7	– 8 + (-7) = _____	Svar: -15	Addend + addend = sum. Unknown sum; two single-digit negative addends; sign rules.
8	___ + (-9) = - 16	Svar: -7	Addend + addend = sum. Unknown addend; the other addend is negative; the sum is negative.

9	– 5 – 4 = ____	Svar: -9	Minuend − subtrahend = difference. Unknown difference; the minuend is negative.
10	2 – (-3) = ____	Svar: 5	Minuend − subtrahend = difference. Unknown difference; the subtrahend is negative.
11	____ - 8 = - 5	Svar: 3	Minuend − subtrahend = difference. Unknown minuend; the minuend is positive.

Table 2. Examples of tasks from the test

4. Analysis and Results

The analysis was conducted by following the deductive–thematic approach described by Braun and Clark (2013) ([3]). Each student solution was coded using the predefined strategy categories from Hickendorff et al. (2019) ([4]). This step allowed us to describe the distribution of strategy types across the 22 tasks. After that we examined patterns in the coded material to identify broader themes in the pre-service teachers' strategy use.

We entered the categories and codes we used from Hickendorff et al. (2019) ([4]) into an analysis form, Table 3. The analysis form provided an overview of how the pre-service teachers responded to the different tasks.

Category	**Not answered**	**Number-based strategies**				**Digit-based algorithm**
Code		**Seq**	**Dec**	**Var**	**Col**	**Algorithme**
Numerical value	0	1	2	3	4	5

Table 3. The categories in the analysis

The results of the analysis are summarized in Table 4, which presents the percentage distribution across the different categories for each task.

Tasks	**1**	**2**	**3**	**4**
1	18,18 %	0	54,54 %	22,27 %
2	13,63 %	0	63,63 %	18,18 %
3	9,10 %	4,54 %	59,1 %	18,18 %
4	68,18 %	0	13,63 %	4,54 %
5	63,63 %	0	22,72 %	9,1 %
6	50 %	0	18,18 %	18,18 %
7	40,9 %	0	27,27 %	18,18 %
8	40,9 %	0	18,18 %	22,27 %
9	59,1 %	0	18,18 %	9,1 %
10	31,81 %	0	13,63 %	27,27 %
11	45,45 %	0	13,63 %	18,18 %
12	27.27 %	0	4,54 %	50 %
13	4,54 %	9,1 %	45,45 %	31,81 %
14	4,54 %	18.18 %	40,9 %	31,81 %
15	9,1 %	0	22,27 %	45,45 %
16	9.1 %	0	22,27 %	45,45 %
17	13,63 %	0	4,54 %	77,27 %
18	13.63 %	0	4,54 %	77,27 %
19	18,18 %	0	0	77,27 %
20	13,63 %	4,54 %	4,54 %	68,18 %
21	13,63 %	4,54 %	4,54 %	68,18 %
22	9,1 %	9,1 %	0	68,18 %

Table 4. Response rate to the different tasks

4.1 Distribution of Strategies Across Tasks

Table 4 summarizes the frequency of strategy categories for each of the 22 tasks. Across the dataset, a clear pattern emerged: an algorithm-based strategy was the most frequently used, particularly in the later tasks of the test. Several tasks (e.g., tasks 17–22) show algorithm use by more than two thirds of the students, with task 19 reaching 77.27%.

In contrast, number-based strategies were used far less frequently. Among these, sequential strategies (which often indicate a number line approach) were almost absent. Only a small number of students used sequential reasoning on any tasks, and often only 1–2 students per task (e.g., 4.54% on several tasks). Importantly, for the tasks specifically suited to number line reasoning (tasks 4–11), the expected increase in sequential strategies did not occur. For instance, task 4 (−20 + 15) and task 5 (−4 + __ = 10) yielded 0% sequential strategies, despite being tasks that could naturally invite a jump strategy on the number line. A second notable pattern is the high rate of non-response on early negative-number tasks (tasks 4–11). Several of these tasks show non-response rates between 40% and 68%.

The structure of the tasks appears to influence the strategy choices. In the early tasks (especially tasks 1–3), a noticeable subset of pre-service teachers applied decomposition strategies (e.g., 54.54% in task 1), but as the test progressed, this category decreased sharply. In contrast, the last six tasks (17–22) show a strong concentration of algorithm use (68–77% across all tasks), suggesting that many students rely on procedural fluency rather than conceptual flexibility. The near absence of varied strategies (e.g., compensation) reinforces this interpretation.

5. Discussion and Didactics Implications

This study examined the solution strategies used by first-year pre-service teachers when solving addition and subtraction tasks. Using a deductive thematic analysis based on the framework of Hickendorff et al. (2019) ([4]), we found that the majority of students relied primarily on algorithmic-digit based strategy across the 22 tasks. Number-based strategies, including sequential, decomposition, and varied approaches, were used far less frequently, and strategies directly connected to number line reasoning appeared only sporadically.

The clear predominance of algorithm-digit based strategy across the majority of tasks indicates that the pre-service teachers, at the beginning of their teacher education, have an poor flexibility and adaptive solution strategies in addition and subtraction tasks. This agrees well with what Körner et al. (2026) ([15]) found in their study. While fluency with standard algorithms is an important component of a teacher's mathematical repertoire, it does not, by itself, constitute specialized content knowledge ([1]). Such knowledge includes the ability to explain procedures conceptually, use and justify alternative strategies, and make explicit connections between representations.

The analysis across tasks shows that the vast majority of students stuck consistently to one strategy type, most often the algorithm-digit based strategy, regardless of task structure. Very few students demonstrated intra-individual variation, an indicator of adaptive expertise in arithmetic. This reinforces the interpretation that many students rely on memorized procedures rather than conceptual reasoning or strategic adaptation. The almost complete absence of intra-individual flexibility across the tasks reinforces this interpretation. Most students used the same strategy type throughout the test, regardless of the structure or complexity of the problem. This is in contrast to earlier research showing that flexible strategic adaptation is characteristic of mathematically proficient learners and a hallmark of expert teaching ([2]). Limited flexibility suggests that these students may struggle to choose strategies purposefully or to demonstrate the variety of methods they will be expected to model in their future classrooms.

We found that number-based strategies were used far less frequently than an algorithm-based strategy. Despite the documented importance of the number line as a foundational representational tool, supporting understanding of magnitude, structure, continuity, and relationships among numbers, only a small number of pre-service teachers employed number line, related (sequential) strategies. This finding is notable because several of the tasks (especially items 4–11 involving negative numbers and missing addends) were well-suited to such reasoning.

Despite the central role of the number line in the Norwegian curriculum ([7]) and in conceptual understanding of number and operations ([19]), only a handful of pre-service teachers used sequential (jump) strategies. Even when tasks were structurally suited for number line reasoning (tasks 4–11), students overwhelmingly chose algorithms or left the task blank. This suggests that the students may not consider the number line a natural or accessible representation when solving arithmetic tasks involving negative numbers. This interpretation aligns with previous studies reporting that pre-service teachers often associate the number line with early counting activities rather than with general mathematical reasoning or structure. If the number line is not perceived as a meaningful tool in their own reasoning processes, it is unlikely that they will use it purposefully in classroom instruction.

The high non-response rates on several items involving negative numbers suggest additional conceptual difficulties. Previous research has identified operations with negative integers as a persistent challenge even for teachers and pre-service teachers' ([22]; [21]; [20]; [16]), and the pre-service teachers' omission patterns in this study reflect this. A lack of representational strategies, such as using a number line to visualize additive inverses or directionality on the continuum, may contribute to this difficulty. This also connects to Ball et al.'s (2008) ([1]) notion of specialized content knowledge: teachers must not only be able to compute with negative numbers but also to explain *why* their procedures work and to choose representations that make the concepts accessible to students. Weaknesses in the students' use of the number line therefore indicate more than a representational gap; they suggest limitations in deeper structural understanding of integer arithmetic ([9]; [10]; [11]; [8]).

The findings highlight the importance of strengthening representational competence and conceptual understanding in the early stages of teacher education ([5]; [2]). In particular, the number line, central in the Norwegian curriculum and foundational for understanding number relations, should be emphasized not only as a pedagogical tool for young students but also as a representational resource for future teachers' own mathematical reasoning. Developing flexible strategy use, and a deeper understanding of the structure of operations with both positive and negative numbers, should be integral goals within the mathematics education of pre-service teachers.

Overall, this study suggests that while pre-service teachers may enter teacher education with strong procedural fluency, they require further support to develop the conceptual and representational knowledge necessary for high-quality mathematics teaching. Future research could examine how targeted instruction on number line representations and flexible strategy use influences pre-service teachers' mathematical reasoning over time.

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