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Circular Functions: Unravelling the Mystery of Music

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Abstract

This work is based on a CLIL Unit realized in a class at the third year of senior high school, an IT technical institute. The Unit lasts 10 hours and deals with the relation between trigonometric functions and sound waves, focusing on the mathematical representation of notes and chords. This approach has a two-fold advantage: on one side it sparks students' interest, on the other side it allows to make clear at least one among all the practical implementation from the very beginning, in order to make more meaningful their learning. The plan has been developed by using authentic material, mainly from Khan Academy. For each lesson the students have been provided with a worksheet with instructions and activities for each phase of the one or two-hour lessons. Useful vocabulary for each lesson has been provided, as well. Teacher's notes for each lesson have been prepared.

The Unit has been developed in six lessons, including the final assessment with a two-hour written test. First lesson is devoted to a warm-up phase and some group activities on a video on sinusoids in music: periodic features, mathematical representation and manipulation. Then, by means of flipped class methodology, the students are asked to practice with trigonometric ratios and right triangles. After that, as a classroom activity, the trigonometric definitions of sine, cosine and tangent are generalized for all angles through the unit circle. The subsequent lesson is devoted to graphing trigonometric functions by deducing their values for remarkable angles. The last two lessons have been spent to analyze periodicity properties of sinusoids: period, midline, amplitude, phase shift. This topic is developed both by reading the graph and deducing the function and the reverse.

In the final part, the first video has been re-proposed to reanalyze notes and chords, sound in general, through their mathematical representation.

Keywords: Circular functions, waves, sinusoids, periodic properties

1. Introduction

The experience presented here is the result of a teaching learning path based on the CLIL methodology. CLIL acronym stands for 'Content and Language Integrated Learning'. This means that the foreign language is not the direct purpose of education but a natural part of the whole learning process, whose aims are wider and related to culture, environment and language content as well. Nonetheless, the foreign language used has an effect on the entire learning situation. It opens a broader view to other kinds of society and cultures that the learners interpret in a very personal way [1]. This main key of the methodology has been the founding idea of the maths Unit developed on the circular functions, as a final result of a training course on CLIL methodology.

The methodology and the linguistic strategies that the teacher employs in order to assist students to develop their cognitive skills, extend their understanding and become competent and independent second language learners are all included in the notion of scaffolding [2]. The metaphor of scaffolding is directly connected to the Zone of Proximal Development [3], which refers to the student's gradual development towards independence as a learner. This practice translates into the math Unit presented here in terms of an accurate time schedule, cooperative work, peer tutoring, reinforcement activities.

As mentioned above, the maths Unit on circular functions presented here has been developed with a special look at the social nature of CLIL methodology. In particular, it has been realised in a third year class of an IT technical institute, about 16 years' old students. The topic is closely related to electromagnetic waves and signals, that are part of the teaching program of a specific discipline of these studies, and this can suggest a cross-curricular approach. Nevertheless, it has been preferred not to relate the Unit to a strictly scholastic context. Rather, it has been developed around the music, a relevant aspect of everyday life of the students.

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Lesson	Time	Topic
1	2 hours	Trigonometric function and music: video from Khan + comprehension activities The sine of music
2	2 hours	From trigonometry in right triangles to the Unit Circle definition. Production, practice. Video from Khan Unit Circle Definitions
3	2 hours	Flipped lesson about graphing sine, cosine and tangent functions.
4	1 hour	Listening and practice activities about properties of sinusoids from graphs. (period, amplitude, frequency, midline, phase shift) Video from Khan: Midline-Amplitude-Period
5	1 hour	Flipped lesson about determining properties of trigonometric functions from equation. Reading and practice activities.
6	2 hours	Final test and assessment

Table 1: Plan of the CLIL Unit

2. CLIL Module

The module lasts 10 hours and is divided in lessons of one or two hours. According to CLIL philosophy, native speaker material has been used throughout, and specific attention has been paid in choosing different authentic material such as reading texts, videos, visuals. Indeed, the input provided by the materials should be as rich and varied as possible to accommodate various learning styles, and help students develop their language skills [5]. A lot of material has been selected from Khan Academy, which is a non-profit educational organization created in 2006 by Salman Khan [6]. The introductory video, 'The sine of music' is a YouTube video [7]. It has a dual role of both introduce students to the link between music and trigonometric functions and to present the technical aspects of the mathematical structure and how they translate into the musical language, that is part of everyday life.

Table 1 represents the plan of the Unit, divided into lessons. The objectives can be stated as follows.

At the end of the path, the students will be able to:

1. define circular functions - sine, cosine and tangent – by recognizing periodicity properties, symmetries and specific features (period, amplitude, frequency) and sketching their graphs using comparative and superlatives
2. use the fundamental goniometric relations by describing periodic phenomena with the appropriate math tools using technical vocabulary
3. model using trigonometry by translating real problems in maths using paraphrasing

Teaching strategies applied span a wide class of methods, mainly based on the *scaffolding*, also in the notion of Vygotsky, who emphasized the role of social interaction as being crucial to cognitive development. Then, from one side each authentic material is analysed with the help of pre/during/post activities and, on the other side, pair-work, peer tutoring and cooperative work are the main bricks of the lesson, as shown in Table 2.

Also the flipped classroom model is widely applied, leaving at home the first levels of the educational learning objectives of Bloom's taxonomy [8]: as homework the student deals with knowledge and comprehension while in class he starts to work on application and analysis. This allows to spend more time to address misconceptions and consolidate knowledge, that is particularly important in maths and science.

To manage the lesson both students and teacher are supplied with worksheet for each lesson, with all the phases with useful instructions and visuals as well as a final table with 'Useful expressions'.

All these principles and practice are summed up in Table 2, that provides the phase template of the first lesson, with phase declination with respect to time, type of activity, description of the activity, type of work and materials.

The Appendix provides a student worksheet, as an example of the tools used and the realization of the methodology.

Phase	Time	Type of Activity	Description of activity	Grouping	Material/Tools
1	10-15 mins	Warm up: contextualising	The students discuss on the relation between music and mathematics. They will make a list of the math tools needed to play music and to listen to it. They will write the list in their exercise books and then share it with the whole class.	group	Students' exercise books
2	10-15 mins	Pre-watching : focusing on the meaning of common-use terms	The students will discuss on the mathematical formulation of music by answering questions	Pairwork	Worksheet 1
3	25-30 mins	While-watching/listening	• Before listening/reading, they will read the text with blanks to make predictions about the missing words (in a text on the mathematical representation of sound) • Then the students will watch the video and fill the blanks	Pairwork	Worksheet 1 The sine of music
4	15 mins	Post-reading/listening	The students will answer true/false questions on text on the mathematical representation of sound and correct the false ones.	Pairwork	Worksheet 1
5	15 mins	Production: information gap activity	In pairs, students are given words or definitions and should correctly associate all of them by asking the other student the missing information.	Pairwork	Worksheet 1
6	10 mins	K-W-L activity	The students fill in a K-W-L chart by brainstorming their ideas on the Unit they are going to study.	group	Worksheet 1
7	5 mins	Assigning homework	At home, the students will watch a video. Moreover the students are asked to practice with trigonometric ratio in right triangles. The material is shared with Classroom (Flipped lesson to recall and strengthen the requirements)	solo work	Khan Academy Soh-Cah-Toa

Table 2: Phase Template of Lesson 1

3. Conclusion

As a conclusive section of the training course a questionnaire about their feelings on the CLIL experience. Among the others, some answers are really significant. When students were asked about their initial motivation in learning a technical discipline in English, 64% of the class has claimed 'enough, a lot', while their interest in math was usually really poor. Also, 85% has declared to be active with some difficulties/ active and participating to the activities; no one answered 'passive and not involved'. Again 71% has chosen the variety of material – videos, images and schemes – for supporting the learning process.

As a final evaluation of the difficulty of studying math in English, 50% responded 'rather difficult', the others 'simple/rather simple'. Nonetheless, to the question 'Would you do again this experience', 71% has chosen 'Yes'.

This shows that the very nature of the CLIL methodology has a positive influence on the teaching-learning process, acting on the initial motivation, as a stimulus during the implementation of the Unit and making the students really participating to the effort.

References

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Appendix: Overview of the CLIL Module

Graphing trigonometric function

PRODUCTION ACTIVITY/PRACTICE

Reproduce the graph of the cosine function as you have seen in the video for the sine function.

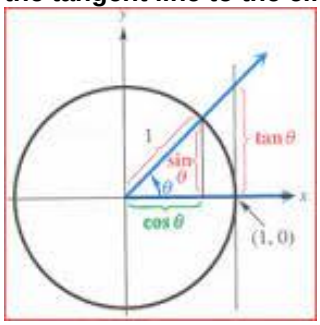
a) Draw a unit circle and –on the same line – a Cartesian plane.

b) Write down the values of the cosine function in the table below and then draw the function on the (θ, $\cos\theta$) plane – a Cartesian plane in which the horizontal axis (x-axis) represents the angle θ and the vertical axis (y-axis) refers to the function cos(θ).

	0	π/2	π	3π/2	2π	5π/2	-π/2
Θ (rad)							
$\cos(\Theta)$							

PRE-WATCHING ACTIVITY:

Recall that the tangent of the angle *θ* , tan (*θ*), is equal to the *y*-coordinate of the intersection of the tangent line to the circle in point (1,0) and the sloped side of the angle *θ*.



It can be demonstrated that the tangent of an angle is also equal to the ratio between sine and cosine of the same angle: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

This is a fundamental trigonometric relation.

Read the questions carefully and guess the answers.

1. What is the domain of the tangent function?
2. When you draw the graph of the tangent function, what do you graph on the horizontal axis (x-axis)?
3. What is the range of the tangent function? (The range is the set of all the values that the function can actually take on)
4. Which is the independent variable?
5. Is the tangent function defined also for π/2?
6. Reproduce the table below in your exercise book and complete it.

	0	π/2	π	3π/2	2π	5π/2	-π/2
Θ (rad)							
$\tan\theta$							

WHILE-WATCHING ACTIVITY

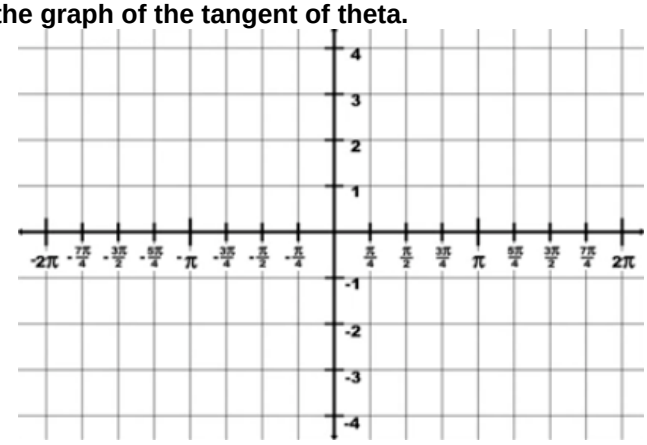
Read carefully true/false questions. Then, answer them while listening to the video. After the correction, copy the correct version of the sentences in your exercise book.

1. The tangent of theta is also equal to cos(Θ) over sin(Θ) T/F
2. The tangent of theta represents the slope of the oblique side of the angle T/F
3. Does the θ=π/4 rad angle corresponds to θ=90°? T/F
4. Sine and cosine value of θ=π/4 rad are equal to $\sqrt{2}/_2$ T/F
5. tan(π/4)=0 T/F
6. tan(-π/4)=-1 T/F
   When getting closer to pi over two, the slope of the oblique side gets closer to zero T/F
7. When getting closer to negative pi over two, the tangent approaches negative infinite T/F
8. When crossing pi over two, the slope becomes negative T/F

9. At θ =3π/4 rad we have slope equal to -1 T/F
10. The tangent graph has two asymptotes for θ=-3 π/2 and θ=3 π/2 T/F

POST-WATCHING ACTIVITY

Now, working in pairs and taking cues from the previous video and/or exercise, complete the Cartesian plane with the graph of the tangent of theta.



HOMEWORKS

1. At how many points do the graphs of y=sin(θ) and y=cos(θ) intersect for θ between 0 and 2π?

Answer the question and justify your choice by drawing both graphs and completing the table below.

	0	π/4	π/2	3π/4	5π/4	3π/2	7π/2	2π
Θ(rad)								
Sin(θ)								
Cos(θ)								

2. Why do the sine and cosine turn to be equal for θ= π/4?

3. How do you algebraically represent the intersection of the two graphs?

Check your answers by watching the third video at the link below.

SIne and Cosine intersections

USEFUL EXPRESSIONS

These are some useful expressions we are using during this activity. If necessary, look for the meaning and write down in your exercise book. You will need it!

Trigonometry	Description
Domain	**Let's set up…**
Range	**To graph a function**
Brackets	**That looks something like…**
Parenthesis	**Getting closer to…**
Infinity	**It approaches…**
Vertical asymptote	**They add up to…**
Pythagorean theorem	**To rationalise**