Brownian motion: an Interdisciplinary Teaching Proposal

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Abstract
In this work we present an educational proposal for high school students based on Brownian motion, an interesting phenomenon, easily observable but not so simply explainable. Why introduce Brownian Motion into the students curriculum? Because of large educational potential into STEM *education (Science, Technology, Engineering, and Mathematics): it links microscopic to macroscopic world, and allows integrating many disciplines.*
Starting from the Brownian motion's historical aspects, we want to highlight its transdisciplinary and interdisciplinary nature: mathematics, statistics, physics, computer science and biology could be involved and treated choosing the required level of detail, and extends to social and economic phenomena.
According to the new National Guidelines for the Italian high schools, which in turn take into account the strategies suggested by the EU for the construction of the " knowledge society ", [2] our work propose an "action-research" approach, including Mathematics, ICT and Physics laboratories, to teach and learn by discovering.
In order to contribute to revitalize school curriculum and improve digital literacies, *starting from historical Brown's experience, beside some classical experiments, real or virtual, we propose a simple macroscopic model of chaotic motion, and a "random walk" simulation made by using the software "Geogebra". Both experiences are easily implementable in a classroom activity.*
This proposal is addressed to teachers who want to build "learning that was less about acquiring, remembering, and repeating subject content, and more about active participation in scholarly ways of doing and being" [10]

1. A brief history of Brownian motion
The biologist Robert Brown in 1827, studying with a microscope pollen particles floating in water, observed minute pollen grains in continuous motion. He confirmed the existence of this motion in the fluids with other particles, and recognized the physical, not biological, nature of the phenomenon. Its origin remained unexplained, but stimulated other physics understanding [1] It was only in 1905 that Albert Einstein, using a probabilistic model, could right explain Brownian motion. He observed that if the kinetic energy of fluids was adequate, its molecules moved with "random walk". So, small particles staying in a fluid would receive a random number of impacts of random strength and from random directions in any short period of time. This "random bombardment" by the molecules of the fluid would cause a sufficiently small particle to move exactly just as Brown observed and described. [4]
Einstein obtained a relation between the macroscopic diffusion constant D and the atomic properties of matter. This relation is the following.

$$\langle x(t)^2 \rangle \cong 6Dt \qquad (1)$$
$$\text{whith } D = \frac{k_B T}{6\pi\eta a} \qquad (2)$$

where $x(t)$ is the particle displacement in the space during time t, $k_B = R/N_A$ is the Boltzmann's constant, $N_A = 6.06 \times 10^{23}$ particles/mol is the Avogadro's number, T is the temperature, η is the viscosity of the liquid and a is the radius of the Brownian particle.
Einstein found that the displacement of a Brownian particle is not proportional to the elapsed time, but rather to its square root: this is based on a conceptual switch from the "ensemble" of Brownian particles to the "single" Brownian particle. This result links coefficient D, that quantifies macroscopic proprieties, easily experimentally observable, with Avogadro's number, that expresses microscopic matter's nature: so that, measuring $\langle x(t)^2 \rangle$ we can obtain the atomic size. The Einstein explanation of this phenomenon was of fundamental importance because, after its experimental confirmation made by Perrin some years later (with a calculation of the forecasted Avogadro's number N_A) [9], the kinetic

theory was definitely confirmed. Brownian motion explanation was, indirectly, a confirm of the existence of atoms and molecules, and Brownian Motion has also become the paradigm of a large class of stochastic processes and, more generally, of the statistical mechanics of non-equilibrium.
The theory of Brownian motion has been extended to situations where the fluctuating object is not a real particle at all, but instead some collective property of a macroscopic system. In 1908, Langevin [7] introduced in the Brownian motion stochastic concept, founding equation containing both frictional forces and random forces. The random force is a stochastic variable giving the effect of a background noise due to the fluid on the Brownian particle. A Brownian particle in a liquid solution undergoes something of the order of 10^{12} random collisions per second with the particles of the environment. Therefore, we can make each interval $t_i - t_{i-1}$ macroscopically very small even though during it many collisions occur. These numerous impacts destroy all correlations between what happens during the time interval $(t_i - t_{i-1})$ and what has happened before t_{i-1}.
Therefore, all these works on Brownian motion are historically substantial, because they indirectly have confirmed the atomic theory, the kinetic theory and signed the origin of stochastic processes' study. Moreover, Brownian motion, that followed the physics of the twentieth century was connected by Feynman to quantum mechanics and is still relevant in today's physics [8].

2. Methodology

Introducing Brownian motion in a scholar's curriculum is an excellent opportunity to activate an interdisciplinary understanding. The ability to integrate knowledge and ways of thinking in many disciplines establish areas of expertise to produce a cognitive advancement – such as explaining a phenomenon, solving a problem, or creating a product – in ways that would have been impossible or unlikely through single disciplinary means. The skills produce skill in creating connections between knowledge and suggest their new uses and settings, which means "skills".
Following constructive teaching, we suggest to start from concrete states and prefer an inductive experimental hypothetical-deductive approach. Everything we can know is the product of an active construction of the subject; as well as constructivists stand «the learning isn't the result of the development, learning is the development» [5].
Learning by doing is very important: in addition to theoretical explanation, we propose to introduce many experimental activities around the exploration of Brownian motion. Starting from historical Brown's experience, we propose a simple macroscopic model of chaotic motion, easily implementable in a classroom activity.

3. Educational Activities

All the proposed activities were structured through interdisciplinary teaching that could allow students to acquire new points of view, in order to overcome a fragmented knowledge "bounded" within a single discipline. Our proposal does not focus only on the subject, but also on methodological aspects. The teaching content, on which we will not pause in this paper, cover a variety of topics in mathematics, probability and statistics, classical and modern physic, and can be developed according to the interest of teachers and specific educational classroom setting. The topics are proposed emphasizing their historical importance. Students are placed in the core of the whole process of teaching and learning. In this way, they are allowed to acquire a general attitude in asking questions, in treating problems, and in mastering knowledge. Carrying out the project has called upon different knowledge expertise and personal resources to handle situations, while building new knowledge and skills, always with the ultimate purpose of the education of men and citizens. This process of metacognition could be constructed through reflection and reconstruction steps in which a student learns. Experimental practice is necessary beside theoretical activities. In fact, in the same way we cannot simply teach in an abstract, formal and theoretical way, without a context, we cannot even leave students at the very early stage of experience and "doing". The main educational power of the choosed argument is the adaptability to different didactic levels. It's possible by only doing observation and experimentation of the phenomenon or, for the clever high school students, to introduce mathematical and statistical analysis. Sthocastical differential equation require not simple mathematical tools; we propose a simplified Langevin's mathematical analysis, where the time variable is discrete [11]. For the theoretical arguments we invite to refer to the bibliography; about the experiences to be done in laboratory, we would like to suggest the following activities:

3.1 Random walk Simulation by Geogebra

As first laboratory step, the students will construct, with the teacher's support, an application that simulates the Brownian motion in two dimensions, using the educational software "Geogebra". It follows an example of possible activities with ICT: based on Langevin's discussion, we will describe the "random walk" starting from time 0 seconds and increasing with discrete intervals. At time 0 seconds, the position of the "particle" is at the origin of a bidimensional frame of reference is obtained (0.0). On the later instant the position is obtained by adding up at the value of the x-axis and y-axis the value "CasualeUniforme [-1, 1]", which returns a random number between -1 and 1. In this way, the "particle" move from the right to the left, upwards or downwards, randomly. The positions are shown in the "View Chart" in Geogebra.

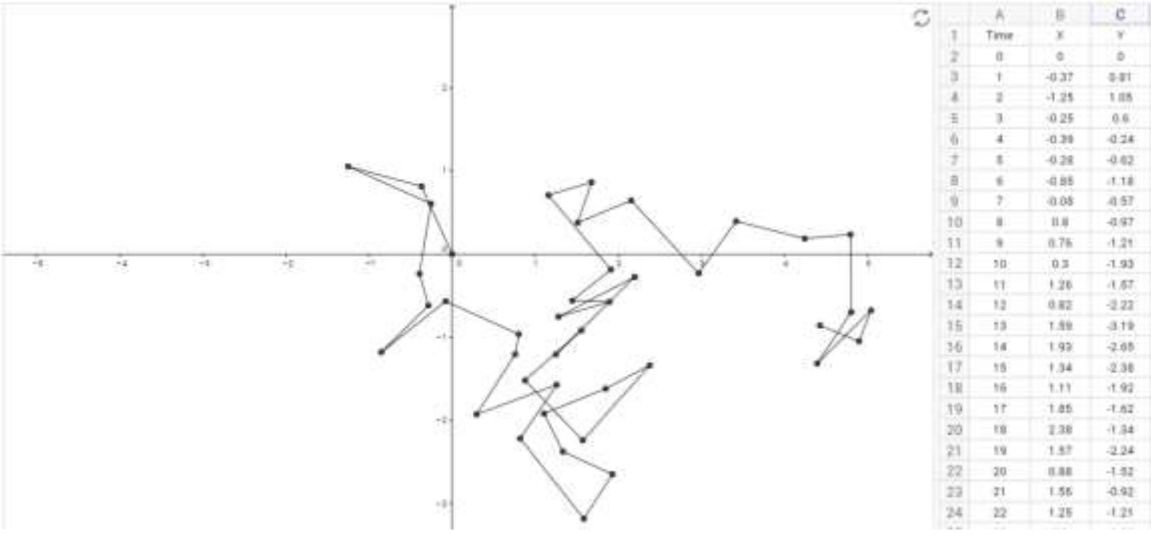

Figure 1: Example of two-dimensional Brownian's motion simulation made by software Geogebra

Legend: In the second and the third column there are the X and Y particle's coordinate .
It is possible to update the construction to obtain different simulations. This Geogebra construction allows students to observe graphically a random motion and so, to create a Brownian's motion simulation. The creation of computer application allows students to work out skills that are the bases for "digital literacy", meant as the skill to understand and use the information in different format, the knowledge of logics and basic functions and available services, the critic thinking for discerning the sense from the contents [6]. This simulation by Geogebra is available at: https://tube.geogebra.org/material/simple/id/2294315.

3.1 Physic laboratory

To introduce students to the study of Brownian motion it seems advisable to use the laboratory teaching. This teaching strategy, based on the use of research methodology, based on "lab" is not necessarily a physical space dedicated and fitted with tools and materials useful for research and production, but a cognitive context, also in the usual classroom, where teachers and students plan, experiences, suggest and compare solutions, searching for activating creativity as a way of working. The laboratory is a place for discovery, observation, action- research around cultural facts [3].
The laboratory activities proposed are:

- physics laboratory (reproduction of the historical Brown's experience on the pollen grains' motion in water, diffusion experiments in fluids)
- use of resources use on the web (watching videos of lectures and experiments on Brownian motion, watching videos of experiments and lessons on fluids' diffusion, use of applets on Brownian motion, using the software TRAKER to describe and quantify Brownian motion).
- The schematization that is usually given to view the molecular thermal agitation is the one of small spheres which move more or less quickly depending on the temperature. The online availability of movies and applet now makes that description for images easier and faster than drawings and photos of the texts.

- In addition, to show concretely the model spheres, we offer a tangible example, the type "hands-on", that provides experience of the phenomenon in question through manipulation and direct observation. Teacher can propose to build a simple macroscopic model achievable with simple materials. In figure 2 and 3 we show some possible exhibits: ping pong balls are caged in a nets container and a jet of air (made by hair dryer, for example) moves them. The motion obtained is not deterministic, due to the casual collision between balls. It is a simple but incisive experiment, that allow student to observe a macroscopic chaotic motion of particles in a fluid, with no apparent explanation.

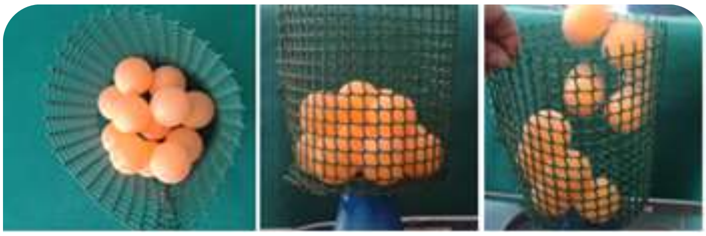

Figure 2: A simple exhibit simulate macroscopically Brownian motion: light balls "agitated" by air flow, are moving chaotically

 a)  b) 

Figure 3: a) light balls of different color are separated by grid. b) when the internal box is removed, the orange balls are diffused by air flow. This simple experiment simulate a diffusion phenomenon, as occurs to ink's drop in a water.

4. Conclusion

The fascination of Brownian motion is due to the connection between macroscopic scale and microscopic world, between the visible (the pollen particle) and the invisible (the water's molecules). Our proposal is to introduce students to the microscopic world and some physical, mathematical, statistical theories that describe it, starting from a simple experiment and doing software simulations besides theoretical study. History of Brownian motion after nearly 200 years remains pertinent today. We think also that this argument really lands itself to teaching avoiding the "sealed compartment of knowledge", that often limits educational systems.

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