Study of the Operation of Dividing Natural Numbers by a One-Digit Number: Analysis of the Current State in Mathematics Textbook Sets for Grade 4

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Abstract

The most recent reform in primary mathematics education in Bulgaria, implemented in 2017, initiated substantial changes in the curricula and led to the approval of seven new textbook sets for the primary stage. The present paper is devoted to a comprehensive theoretical and practical analysis of the methodology for teaching division of multi-digit numbers by a one-digit number in Grade 4. This topic is of critical importance for the development of mathematical literacy and logical thinking among primary school pupils.

The study identifies and substantiates eight key methodological cases arranged in a strict logical sequence according to their cognitive complexity. The system begins with oral division of numbers expressed in place-value units and proceeds through algorithms without regrouping, cases with one or more regroupings in adjacent and non-adjacent place values, and specific difficulties related to zero in the quotient and cases in which the leading digit of the dividend is smaller than the divisor.

A central focus of the research is the comparative analysis of the current textbook sets published by “Arhimed,” “Riva,” “Prosveta Plus,” “Bulvest 2000,” “Anubis,” “Prosveta,” and “Bit i Tekhnika.” The paper examines how each author team implements the proposed sequence and where methodological differences or unjustified jumps in difficulty are observed. The results show varying degrees of correspondence with the logical progression of the material, which directly affects the quality of students’ acquisition of written algorithms. The paper offers valuable guidance for primary school teachers and methodologists, emphasizing the need for a systematic approach to teaching arithmetic operations in Grade 4.

Keywords: *mathematics education; division of natural numbers; Grade 4; textbook analysis; primary school*

Introduction

In recent years, educational reform in Bulgaria has been primarily directed toward the alignment of national educational standards with European Union standards. In this process, curricula have become a fundamental instrument for defining instructional aims, content, and expected learning outcomes for a given subject within a particular grade. The main objective of this process is to ensure that pupils receive high-quality education that is both appropriate and competitive in an international context.

Each topic in the curriculum is associated with specific expected outcomes and key concepts from different competence areas, with emphasis placed on the acquisition of arithmetic skills. These outcomes include knowledge aligned with the subject matter as well as with the age and cognitive capabilities of pupils. The methodological guidelines embedded in the mathematics curriculum for Grade 4 emphasise the use of technologies and strategies for mastering arithmetic operations such as multiplication and division of multi-digit numbers by one-digit numbers. This underscores the need to create an adaptive learning environment resposive to pupils’ individual needs.

1. Method

The aim of this paper is to present the theoretical propositions that form the basis of the methodology for studying the topic “Division of a Multi-Digit Number by a One-Digit Number” and its implementation in the currently available mathematics textbook sets for Grade 4 of primary education.

Division of a multi-digit number by a one-digit number is taught through algorithms for written computation and through methods of oral computation.

A specific feature of division is that, in both oral and written procedures, calculations always begin from the highest place value of the dividend.

Written division is carried out in strict accordance with the following algorithm: activation of prior knowledge and skills concerning the meaning of division; the algorithm for dividing numbers up to 1000 by a one-digit number; the distributive property of division with respect to addition; and the ability to represent multi-digit numbers as sums of their place-value units in different ways.
The cases of division of numbers greater than 1000 by a one-digit number are grouped into two main categories:
Division without regrouping, in which each place value of the dividend is divided exactly by the divisor; and division with regrouping, in which at least one place value requires transformation.
The algorithms for computation should be studied in a specific sequence. The study of the algorithms for written and oral division of a multi-digit number by a one-digit number is organised in the following logical progression:

1. Division of numbers of the type 6000 : 2; 6,000,000 : 2 and 1500 : 3; 15,000 : 3; 15,000,000 : 3
2. Division of a multi-digit number by a one-digit number without regrouping, for example 4682 : 2
3. Division of a multi-digit number by a one-digit number with one regrouping, for example 3268 :
4. Division of a multi-digit number by a one-digit number with more than one regrouping in non-adjacent place values, for example 3674 : 2.
5. Division of a multi-digit number by a one-digit number with more than one regrouping in adjacent place values, for example 4386 : 3.
6. Division of a multi-digit number by a one-digit number when one of the digits in the quotient is zero, for example 6128 : 2.
7. Division of a multi-digit number by a one-digit number when more than one digit in the quotient is zero, for example 61,524 : 3.
8. Division of a multi-digit number by a one-digit number when the number of units in the highest place value of the dividend is less than the divisor, for example 1926 : 3.

This approach to arranging and studying the cases of division by a one-digit number is logically justified. It systematically derives all computational algorithms and organises them according to their complexity. The first of the listed cases is based on the decimal structure of the multi-digit number, with oral methods of division being applied. In deriving the algorithms for the subsequent cases, the distributive property of division with respect to addition is applied.
In order for pupils to recognize the practical necessity of mastering the algorithms for finding the quotients of multi-digit numbers by a one-digit number, it is appropriate to use a word problem as an introductory task, the solution of which requires the application and clarification of the new algorithm.

2. Analysis of Individual Cases

2.1. First Case: Division of Numbers of the Type 6000 : 2; 6,000,000 : 2; 1500 : 3; 15,000 : 3; 15,000,000 : 3

In deriving oral methods for division of numbers of the type 6000 : 2 and 6,000,000 : 2, the following conversions are used:
number ↔ number of thousands, for example 6000 = 6 thousand, 3 thousand = 3000;
number ↔ number of millions, for example 6,000,000 = 6 million, 3 million = 3,000,000.
The computation is reduced to basic multiplication facts.
For example:
6000 : 2 = 6 thousand : 2 = 3 thousand = 3000;
6,000,000 : 2 = 6 million : 2 = 3 million = 3,000,000.
The quotients of the type 1500 : 3; 15,000 : 3; 15,000,000 : 3 are clarified through analogous transformations, together with multiplication facts.

2.2. Second Case: Division of a Multi-digit Number by a One-digit Number without Regrouping (for Example 4682 : 2)

2.3.

An analysis of the approved mathematics textbook sets for Grade 4 shows diverse approaches to teaching the division of a multi-digit number by a one-digit number without regrouping, for example 4682 : 2.
The methodology for introducing this case is as follows:
the dividend is represented as a sum of its place-value units, and the distributive property of division with respect to addition is applied:
4682 : 2 = (4 thousands + 6 hundreds + 8 tens + 2 units) : 2

= 2 thousands + 3 hundreds + 4 tens + 1 unit = 2341.
The written algorithm is explained with the help of a place-value table in which the dividend is recorded, emphasising its decimal structure. The division sign and the one-digit divisor are written to the right of the table. It is emphasized that division begins with the highest place value, in this case the thousands.

Thousands	Hundreds	Tens	Units
4	6	8	2
2	3	4	1

: 2

Written as:
4682 : 2 = 2341

The written algorithm for division of numbers of this type (4682 : 2) is as follows:
1. Divide the thousands of the dividend by 2. We obtain 4 thousands : 2 = 2 thousands. Write 2 thousands in the result.
2. Divide the hundreds of the dividend by 2. We obtain 6 hundreds : 2 = 3 hundreds. Write 3 hundreds in the result.
3. Divide the tens of the dividend by 2. We obtain 8 tens : 2 = 4 tens. Write 4 tens in the result.
4. Divide the units of the dividend by 2. We obtain 2 units : 2 = 1 unit. Write 1 unit in the result.

We obtain 2 thousands + 3 hundreds + 4 tens + 1 unit = 2341.
Finally, the procedure is presented without the table.
In my view, the methods for dividing a multi-digit number by a one-digit number without regrouping can be summarized as follows:

First method:

4682 : 2
= (4000 + 600 + 80 + 2) : 2
= 4000 : 2 + 600 : 2 + 80 : 2 + 2 : 2
= 2000 + 300 + 40 + 1 = 2341

Second

4682 : 2 = 2341

In the first method, the dividend is represented as a sum of its place-value units, and the distributive property of division with respect to addition is applied.
In the second method, the written computation algorithm is applied.
Such cases are presented in all seven approved textbook sets for compulsory mathematics instruction in Grade 4: the set published by “Arhimed,” by R. Alashka and colleagues [1]; by “Riva,” by L. Aleksieva and M. Kirilova [2]; by “Prosveta Plus,” by V. Angelova and Zh. Koleva [4]; by “Bulvest 2000,” by M. Bogdanova, M. Temnikova, and V. Ivanova [5]; by “Anubis,” by T. Vitanov and colleagues [9]; by “Prosveta,” by Yu. Garcheva and A. Manova [6]; and by “Bit i Tekhnika,” by D. Kapitanova and colleagues [7].
In the textbook by V. Angelova and Zh. Koleva [3] the oral method of computation is first clarified, with the dividend represented as a sum of its place-value units and the distributive property applied; for the written method, a place-value table is used, and a clearly formulated algorithm for carrying out division is presented.
In the textbooks by L. Aleksieva and M. Kirilova and by D. Kapitanova and colleagues [7], a place-value table is used to explain the algorithm for performing division, but the algorithm is not explicitly formulated. The oral method of computation is insufficiently elaborated.
In the textbook by M. Bogdanova and colleagues [5], both an oral computation algorithm and a written computation algorithm are included. No tabular form is used.
The authors of the Grade 4 mathematics textbook, L. Vitanov and colleagues [9], first explain the oral method of computation; for the written method, they use a place-value table, but no clearly defined algorithm for written computation is provided.
In the Grade 4 mathematics textbook by Yu. Garcheva and A. Manova [6], no algorithm is formulated for the written division of a multi-digit number by a one-digit number; instead, pupils are directed to apply the corresponding algorithm for division with numbers up to 1000.

2.4. Third Case: Division of a Multi-digit Number by a One-digit Number with One Regrouping (for Example 3268 : 2)

In the third case, division in one of the place values is results in a remainder, which is carried to the next place value. The methodology for introducing this case is analogous to that described in section 2.2. It is important that pupils understand the role of the remainder and its relation to the decimal structure of the number, rather than perceiving regrouping as a purely formal operation.
In the textbook by "Arhimed," these tasks follow immediately after the cases without regrouping; an oral method is demonstrated through decomposition of the dividend and the carrying of the remainder, after which the standard written algorithm is shown. In the textbook by "Riva," cases with one regrouping are also included, but the emphasis is mainly on written division by example, with more limited oral explanations of the written algorithm.
In the textbook by "Prosveta Plus," the methodology is developed in greater detail: an oral method is first presented, and then the written algorithm is introduced with the corresponding steps and comments in a place-value table. In the textbook by "Bulvest 2000," both oral and written methods are presented, but greater emphasis is placed on the standard vertical layout. In the textbook by "Anubis," a place-value table is used, but a clearly formulated step-by-step algorithm is not provided, while in the textbook by "Prosveta," reliance is placed on transferring knowledge of division of numbers up to 1000 by a one-digit number. The textbook by "Bit i Tekhnika" includes a variety of tasks with one regrouping, but the algorithm is again presented mainly through solved examples.

2.5. Fourth Case: Division with More than One Regrouping in Non-adjacent Place Values (for Example 3674 : 2)

The fourth case involves divisions in which more than one remainder appears in different, non-adjacent place values, requiring pupils to follow a longer sequence of regroupings. The methodology for introducing this case is analogous to that described in section 2.2.
In the textbook by "Arhimed," such tasks appear after sufficient practice with simpler cases and are often marked as more difficult or included in sections intended for independent work.
In the textbook by "Prosveta Plus," these cases are introduced gradually, with visual support through a place-value table used purposefully to illustrate the carrying of remainders.
In the textbooks by "Riva," "Anubis," "Prosveta," and "Bit i Tekhnika," similar tasks are included, but they are not always clearly distinguished as a new type of difficulty; they are often distributed among general exercises without specific methodological commentary, which makes it more difficult for pupils to recognise the qualitatively new level of complexity.

2.6. Fifth Case: Division with More than One Regrouping in Adjacent Place Values (for Example 4386 : 3)

The fifth case is even more cognitively demanding, as remainders arise consecutively in adjacent place values and pupils must manage several successive regroupings without losing the logic of the algorithm. As in the previous cases, the methodology for introducing this case is analogous to that described in section 2.2.
The textbook by "Prosveta Plus" presents methodological guidance for such tasks through detailed solutions and intermediate questions that direct pupils' attention to each individual regrouping step.
In the textbook by "Bulvest 2000," tasks involving successive regroupings are also included, along with diagnostic exercises that allow for assessment of pupils' understanding.
In other textbook sets, such cases appear more rarely and are often not explicitly distinguished from cases with one regrouping, which may lead to a more mechanical application of the algorithm.

2.7. Sixth Case: Division of a Multi-digit Number by a One-digit Number When One Digit in the Quotient is Zero (for Example 6128 : 2)

The sixth case encompasses divisions in which one of the digits in the quotient is zero. It is methodologically important because pupils often omit the zero in the quotinent or fail to understand that it is a significant digit in the positional number system. The methodology for introducing this case is analogous to that described in section 2.2.
In the textbooks by "Riva" and "Bulvest 2000," specially selected tasks are included in which the quotient contains a zero, and the explanations emphasise the necessity of writing the zero in the correct place in the quotient. In the textbook by "Prosveta Plus," guiding questions are also used ("How many tens do

we obtain? If there are no tens, what do we write in the quotient?"), which help pupils understand the role of zero.
In the textbooks by "Anubis," "Prosveta," and "Bit i Tekhnika," tasks with a zero in the quotient are included in the general set of exercises, but without specific methodological separation or additional oral or written explanations, which increases the likelihood of errors.

2.8. Seventh Case: Division of a Multi-digit Number by a One-digit Number When More than One Digit in the Quotient is Zero (for Example 61,524 : 3)

The seventh case involves division situations in which more than one digit in the quotient is zero, which places increased demands on pupils' attention and on their understanding of sequences of zeros in the quotient. Such tasks are encountered relatively rarely and more often found in sections for consolidation or supplementary practice. The methodology for this case is analogous to that described in section 2.2.
In most textbook sets, with the exception of isolated examples in "Prosveta Plus" and "Bulvest 2000," there is no targeted work aimed at helping pupils understand a sequence of zeros in the quotient, which leaves this case partially embedded within the general flow of exercises.

2.9. Eighth Case: Division of a Multi-digit Number by a One-digit Number When the Number of Units in the Highest Place Value of the Dividend is Less than the Divisor (for Example 1926 : 3)

The eighth case is conceptually challenging, as it requires pupils to realise that they cannot divide the leading digit of the dividend alone, but must take the first two digits together, which changes the position of the first digit in the quotient. The methodology for introducing this case is analogous to that described in section 2.2.
In the textbooks by "Arhimed," "Prosveta Plus," and "Bulvest 2000," such tasks are introduced after secure mastery of the other cases, accompanied by oral explanations and visual examples showing why the first digit cannot be divided separately and how this affects the place of the quotient.
In the textbooks by "Riva," "Anubis," "Prosveta," and "Bit i Tekhnika," similar tasks appear earlier in the sequence and without sufficient methodological emphasis, which increases the likelihood of errors and a more mechanical application of the procedure.
Conclusions and Methodological Recommendations
Summarising the analysis across the eight cases, the following conclusions can be drawn:
The proposed system of eight cases fully encompasses the range of division situations involving multi-digit numbers by a one-digit number within the primary curriculumand may serve as a criterion for evaluating the completeness and methodological soundness of textbook sets.
Although all textbooks formally comply with the curriculum, there are substantial differences in how consistently they follow the logical progression of the cases and in the way they emphasise difficult situations.
The most effective textbook sets are those that combine oral methods, visual support, and clearly formulated algorithms, and that provide a sufficient number of tasks involving more than one regrouping and zero/s in the quotient.
From a methodological perspective, the following recommendations are proposed:
The logical sequence of the eight cases should be strictly observed.
Oral and written methods should be systematically combined, with oral computation serving as a basis for understanding the written algorithm.
Special attention should be given to difficult cases such as multiple regrouping, zero/s in the quotient, and situations in which the leading digit of the dividend is less than the divisor.
Tasks with a practical orientation should be selected, in which division has real-life meaning, in accordance with the competence-based approach in education.
The role of the teacher is crucial: when deficiencies are identified in a particular textbook, the teacher should compensate through additional exercises, visual aids, and explanations, so as to ensure systematic and conscious mastery of division in Grade 4.

3. Conclusion

The study of division of multi-digit numbers by a one-digit number in Grade 4 plays a key role in developing stable arithmetic skills and in preparing pupils for more complex mathematical concepts in lower secondary education. Systematising the content through eight clearly distinguished

methodological cases ensures logical progression and enables more precise instructional planning at the primary level.

The analysis of the approved textbook sets shows that, although all of them meet the curricular requirements, there are substantial differences in the way algorithms are presented, in the degree of visual support, and in the emphasis placed on difficult cases related to regrouping and zeros in the quotient. This finding requires teachers to adopt a critical and reflective approach to the chosen textbook and, when necessary, to compensate for missing elements through additional tasks, oral methods, and visual models.

The proposed logical system of eight cases can serve not only as a tool for analysing existing textbook sets, but also as a guide for developing new teaching materials, supplementary aids, and methodological resources for primary mathematics education. In this sense, the results of the study are relevant both to classroom practice and to future research in the methodology of mathematics teaching.

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