Diagnostic Analysis of Students' Errors in Integral Calculus: Method Selection as a Source of Difficulty in Undergraduate Mathematics

Siditë Duraj[1], Lekë Pepkolaj[2]

[1]University of Shkodra "Luigj Gurakuqi", Albania
[2]University Metropolitan Tirana, Department of Computer Science, Albania

Abstract

Throughout several years of teaching Mathematical Analysis to first-year students in the Bachelor program in Informatics, we have observed that students encounter significant difficulties in selecting an appropriate integration method for a given problem. Our study aims to investigate the major difficulties related both to the selection of integration techniques and to the conceptual understanding of the structure of the integral itself, with the purpose of identifying instructional interventions that may help address these difficulties, which subsequently affect students' ability to correctly apply integration methods.The study was conducted with first-year students at the University of Shkodra "Luigj Gurakuqi" enrolled in the Bachelor program in Informatics, focusing on the transition from procedural methods of integration learned at the pre-university level toward more advanced reasoning grounded in structural understanding. The analysis revealed a tendency among students to apply familiar integration methods without adequately evaluating their suitability for the given problem. The findings indicate that these difficulties are mainly related to misconceptions regarding the structure of integrals and to difficulties in decision-making during the selection of an appropriate method.The study emphasizes the importance of engaging students in the comparison of alternative solution strategies and in the integration of diagnostic tasks in order to systematically address these deficiencies.

***Keywords**: Integral calculus; students' errors; method selection; diagnostic assessment; mathematical understanding*

1. Introduction

The transition from secondary to university education represents a major challenge, first in terms of learning approaches and secondly in relation to the academic requirements that demand a higher level of abstraction in scientific subjects. Students often face difficulties adapting to new teaching approaches, the large volume of course material, and the increased emphasis on analytical and independent thinking. Mathematical Analysis constitutes one of the greatest challenges for first-year students enrolled in study programs with a mathematical component. These difficulties are also evident in the understanding and application of integration methods [1], [2].

A fundamental issue that has remained relatively underexplored is that students' difficulties do not lie solely in the technical execution of integration procedures. Many students are able to correctly apply familiar methods when these are presented explicitly, yet they encounter difficulties when they must independently decide which method should be used and why. This moment of decision-making, which precedes every solution process, constitutes the gap that the present study aims to investigate.

First-year students enrolled in the Bachelor program in Informatics study Mathematical Analysis as a compulsory subject and generally come from a pre-university educational background oriented mainly toward algorithms and familiar exercise types, an approach that tends to favor procedural rather than structural reasoning.

Orton (1983) documented that students tend to treat integration as a sequence of mechanical steps without connecting procedures to conceptual understanding [1]. Mahir (2009) confirmed that procedural performance and conceptual understanding constitute two distinct dimensions [2]. Radatz (1979) showed that mathematical errors are systematic symptoms of the way students construct their understanding [3]. Nursyahidah and Albab (2017) linked difficulties in mastering integration to limitations in critical thinking skills [4].

This study contributes empirical evidence from the Albanian context on error patterns in the choice of integration method. The main theoretical contribution lies in the identification of a recurring pattern: premature activation of the method, without prior analysis of the problem structure, as the dominant source of students' errors. The study is guided by three research questions:

RQ1: How do first-year students identify the structure of the integral when choosing a method?
RQ2: What types of errors characterize method selection and what are their structural and conceptual sources?
RQ3: What metacognitive patterns emerge in students' decision-making during the integration process?

2. Theoretical Framework

2.1 Procedural and Conceptual Understanding

Hiebert and Lefevre (1986) proposed a foundational distinction between procedural understanding the capacity to execute learned algorithmic steps correctly and conceptual understanding, conceived as an interconnected network of ideas that confers meaning upon those steps [5]. In the context of integral calculus, the absence of conceptual grounding produces a characteristic fragility: students perform adequately in familiar problem types, yet encounter difficulties as soon as the structural configuration of the integral changes, even marginally.

2.2 Process-Object Duality

Sfard (1991) advanced the theory of process-object duality, according to which mathematical concepts may be apprehended either as processes or as objects [6]. Structural, object-based understanding is precisely what enables type recognition and, consequently, method selection. According to Tall (1991), many students entering university continue to rely mainly on operational forms of reasoning and experience difficulties in developing a more structural understanding of mathematical concepts [7].

2.3 Classification of Errors

Radatz (1979) classified mathematical errors as systematic symptoms of underlying misconceptions, identifying three principal categories: errors arising from misinterpretation of symbolic notation, errors from the context-independent application of algorithms, and errors attributable to gaps in conceptual knowledge [3]. Orton (1983) further distinguished structural errors rooted in a failure to recognise the integral's underlying form from executive errors, which emerge during the execution of a correctly identified method [1]. Broader frameworks in the literature [9] extend this classification to encompass the distinction between systematic errors reflecting conceptual deficits and random errors produced by carelessness or situational factors, with some analyses drawing on cognitive psychology to illuminate the role of mental processes in the emergence and resolution of such difficulties.

2.4 Metacognition and Decision-Making

Schoenfeld (1985) documented that lower-achieving students tend to persist along unproductive solution paths without monitoring their progress, whereas expert problem-solvers flexibly revise their approach in response to emerging evidence [8]. Related research [10] underscores the contribution of formative self-assessment to the development of metacognitive regulation, enabling students to monitor and adjust their mathematical reasoning more effectively. From this perspective, choosing an integration method is not only a technical issue. Students must also evaluate whether the chosen strategy is appropriate and whether it is leading toward a valid solution

3. Methodology

3.1 Research Design

The study adopted a qualitative diagnostic design, oriented towards the analysis of structural reasoning and decision-making patterns. The diagnostic approach enables the exposure of mental processes, not just outcomes, and provides a direct basis for instructional interventions.

3.2 Participants and Context

The study was conducted with $n = 35$ first-year students, enrolled in the Mathematical Analysis course in the Bachelor's program in Informatics of the University of Shkodra "Luigj Gurakuqi" (academic year 2025–2026). Of these, 29 students submitted tests with at least partial answers, while 6 students did not complete any section, classified as level L0 (structural disengagement). The test was administered during class, without time pressure and without impact on students' grades. Participation was voluntary and anonymous.

3.3 Instrument

The diagnostic test "Integration Methods" was developed specifically for this study (100 points, open-ended, eight sections, 22 problems). The five integration methods are clearly labeled: M1 Direct, M2 Substitution, M3 By Parts, M4 Manipulation, M5 Standard. For each exercise, the student is required to: (a) identify the structure of the integral, (b) choose the appropriate method, and (c) justify the choice in a structural manner.
Section I (Structural Analysis) contains nine exercises selected to test structural differences, particularly the pairs 2a/2b, 4/9, and 5/7.
Section II (Decision Making) explores the initial method and structural indicators.
Section III (Abstract Structure) tests the transfer of reasoning to the forms $\int \frac{f'(x)}{f(x)} dx$, $\int f(x)\, dx$ and $\int f'(x) e^{f(x)} dx$.
Section IV (Error Analysis) requires critical explanation of the erroneous rule $\int f(x) \cdot g(x) d\,x \neq \int f(x) dx \cdot \int g(x)\, dx$.
Sections V–VIII collect metacognitive data and explore students' heuristics in choosing a method.

Table 1. Exercises of Section I Structural Analysis

No.	Integral	Diagnostic Purpose
1	$\int xe^x\, dx$	Intended to assess recognition and application of integration by parts in a product structure involving algebraic and exponential functions (M3).
2a	$\int \frac{2x}{x^2+1}\, dx$	Intended to assess recognition of the logarithmic derivative structure $\frac{f'(x)}{f(x)}$, leading to logarithmic integration (M2).
2b	$\int \frac{x}{x^2+1}\, dx$	Designed to assess students' ability to adapt the logarithmic derivative structure when the exact derivative factor is not explicitly present (M2).
3	$\int \ln x\, dx$	Intended to assess recognition of implicit reformulation as 1·ln x, requiring integration by parts (M3).
4	$\int \frac{x^2}{x^2+1}\, dx$	Requires prior algebraic restructuring before integration can proceed (M4).
5	$\int \frac{x}{x^2 - 1}\, dx$	Designed to examine premature transfer of the $\frac{f'(x)}{f(x)}$structure without sufficient structural verification (M2/M4).
6	$\int \frac{\sin x}{1+\cos\, x}\, dx$	Intended to assess transfer of the logarithmic derivative structure to a trigonometric context (M2).
7	$\int \frac{2x}{x^2-1}\, dx$	Intended to assess recognition of the logarithmic integration structure in a structurally similar but non-identical algebraic context (M2).
8	$\int \frac{1}{\sqrt{1-x^2}}\, dx$	Intended to assess recognition and retrieval of a standard inverse trigonometric integral form (M5).
9	$\int \frac{x^2+1}{x^2-1} dx$	Requires algebraic decomposition prior to the application of standard integration techniques (M4 + M5).

3.4 Coding System

Each response was coded according to four dimensions: (S) identified structure; (M) chosen method; (E) type of error: E0 = no error, E1 = weak reasoning, E2 = unclear criterion, E3 = wrong method, E4 = deep conceptual error, E5 = missing; (K) cognitive level (K1–K4). Based on the coding profiles, students were classified into six cognitive levels (L0–L5).

3.5 Data Analysis

The data were analyzed at three complementary levels: (1) analysis of error frequencies by sections and exercises; (2) categorization of reasoning at three levels: surface, procedural, and structural; (3) analysis of metacognitive models. The categories were constructed inductively and validated by two independent coders.

4. Results

4.1 Distribution of Students by Cognitive Level

Analysis of the 35 diagnostic tests identified six distinct cognitive profiles (Table 2). The L0 and L3 groups each represented 28.6% of the sample, together comprising 57.2% of the participants. The L4 and L5 levels represented only 14.3%, indicating that full functional competence in the structure–method link was rare. Only one student (2.9%) demonstrated a stable structure–method link (L5).

Table 2. Distribution of students by cognitive level ($n = 35$)

Level	Student Profile	n	%	Dominant Error Pattern
L0	Pre-structural / Non-engagement	10	28.6%	E5 (absence of response)
L1	Formula recall / Low structural activation	5	14.3%	E5 + E3
L2	Surface-level recognition / Decoupled reasoning	5	14.3%	E3 + E1
L3	Overgeneralization / Method fixation	10	28.6%	E3
L4	Partially functional / Structurally conflicted	4	11.4%	E2 + E1
L5	Selectively functional structural reasoning	1	2.9%	E2 (selective)
	Total	**35**	**100%**	

4.2 Structural Errors: Not Knowing the Structure of the Integral

The dominant error category was structural: students activated a method before analyzing the structure of the integral. The contrasting pair 2a/2b, $\int \frac{2x}{x^2+1}\,dx$ and $\int \frac{x}{x^2+1}\,dx$, showed that (19 out of 29) responding students applied the same procedure to both integrals, without identifying that $\int \frac{x}{x^2+1}\,dx$ requires prior adjustment. These results suggest that structure recognition is based on surface features, not deep analysis, in line with Mahir's (2009) characterization [2].

Exercise 4, $\int \frac{x^2}{x^2+1}\,dx$, confirmed this pattern. Most participants attempted the substitution without verifying whether x^2 is a derivative of x^2+1. Only students at levels L4–L5 performed the preliminary manipulation $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$. Section III showed that the transfer to the abstract form $\frac{f'(x)}{f(x)}$ did not occur spontaneously: most students left these exercises unanswered.

4.3 Procedural Errors: Incorrect Implementation after Correct Choice

A second category of errors was identified in students who chose the correct method but made errors during execution. Procedural errors appeared mainly in exercises requiring double application of

integration by parts ($\int x^2 e^x \, dx$) and in preliminary algebraic reformulation. According to Radatz (1979), these constitute execution errors, distinct from structural errors [3].
Exercise 17 showed that most students knew that the statement $\int f(x)g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx$ is false, but they failed to justify why, a phenomenon consistent with Hiebert and Lefevre (1986) [5].

4.4 Metacognitive Failures: Failure to Monitor the Process

Most students did not change their method during the process, or only changed it when they were completely stuck. Only a small percentage reported actively monitoring their progress, a finding consistent with Schoenfeld (1985) [8]. From the reflection section, the first step most reported was "try substitution" or "check if it is direct," not "analyze structure." This confirms that students' spontaneous approach is primarily procedural, not structural.

Table 3. Main error patterns by category and theoretical basis

Exercises	Observed Error	Category	Theoretical Basis
2a vs 2b	Identical procedures applied to structurally distinct integrals	Structural	Werner Radatz (1979)
4	Substitution attempted without prior algebraic restructuring	Structural	Anthony Orton (1983)
9	Direct substitution attempted without the required algebraic reformulation	Structural	Anna Sfard (1991)
10–14	No change of method observed during the solution process	Metacognitive	Alan Schoenfeld (1985)
10–14	High confidence reported alongside incorrect solutions	Metacognitive	Nizam Mahir (2009)
17	Inability to justify why $\int f(x)g(x)\,dx \neq \int f(x)\,dx \cdot \int g(x)\,dx$	Procedural	Werner Radatz (1979)
18	Failure to verify structural conditions prior to method selection	Structural	Anthony Orton (1983)

5. Discussion

5.1 Premature Procedural Activation Without Structural Analysis

The central finding of this study, that students activate a method before analyzing the structure of the integral, constitutes the main theoretical contribution. This phenomenon, which we term premature procedural activation, explains most structural errors and can be interpreted through Sfard's (1991) process-object framework: students operate with an operational, not a structural, conception of the integral [6]. The transition that Tall (1991) identifies as essential from operational to structural thinking had not yet been realized for most of the sample [7].
The pedagogical implication is clear: it is not enough to learn the application of a method; it is essential to develop the ability to recognize the structure that requires it. In this sense, the sequencing of teaching must be reoriented, placing structural analysis before procedures.

5.2 Consistency with Existing Literature

The results are in full agreement with Mahir (2009), who documented that students with a dominant procedural understanding identify the type of problem based on its appearance [2]. The pair 2a/2b confirms this mechanism clearly. The metacognitive failure documented by Schoenfeld (1985) is reflected in students' insistence on using unproductive methods [8]. Overall, these findings suggest that the fundamental difficulty in method selection is not technical, but epistemological.

6. Implications For Teaching

First, the systematic use of contrasting pairs (e.g., 2a/2b), characterized by superficial similarities and structural differences, promotes the development of structural recognition and requires deliberate, ongoing exposure.
Second, requiring a written justification as a mandatory step, “Why did you choose this method?”, encourages structural engagement before selection. This practice can be integrated as a routine component of the exercises.
Third, teaching should position structural analysis as an explicit phase of the solution. The question “What do I notice in this integral?” should be the first step, clearly articulated, not assumed.
Fourth, treating typical errors (e.g., $\int f(x)g(x)\,dx \neq \int f(x)\,dx \cdot \int g(x)\,dx$) as case studies supports the transition from memorization to conceptual understanding.

7. Study Limitations

Despite the diagnostic value of the findings, this study should be interpreted within several limitations. First, the research was conducted within a single higher education institution and involved a relatively small number of students who agreed to participate in the study. In fact, this work initially emerged from the need to identify the main difficulties encountered by Bachelor students in Informatics in selecting integration methods, with the aim of focusing more directly on students’ difficulties rather than on a traditional linear approach to teaching.
In addition, since students were informed that the test would not be graded, not all participants engaged with the same level of seriousness and commitment throughout the diagnostic process. Although several students voluntarily agreed to participate, some of them left sections incomplete or provided only partial responses.
Furthermore, the present study represents primarily a diagnostic investigation of students’ difficulties and proposes a framework for pedagogical intervention. However, the effectiveness of these interventions was not empirically tested within the scope of this research and therefore remains open for future studies and broader comparative investigations. Future work might also explore whether targeted structural instruction, even in brief form, leads to measurable shifts in students’ method-selection strategies.

8. Conclusions And Recommendations

This study provides empirical evidence that the dominant difficulty faced by first-year students in solving integrals is not related to the technical execution of integration techniques, but rather to the structural analysis of the integral and the decision-making process involved in selecting an appropriate method of solution. The three categories of errors, structural, procedural, and metacognitive, reflect distinct dimensions of the same fundamental gap: the lack of transition from operational to structural understanding.
The diagnostic instrument used in this study provides not only a research tool, but also a pedagogical tool. The study has important limitations, since it was conducted within a single institution and the sample size is also relatively modest.
Future research should test the proposed interventions through comparative designs and expand the sample to other university contexts. A practical starting point would be to require students, as a regular habit, to write one sentence before beginning any solution: what they notice about the structure of the integral and why that observation matters for choosing a method of solution.

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