



# An Innovative Model for Mastering the Activity of Finding an Unknown Component in Equations in Primary School

Vladimira Angelova<sup>1</sup>, Aleksandra Nikolova<sup>2</sup>

<sup>1,2</sup>Plovdiv University 'Paisii Hilendarski', Bulgaria

## Abstract

*One of the expected outcomes set out in the mathematics curricula for second, third, and fourth grades (general education) in the Republic of Bulgaria is the acquisition of skills for finding unknown addends and unknown multipliers (second grade), unknown minuend and unknown dividend (3rd grade), and unknown subtrahend and unknown divisor (4th grade).*

*This publication focuses on the acquisition of skills by 8-10-year-old students for finding unknown components in equations. Its aim is to present the author's model for teaching primary school pupils (aged 8-10) to effectively master the activity of finding unknown components in equations. To test the effectiveness of the model, a long-term pedagogical experimental study was conducted in the period 2022–2025, covering three consecutive school years. It involved 250 students, 125 of which in a controlled group and 125 students in an experimental group tracked from second to fourth grade, which allowed for an analysis of the sustainability and accumulation of results over time.*

*The analysis of the results obtained from this study shows that the implementation of the author's model in mathematics education of the experimental group from 2nd to 4th grade supports a more conscious and in-depth mastery of competencies for finding unknown components in equations.*

*The application of systematic training in finding unknown components also helps students aged 8–10 to apply these competencies more easily and effectively when solving problems for composing equations with unknown numbers in mathematical texts and when solving word problems.*

**Keywords:** *primary school, mathematics, unknown component*

## 1. Introduction

The mathematics curricula for the primary stage of education in Bulgaria [1], [2], [3] provide for the derivation of rules for:

- finding an unknown addend and an unknown multiplier in second grade;
- finding an unknown minuend and an unknown dividend in third grade; and
- finding an unknown subtrahend and an unknown divisor in fourth grade.

The aim of the present publication is to present the author's approach to teaching primary school students to effectively master the activity of finding an unknown component in equations.

Different symbols, letters, etc. may be used to denote the unknown component. In primary school practice in Bulgaria, the most common way of denoting it is by means of an empty square ( $\square$ ).

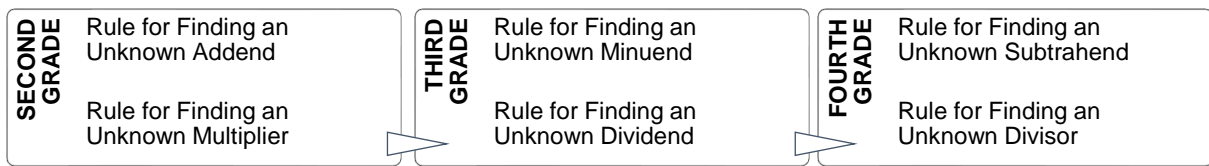
Primary school students determine the unknown component in equations in two ways:

1. Based on their knowledge of arithmetic operations, by "guessing" the unknown number and checking whether their choice is correct;
2. Based on rules for finding the unknown component. These rules are formulated on the basis of the relationships between the components and the results of arithmetic operations that students have previously mastered.

## 2. Method

The aim of the present paper is to present an innovative teaching model for mastering the activity of finding an unknown component in equations by students at the primary stage of education.

The components of this innovative teaching model are presented in **Fig. 1**.



**Fig. 1.** Innovative teaching model for mastering the activity of finding an unknown component in equations.

## 2.1. Methodological Approach to Teaching the Rule for Finding an Unknown Addend

Finding an unknown addend by using the relationship between the components and the result in the operation of addition is a competence defined as an expected learning outcome in the mathematics curriculum for second grade [1]. The derivation of the rule is planned to take place within the general topic *Addition and Subtraction of Numbers up to 100 without Regrouping*.

It is appropriate for the methodological work to begin with the consideration of a problem on the basis of which the relationship between the components and the result in the operation of addition can be reached. To discover this relationship, the following problem is proposed:

Example 1. Compose a word problem based on the picture.



The picture is examined and analyzed. The teacher needs to emphasize the number of doughnuts that the boy and the girl place on the table and guide the students toward composing problems that are modeled by the expressions shown to the right of the picture.

Clarification: There are 23 blue doughnuts on the boy's tray and 10 pink doughnuts on the girl's tray. Based on the illustration and the mathematical expressions, three word problems can be composed: one that is solved by addition and two that are solved by subtraction. The problems are as follows:

- Georgi serves 23 blue doughnuts on the table, and Ani serves 10 pink doughnuts. How many doughnuts are there on the table altogether?
- There are 33 doughnuts on the table. Of these, 10 are pink, and the rest are blue. How many blue doughnuts are there?
- There are 33 doughnuts on the table. Of these, 23 are blue, and the rest are pink. How many pink doughnuts are there?

The solutions to these word problems are as follows:

- $23 + 10 = 33$  (doughnuts altogether);
- $33 - 10 = 23$  (blue doughnuts);
- $33 - 23 = 10$  (pink doughnuts).

The three resulting interconnected equations are examined and analyzed. The discussion, aimed at identifying the position of the same numbers, leads to the following generalization: *If one addend is subtracted from the sum, the other addend is obtained.*

The derived relationship is applied when finding the unknown number in the problem from the following example.

Example 2. Find the unknown addend.

$$16 + \square = 48$$

$16 + \square = 48$ $48 - 16 = \boxed{32}$ <i>Check:</i> $16 + \boxed{32} = 48$
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**Fig. 2.** Layout of the solution for problems involving the finding of an unknown addend.



By using the relationship between the components and the result in addition, the rule for finding an unknown addend is derived: *The unknown addend is found by subtracting the known addend from the sum.*

It is appropriate to reinforce this rule through problems in which an equation with an unknown addend is composed on the basis of a given mathematical text (Example 3) and through word problems that are modeled by an equation with an unknown addend (Example 4).

Example 3. Find the number that was thought of.

Neli thought of a number. She added 42 to it and obtained 68. What number did Neli think of?

Example 4. There were several boxes of chocolates on a shelf. After 3 more were placed there, they became 25. How many boxes of chocolates were there on the shelf initially?

Solving mathematical and practical problems by composing an equation with an unknown addend is an activity suitable for improving students' ability to use the derived rule in different situations.

For the application of students' mathematical knowledge, it is appropriate to solve problems with a more complex structure, such as the one proposed in the following example.

Example 5. Find the number.

To which number should 32 be added in order to obtain the difference between the numbers 96 and 54?

The organization of problems of this type should be carried out in two stages: first, establishing the correctness of the equations written by the students, and then checking the correct application of the rule for finding an unknown addend and the formatting according to the model in Example 2.

## 2.2. Methodological Approach to Teaching the Rule for Finding an Unknown Multiplier

Finding an unknown multiplier by using the relationship between the components and the result in the operation of multiplication is a competence defined as an expected learning outcome in the mathematics curriculum for second grade [1]. The teaching of the rule for finding it is planned to take place within the general topic *Multiplication and Division Tables*.

It is appropriate for the methodological work to begin with the consideration of a problem on the basis of which the relationship between the components and the result in multiplication can be defined. To discover this relationship, the following problem may be used:

Example 6. Compose word problems based on the picture.



Based on the illustration, three problems can be composed: one that is solved by multiplication and two that are solved by division. Here are sample formulations of the problems:

- There are 3 daffodils in each of 4 vases. How many daffodils are there in the vases altogether?*
- Twelve daffodils are placed in vases, with 3 daffodils in each vase. In how many vases are the daffodils placed?*
- Twelve daffodils are placed equally in 4 vases. How many daffodils are there in each vase?*

The solutions to these word problems are as follows:

- $4 \times 3 = 12$  (daffodils altogether);
- $12 \div 3 = 4$  (vases);
- $12 \div 4 = 3$  (daffodils in one vase).

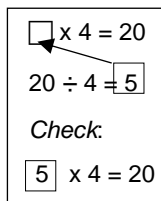
The three resulting interconnected equations are examined and analyzed. The discussion, aimed at identifying the position of the same numbers, leads to the following generalization: *If we divide the product by one of the multipliers, we obtain the other multiplier.*

The derived relationship is applied when finding the unknown number in the problem from the following example.



Example 7. Find the unknown multiplier.

$$\square \times 4 = 20$$



$$\square \times 4 = 20$$

$$20 \div 4 = \boxed{5}$$

Check:

$$\boxed{5} \times 4 = 20$$

Fig. 3. Layout of the solution for problems involving the finding of an unknown multiplier.

By using the relationship between the components and the result in multiplication, the rule for finding an unknown multiplier is derived: *The unknown multiplier is found by dividing the product by the known multiplier.*

This is followed by solving a sufficient number of problems, such as those in Example 7, in order to help students make sense of the knowledge related to finding an unknown multiplier.

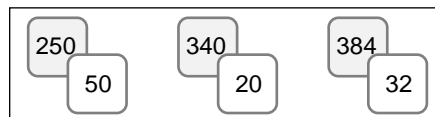
The problems for reinforcing and improving the skill of finding an unknown multiplier are analogous to those presented in section 2.1.

### 2.3. Methodological Approach to Teaching the Rule for Finding an Unknown Minuend

Finding an unknown minuend on the basis of the relationship between the components and the result in the operation of subtraction is a competence defined as an expected learning outcome in the mathematics curriculum for third grade [2]. The teaching of the rule for finding an unknown minuend is planned to take place within the general topic *Addition and Subtraction of Numbers up to 1000 without Regrouping*.

It is appropriate for the methodological work to begin with the consideration of a problem on the basis of which the relationship between the components and the result in the operation of subtraction can be clarified. To discover this relationship, the reasoning involved in solving the problem from the following example is proposed.

Example 8. Calculate the difference between the two numbers written on the cards and check it by addition.



As early as first grade, students know that the operation of addition can be used to check the operation of subtraction. According to the instructions for this problem, students should write three pairs of equations:

$$250 - 50 = 200, \quad 340 - 20 = 320, \quad 384 - 32 = 352,$$

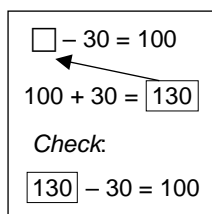
$$200 + 50 = 250, \quad 320 + 20 = 340, \quad 352 + 32 = 384.$$

On this basis, after discussing the position of the same numbers in the equations, the following relationship is formulated: *If the difference is added to the subtrahend, the minuend is obtained.*

The derived relationship is applied when finding the unknown number in the problem from the following example.

Example 9. Find the unknown minuend.

$$\square - 30 = 100$$



$$\square - 30 = 100$$

$$100 + 30 = \boxed{130}$$

Check:

$$\boxed{130} - 30 = 100$$

Fig. 4. Layout of the solution for problems involving the finding of an unknown minuend.

By using the relationship between the components and the result in subtraction, the rule for finding an unknown minuend is derived: *The unknown minuend is found by adding the difference and the subtrahend.*



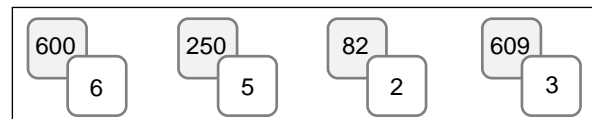
This is followed by solving a sufficient number of problems, such as those in Example 9, in order to help students make sense of the knowledge related to finding an unknown minuend. The problems for reinforcing and improving the skill of finding an unknown minuend are analogous to those presented in section 2.1.

#### 2.4. Methodological Approach to Teaching the Rule for Finding an Unknown Dividend

Finding an unknown dividend on the basis of the relationship between the components and the result in the operation of division is a competence defined as an expected learning outcome in the mathematics curriculum for third grade [2]. The teaching of the rule for finding the unknown dividend is planned to take place within the general topic *Division of Two-Digit and Three-Digit Numbers by a One-Digit Number*.

It is appropriate for the methodological work to begin with the consideration of a problem on the basis of which the relationship between the components and the result in the operation of division can be clarified. To discover this relationship, the problem from the following example is suitable:

Example 10. Calculate the quotient of the two numbers written on the cards and check it by multiplication.



As early as second grade, students know that the operation of multiplication can be used to check the operation of division. According to the instructions for this problem, students should write four pairs of equations:

$$\begin{array}{cccc} 600 \div 6 = 100, & 250 \div 5 = 50, & 82 \div 2 = 41, & 609 \div 3 = 203, \\ 100 \times 6 = 600, & 50 \times 5 = 250, & 41 \times 2 = 82, & 203 \times 3 = 609. \end{array}$$

On this basis, after discussing the position of the same numbers in the equations, the following relationship is formulated: *If the quotient is multiplied by the divisor, the dividend is obtained.*

By using this relationship, the rule for finding an unknown dividend is derived in the following example.

Example 11. Find the unknown dividend.

$$\square \div 3 = 40$$

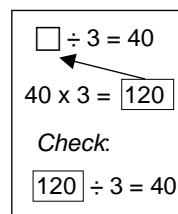


Fig. 5. Layout of the solution for problems involving the finding of an unknown dividend.

By using the relationship between the components and the result in division, the rule for finding an unknown dividend is derived: *The unknown dividend is found by multiplying the quotient by the divisor.* This is followed by solving a sufficient number of problems, such as those in Example 11, in order to help third-grade students make sense of the knowledge related to finding an unknown dividend. The problems for reinforcing and improving the skill of finding an unknown dividend are analogous to those presented in section 2.1.

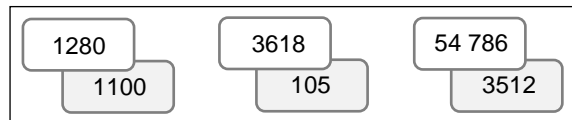
#### 2.5. Methodological Approach to Teaching the Rule for Finding an Unknown Subtrahend

Finding an unknown subtrahend by using the relationships between the components and the result in the operation of subtraction is a competence defined as an expected learning outcome in the mathematics curriculum for fourth grade [3]. The teaching of the rule for finding an unknown subtrahend is planned to take place within the general topic *Addition and Subtraction of Multi-Digit Numbers without Regrouping*.

The methodological work begins with the consideration of problems on the basis of which the relationships between the components and the result in the operations of addition and subtraction, studied in the previous grades and derived in the present paper in sections 2.1 and 2.3, respectively, can be updated. For this purpose, the problems from the following two examples are proposed.



Example 12. Calculate the difference between the two numbers written on the cards and check it by addition.



Example 13. Fill in the blanks by using the relationship between the operations of addition and subtraction.

$$6000 + 20 = 6020$$

$$15\ 204 + 3152 = 18\ 356$$

$$6020 - 20 = \underline{\hspace{2cm}}$$

$$18\ 356 - 3152 = \underline{\hspace{2cm}}$$

$$6020 - 6000 = \underline{\hspace{2cm}}$$

$$18\ 356 - 15\ 204 = \underline{\hspace{2cm}}$$

By using the relationships recalled in Example 12 and Example 13, two approaches to finding an unknown subtrahend are derived.

Example 14. Find the unknown subtrahend.

$$3500 - \square = 1000$$

It is appropriate to present the solution to this problem through the following two approaches:

*First approach*

$$\begin{aligned} 3500 - \square &= 1000 \\ 1000 + \square &= 3500 \\ \\ 3500 - 1000 &= \boxed{2500} \\ \\ \text{Check:} \\ 3500 - \boxed{2500} &= 1000 \end{aligned}$$

*Second approach*

$$\begin{aligned} 3500 - \square &= 1000 \\ \\ 3500 - 1000 &= \boxed{2500} \\ \\ \text{Check:} \\ 3500 - \boxed{2500} &= 1000 \end{aligned}$$

In the first approach, the problem of finding an unknown subtrahend is reduced to a problem of finding an unknown addend, while in the second approach the following rule is derived: *The unknown subtrahend is found by subtracting the difference from the minuend.*

This is followed by solving a sufficient number of problems, such as those in Example 14, in order to help students make sense of the knowledge related to finding an unknown subtrahend.

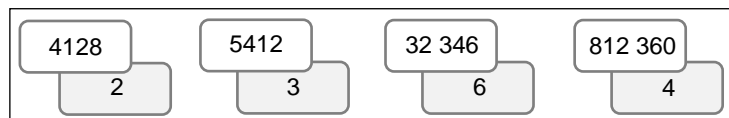
The problems for reinforcing and improving the skill of finding an unknown subtrahend are analogous to those presented in section 2.1.

### 2.6. Methodological Approach to Teaching the Rule for Finding an Unknown Divisor

Finding an unknown divisor by using the relationships between the components and the result in the operation of division is a competence defined as an expected learning outcome in the mathematics curriculum for fourth grade [3]. The study of the rule for finding an unknown divisor is planned to take place within the general topic *Division of a Multi-Digit Number by a One-Digit Number*.

It is appropriate for the methodological work to begin with the consideration of problems on the basis of which the relationships between the components and the result in the operations of multiplication and division, studied in the previous grades and derived in the present paper in sections 2.2 and 2.4, respectively, can be updated. For this purpose, the problems from the following two examples are proposed.

Example 15. Calculate the quotient of the two numbers written on the cards and check it by multiplication.



Example 16. Fill in the blanks by using the relationship between the operations of multiplication and division.

$$328 \times 5 = 1640$$

$$3050 \times 3 = 9150$$

$$10\ 900 \times 7 = 76\ 300$$

$$65\ 020 \times 8 = 520\ 160$$

$$1640 \div 5 = \underline{\hspace{2cm}}$$

$$9150 \div 3 = \underline{\hspace{2cm}}$$

$$76\ 300 \div 7 = \underline{\hspace{2cm}}$$

$$520\ 160 \div 8 = \underline{\hspace{2cm}}$$

Example 17. Find the unknown divisor.

$$8600 \div \square = 2$$

It is appropriate to present the solution to this problem through the following two approaches:

*First approach*

*Second approach*



$$8600 \div \square = 2$$

$$2 \cdot \square = 8600$$

$$8600 \div 2 = \boxed{4300}$$

$$8600 \div \square = 2$$

$$8600 \div 2 = \boxed{4300}$$

In the first approach, the problem of finding an unknown divisor is reduced to a problem of finding an unknown multiplier, while in the second approach the following rule is derived: *The unknown divisor is found by dividing the dividend by the quotient.*

This is followed by solving a sufficient number of problems, such as those in Example 17, in order to help students make sense of the knowledge related to finding an unknown divisor.

The problems for reinforcing and improving the skill of finding an unknown divisor are analogous to those presented in section 2.1.

### 3. Organization of the Study

A longitudinal pedagogical experimental study was conducted in the period 2022–2025. It covered three consecutive school years, with the students participating in the study being in second grade during the first year and in fourth grade during the final year. A total of 250 students from three schools in Plovdiv, Bulgaria, took part in the experiment: 125 in the control group (CG) and 125 in the experimental group (EG). The author's innovative model for the effective mastery of skills for finding an unknown component in equations was applied in the instruction of the students from the experimental group.

The aim of the present study is to create, apply, experiment with, and establish the effectiveness of the developed innovative model of mathematics instruction in second, third, and fourth grade.

Hypothesis: If the author's innovative model of mathematics instruction is applied in grades 2–4, it would contribute to mastering and deepening students' knowledge and skills for finding an unknown component in equations, as well as to developing the mathematical and logical thinking of students at the primary level of education.

The object of the study is the process of mastering the knowledge and skills for finding an unknown component in equations by students from second to fourth grade.

The subject of the study is the identification of the relationships between the specific features of the activities for finding an unknown component in equations and the degree to which students master them, depending on the developed and applied author's innovative teaching model.

In the course of the experimental study, the students from both groups were subjected to a diagnostic assessment at the beginning of the study. This assessment was used to determine the level of their mathematical competencies at the beginning of second grade, which made it possible to divide the students into a control group and an experimental group. One diagnostic assessment was also conducted at the end of second, third, and fourth grade, respectively, in order to determine the level of mastery of the skills for finding an unknown component in equations acquired in the respective grade.

The effectiveness of the innovative model created and applied in the instruction was assessed at the end of each school year according to the following criterion: *Mastery of competencies for finding an unknown component in equations.*

The evaluation of the obtained results was carried out according to the following indicators:

First indicator. Knowledge of the rules for finding an unknown component in equations.

Second indicator. Composing an equation with an unknown component based on a given mathematical text.

Third indicator. Solving a word problem that is modeled by an equation with an unknown component.

The present publication focuses on the final diagnostic assessment conducted in fourth grade, after the completion of the instruction of the fourth-grade students from the experimental group according to the innovative model. This assessment was used to diagnose the level of students' mathematical competencies according to the three indicators defined above, both in the control group and in the experimental group. The diagnostic assessment was carried out after the study of the general topic *Division of a Multi-Digit Number by a One-Digit Number.*

### 4. Results

The following section presents and analyzes the results of the conducted final diagnostic assessment according to the three indicators formulated above.

With regard to the first indicator, "Knowledge of the rules for finding an unknown component in equations," the analysis of the results shows that 93.6% of the students from the EG know the specified



rules, whereas the percentage of students from the CG who have acquired this knowledge is 75.2%. Knowledge of the rules for finding an unknown component in equations is a prerequisite for the successful solving of various types of problems involving the specified learning content.

As regards the second indicator, “Composing an equation with an unknown component based on a given mathematical text,” the analysis of the results from the independent work shows that 87.2% of the fourth-grade students from the EG who participated in the study have mastered the specified skills. By comparison, the percentage of students from the CG who are able to apply their skills in the specified context is 68.8%.

With regard to the most complex activity, “Solving a word problem that is modeled by an equation with an unknown component,” which is also the third indicator used to assess students’ mathematical competencies, the students from the EG demonstrated high results, with 75.2% of the students having mastered this activity. By comparison, the specified competence was mastered by 56.8% of the fourth-grade students from the CG, who were not taught according to the presented innovative model.

## 5. Conclusion

The pedagogical observations carried out during the experimental study, which lasted for three consecutive school years, as well as the presented results from the conducted final diagnostic assessment, confirm the previously formulated hypothesis: “If the author’s innovative model of mathematics instruction is applied in grades 2–4, it would contribute to mastering and deepening students’ knowledge and skills for finding an unknown component in equations, as well as to developing the mathematical and logical thinking of students at the primary stage of primary education.” The analysis of the results clearly shows that instruction supported by the author’s innovative model helped the students from the EG apply the acquired competencies more easily and effectively when composing an equation with an unknown component based on a given mathematical text and when solving word problems that are modeled by an equation with an unknown component.

The implemented longitudinal pedagogical experimental study shows that systematic work involving the solving of a sufficient number of tasks, diverse in type, for applying knowledge and skills related to finding an unknown component in equations leads to a higher level of mastery of the specific mathematical competencies. This will undoubtedly have a positive impact on students’ confidence and self-esteem in mathematics lessons.

## ACKNOWLEDGEMENTS

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