Study of Cases Involving Addition and Subtraction of Numbers Up To 100 without Crossing the 100-Mark Analysis of the Current State of Mathematics Textbooks for Secondary Schools Grade in Bulgaria

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Abstract

The recent reforms in the educational system in the Republic of Bulgaria and the subsequent updates to the mathematics curricula for grades 1–4 (general education) have led to the development and approval of textbooks aimed at building sustainable mathematical competencies in students. In this regard, the Ministry of Education and Science (MES) has approved nine mathematics teaching kits for compulsory education in second grade, developed by various teams of authors.

The aim of this article is to analyze the theoretical frameworks that substantiate the methodology for teaching the global topic "Addition and Subtraction of Numbers Up to 100 Without Crossing the Hundred-Mark," and to examine their implementation in the current mathematics teaching materials for second grade in the primary education stage. Emphasis is placed on the sequence of introducing new knowledge, the approaches used to clarify the algorithms for adding and subtracting numbers up to 100 without crossing the 100-mark, and the means of illustrating them. With a view to optimizing the learning process and assisting teachers in planning and implementing instruction, the study aims to highlight best practices and methodological solutions presented by the author teams. It also seeks to identify similarities and differences in the presentation of the global theme of mathematics in second grade.

Keywords: *elementary mathematics education; addition and subtraction of numbers up to 100 without carrying*

1. Introduction

In the early stages of primary education, mathematics instruction plays a fundamental role in developing logical and analytical thinking in students. More specifically, this involves mastering basic arithmetic operations (addition and subtraction), as well as the knowledge and skills acquired through them. These are key factors in developing confidence, a systematic approach, and competencies for solving practical real-life problems.

The curricula developed following the educational reform aim to bring Bulgarian educational standards into alignment with European ones. They serve as a regulatory framework that defines the learning objectives, curriculum content, and expected outcomes for each subject in every grade. Their primary task is to ensure a higher-quality and more competitive education.

The topics in the second-grade mathematics curriculum include specific expected outcomes and key concepts from various areas of competence. The focus is on building upon the arithmetic operations of addition and subtraction learned in first grade and developing knowledge and skills for multiplication and division using tables. One of the key focuses is the application of algorithms and techniques for performing addition and subtraction with numbers up to 100 without crossing the tens place.

2. Method

According to V. Angelova, "Cases involving addition and subtraction of numbers up to 100 can be grouped into two main categories:

First group. Adding a two-digit number to a one-digit or two-digit number when the sum of their ones and tens is less than ten; Subtracting a one-digit or two-digit number from a two-digit number when the sum of the ones and tens of the minuend is greater than that of the subtrahend.

Second group. Adding a two-digit number to a one-digit or two-digit number when the sum of their one digits is greater than or equal to ten, or the sum of their tens digits is ten; subtracting a one-digit or two-digit number from a two-digit number when the number of ones in the minuend is less than the number of ones in the subtrahend; subtraction from 100." [5]

This article/publication aims to present the theoretical foundations and contemporary methodological approaches to teaching the topic *"Addition and Subtraction of Numbers Up to 100 Without Carrying*," as well as to analyze how these concepts are applied in current second-grade mathematics textbooks.

The methodology for teaching the arithmetic operations of addition and subtraction without crossing the 100-mark encompasses both oral and written techniques for performing calculations with natural numbers up

to 100.
Written addition (and *subtraction*) is performed according to the following algorithm: first, the ones digits of the numbers are added (or subtracted), and then their tens digits.
Oral addition (or *subtraction*) starts with the tens, and there is some variation in the calculations. Arithmetic algorithms are taught within a specific system, with each subsequent concept being explained based on the previous ones.
First group. Adding a two-digit number with a zero in the ones place to a one-digit number; subtracting the ones or tens from a two-digit number (20 + 3; 23 — 3; 23 — 20).
Second group. Adding a two-digit number to a one-digit number and subtracting a one-digit number from a two-digit number (21 + 5; 26 — 5).
Third group. Adding two-digit numbers, one of which has a zero in the ones place. Subtracting a two-digit number with a zero in the ones place from any two-digit number (27 + 10; 37 — 10).
Fourth group. Adding and subtracting arbitrary two-digit numbers (32 + 26; 58 — 26).
Let's consecutively examine how these four groups of cases are implemented in the current second-grade mathematics textbooks. These are the mathematics textbooks published by:

- "Prosveta Plus" by authors V. Angelova and S. Doichinova [1] ;
- "Prosveta" by authors Y. Garcheva and A. Manova [4];
- "Bulvest" by M. Bogdanova, M. Temnikova, and V. Ivanova [3];
- "Anubis" by T. Vitanov, G. Kirova, Z. Sharkova, I. Pushkarova, and D. Parusheva [9];
- "Pythagoras" by I. Mincheva, M. Dimitrova, and R. Gernat [6];
- "Scorpio" by R. Petrova, R. Stoyanova, and P. Daskova [8] ;
- "Riva" by L. Alexieva and M. Kirilova [2];
- "Bit and Technology" by T. Vulkova, M. Vanikova-Rukhova, D. Stoyanova, D. Dimitrova, I. Dimitrova, V. Damaskova, and T. Lazarova [10];
- "Archimedes" by Z. Paskaleva, M. Alashka, P. Paskalev, and R.Alashka [7].

The reasoning involved in studying the **first group** of cases is as follows:

- The number consisting of 2 tens and 3 ones is 23. Therefore, 20 + 3 = 23.
- In the case of 23 — 3, we use the fact that the number 23 consists of 2 tens and 3 ones. Therefore, if we subtract 3 (i.e., 3 ones) from 23, we get 2 tens, i.e., the number 20.
- In the case of 23 — 20, we also use the fact that the number 23 is the sum of 2 tens and 3 ones. When 2 tens are subtracted from 23, 3 ones remain.

These cases are presented as separate lessons in the textbooks of the following publishers: "Prosveta Plus" [1], "Pythagoras" [6], "Bulvest" [3] and "Scorpio" [8] which allows for a more targeted introduction and understanding of the new knowledge.
In the textbook published by "Anubis" [9] these cases are combined with others from different groups into a single lesson, resulting in a more compact presentation but requiring a higher degree of abstraction and potentially making it more difficult for students to initially comprehend material.
In the textbook published by "Prosveta" [4], the content is divided into two separate lessons—one presenting the addition operation and the other the subtraction operation—which creates a clear distinction between the two operations and supports the step-by-step development of algorithms.
In some of the textbooks published by "Bit i Technika" [10], "Riva" [2] and "Archimedes" [7] these topics are not treated as separate lessons, indicating a different, integrated approach to structuring the curriculum content rather than a differentiated one.
In the textbooks published by "Prosveta Plus" [1], "Bulvest" [3], and "Anubis" [9], new knowledge is formed through mathematical visualisation based on the place-value principle. The methodology relies on an analytical-synthetic approach, in which numbers are viewed as composed of tens and ones. The presentation through tables of decimal composition, as well as the visualization through colored columns and squares in the textbook by "Prosveta Plus" [1] and through the grouping of colored sticks and coins in the textbooks by "Anubis" and "Bulvest" [3] creates a clear, concrete-visual understanding of the structure of numbers and the process of performing the operation itself, facilitating the transition from concrete to abstract thinking. This approach helps students realize that operations are performed within the context of the given sequence, which is key to understanding algorithms without carrying.
In contrast, the textbook published by "Pythagoras" [6] uses a more limited and less developed visual model (using coins). The emphasis is on the type of problems that help students understand the correlation addition and subtraction: *if we subtract one addend from the sum of two numbers, we get the other.*
In the textbook published by "Scorpio" [8], the introduction is primarily implemented through story-based and text-based problems, which points toward a situational and contextual approach. Although this increases motivation and connects knowledge to students' real-world experiences, the lack of sufficient concrete visual aids during the initial introduction may hinder the comprehension of algorithms, especially for students with less developed abstract thinking.
To clarify **the second group** of cases, it is advisable to use a concrete, schematic, and mathematical model in sequence. The numbers are then arranged in a table with rows, thus forming algorithms of the specified

type.
The cases in the second group are presented differently in the various teaching kits, both in terms of lesson structure and methodological approaches.
In the textbook published by “Prosveta Plus” [1] these cases are presented in a separate lesson, which allows a clear focus on the new type of operations and their comparison, as they are covered in the same lesson.
In the textbooks from the following publishers: “Prosveta” [4], “Pythagoras” [6], “Archimedes” [7] and “Riva” [2] the content is divided into separate lessons in addition and subtraction, which supports the step-by-step development of algorithms. This allows students to focus on the algorithms for the individual operations. In the textbooks published by “Anubis” [9], “Bit and Technika” [10] and “Bulvest” [3] cases from the different groups are combined into a single lesson, which implies a higher degree of generalization and saves instructional time, but at the same time places higher demands on students’ cognitive readiness.
In the textbook published by “Scorpio” [8] the content is more integrated; new knowledge is not introduced through a clearly defined stage but is incorporated into a system of problems. This would make it difficult for students to discover the algorithm on their own and could lead to incomplete and superficial learning.
Ultimately, the presentation of new knowledge is again based on the analytical-synthetic approach, which is implemented by breaking numbers down into tens and ones. This approach is particularly evident in the textbooks published by “Prosveta Plus” [1], “Anubis” [9], and “Archimedes” [7]. Through it, students realize that a two-digit number has an internal structure (for example, 24 = 2 tens + 4 ones), which allows operations to be performed separately in the order of the ones or tens. Thus, algorithms are constructed not mechanically, but as a consequence of understanding the decimal positional system.
Mathematical operations are presented as actions on objects or movement along the number line. This approach is characteristic of textbooks published by “Prosveta” [4], “Prosveta Plus” [1], and “Bit i Tekhnika” [10]. Through this approach, addition is perceived as “adding” or “moving forward,” and subtraction as “taking away” or “moving backward.” This facilitates the initial understanding of the operations and creates a connection to students’ real-world experiences.
All of the textbooks listed, with the exception of those published by “Scorpio” [8] and “Pythagoras” [6], use an algorithmic approach in which the operations are presented as a sequence of steps ––writing the numbers in rows in a number chart, performing the operations in the ones place, writing down the result, then performing the operations in the tens place, and writing down the result again. This approach helps develop accurate computational skills and prepares students for more complex algorithms. At the same time, the textbooks published by “Pythagoras” [6] and “Scorpio” [8] do not present the algorithm using a row table; only a version involving mental calculation is provided. This may hinder the development of precise algorithmic concepts.
When studying the cases from **the third group**, an appropriate visual aid can be used, after which the numbers are arranged in the row table, and finally, the algorithms for written addition and subtraction of numbers of the type under consideration are formulated.
In the textbooks published by “Prosveta Plus” [1], “Pythagoras” [6], “Bulvest” [3], “Archimedes” [7], and “Bit i Technika” [10], the cases under consideration are separated into independent or clearly differentiated sections, which facilitates the targeted introduction of the algorithm. In contrast, in the textbooks published by “Anubis” [9], “Riva” [2], “Scorpio” [8] and “Prosveta” [4] these cases are not presented as separate lessons but are included in a broader context, which may make it difficult to highlight their specific features—namely, that the operations are performed only in the tens place, while the ones remain unchanged.
Particular attention should be paid to the textbooks published by “Bulvest” [3], “Bit i Technika” [10] and “Prosveta Plus” [1], where new knowledge is introduced through highly concrete, object-based visual aids. Models of tens and ones (colored cubes, grouping, and breaking down numbers) are used to clearly demonstrate that in operations such as (27 + 10; 37 — 10), changes occur only in the tens place. This approach helps students build a solid understanding of the structure of numbers and grasp the algorithm through observation and action, rather than through rote memorization.
Here again, the primary methodological approach used in most textbooks is the analytical-synthetic approach, implemented by breaking numbers down into tens and ones:

- when adding a round ten, the number of tens increases;
- when subtracting a round ten, the number of tens is reduced;
- the ones remain unchanged.

In the textbook published by “Archimedes” [7] the new concept is presented through visual aids and examples, but it is noted that there is no clearly formulated algorithm for the case (35 — 20). Students arrive at the result through models and examples, but without an explicitly generalized rule. This may make it difficult for some students to apply the knowledge independently, as there is no clear illustration of the algorithm.
The textbook published by “Bit and Technika” [10] employs an inductive approach, in which new knowledge is introduced through concrete problems and situations. Through observation and working with models,

students arrive at the method of calculation, but the algorithm is not presented separately; rather, it is presented alongside examples of adding and subtracting a two-digit number from a one-digit number without carrying.

In the textbooks published by “Pythagoras” [6] “Prosveta Plus” [1] and “Bulvest” [3] the exercises for reinforcing knowledge and skills are particularly diverse and include both basic and advanced cases, which aids in a more thorough mastery of the material. Only the “Bulvest” [3] textbook lacks word problems.

The reasoning behind studying the problems in **the fourth group** is as follows: initially, the numbers are modeled using concrete models, then schematic models are used, and finally, the numbers are arranged in the rows of the table, with algorithms for written addition and subtraction of numbers of the type under consideration being formulated.

The introduction of new knowledge most often begins with a real-life word problem that creates a problem situation. For example, in textbooks published by “Anubis” [9] and “Archimedes” [7], students encounter real-life situations (money, objects) that require performing the arithmetic operations of addition or subtraction. This is followed by presenting the solution by breaking down the numbers into tens and ones and recording them in a table across the rows.

In the analyzed textbooks, visual aids vary in degree and type; in the textbooks published by “Prosveta Plus” [1] and “Bulvest” [3] a rich variety of concrete and visual aids is observed–colored blocks, tokens, coins, and other models are used to illustrate tens and ones. This is important for the transition from concrete to abstract thinking. The textbook from the “Bit i Technika” [10] publishing house also uses a variety of visual aids.

In the textbooks published by “Anubis” [9] and “Prosveta” [4] the visual aids are more structured and are primarily implemented through a number chart (tens and ones). This serves as a means of presenting the algorithms for the operations and clearly shows the order in which the operations are performed.

In the textbooks published by “Pythagoras” [6] and “Riva” [2] the visual presentation is more schematic–through the decomposition of numbers and the use of arrows and diagrams, which encourages the development of thinking strategies.

In the textbook published by “Scorpio” [8], visual aids are more limited and consist mainly of the use of an abacus and a table, which is not always sufficient for all students.

In terms of algorithmic approach, no significant differences are observed. In all textbooks, the algorithm is clearly formulated using number tables and the order of operations (“add the ones, add the tens,” “first subtract the ones, then the tens”), which aids in mastering the standard method of calculation. Only in the textbook published by “Bulvest” [3] is the algorithm formulated based on mental calculation methods, specifically breaking down the addends into sums of tens and ones and adding them sequentially in rows: $32 + 26 = (30 + 20) + (2 + 6) = 50 + 8 = 58$.

Representing one addend as the sum of its tens and ones and adding them sequentially to the other: $32 + 26 = (32 + 20) + 6 = 52 + 6 = 58$ or $32 + 26 = (32 + 6) + 20 = 38 + 20 = 58$. This can lead to students learning the process mechanically without sufficient understanding. The algorithms for mental calculation are also covered by the other author teams, but not all of them include them in the lesson introducing the new concept.

The types of consolidation problems also show variety. All textbooks include arithmetic problems for direct calculation. In addition, word problems are widely used to develop mathematical modeling skills. The textbooks from the publishers “Pythagoras” [6] “Riva” [2] and “Prosveta Plus” [1] also include more challenging problems–composing word problems, finding an unknown number, and verifying equalities and inequalities. The textbook published by “Prosveta” [4] includes problems with number lines and visual models that aid in understanding the relationship between addition and subtraction. In the textbook published by “Riva” [2], problems involving relations (comparisons and filling in tables) play a special role in developing logical thinking.

To reinforce new knowledge, a diverse system of problems is used:

- calculation problems for automation;
- table-filling problems;
- practical word problems;
- problems for identifying relationships;
- combined problems requiring deeper thinking.

3. Conclusion

An analysis of the nine second-grade mathematics curriculum sets approved by the Ministry of Education and Science (MES) shows that they offer a structured and multifaceted approach to learning, particularly regarding addition and subtraction of numbers up to 100 without carrying. The variety of methods, examples, and visual aids facilitates students’ understanding of the material and stimulates the development of their analytical and research skills. It can be concluded that the most effective way to clarify

the algorithms for addition and subtraction without crossing the tens place is to combine several approaches:

- concrete visual aids → for initial understanding
- breaking down into tens and ones → to grasp the structure
- algorithmic steps → for skill development
- exercises and problems → for reinforcement.

To summarize, all textbooks cover addition and subtraction of two-digit numbers without carrying, but they differ in their method of introduction, the degree of visual representation, and the variety of problems. The most effective mastery of the cases covered is achieved by combining visual aids, an analytical-synthetic approach, and a clearly formulated algorithm. Of particular importance is the use of concrete models, which aid students' understanding of the material and facilitate the transition from concreteness to abstraction. The differences, in turn, reflect various didactic approaches and can be effectively utilized in practice by combining their advantages. Since every student learns differently, the materials must meet diverse needs—ranging from strong object-based visual aids for some to more abstract schematic models for others.

Educational standards are changing to align with European requirements, so teaching materials must be constantly updated and strive for innovations that enrich the learning process and make it even more effective and accessible to all students.

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