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From Logic Puzzles to Logic Gates – Combining Fun and Practice in Teaching Introduction to Computer Science

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Abstract

Introduction to Computer Science is a foundation course in undergraduate programs in computer and computational sciences. It covers wide spectrum of topics in hardware and software systems, including evolution of computer architecture and its basic components, numbers and operations, binary and hexadecimal numeral systems, data representation as bit patterns, logic gates, etc. The course provides not only with the overview of the field, but also contributes to the development of general problem solving skill and prepares students for subsequent programming disciplines. We prioritize careful selection of problems and exercises in the teaching of Introduction to Computer Science to freshmen students in the first semester of their studies. In the current paper the approach is applied to the module of logic gates. We show that puzzles taken from the text "The Lady or the Tiger?: and Other Logic Puzzles, Including a Mathematical Novel that Features Godel's Great Discovery" by Raymond Smullyan (Times Books, New York, 1982) can be used for introducing the concepts of logic truth-tables, gates and circuits, and highlighting their main properties. As pointed by the author, these puzzles are tightly related to the fundamental theory and are of equal interest to logicians, mathematicians and computer scientists. We implement the gate-based solutions of selected puzzles and categorize them according to teaching activities, such as in-class exercises, home tasks and course projects. Finally, we outline the ties between different sections of the course due to the adopted puzzle format and discuss its impact on students' motivation and productivity.

1. Introduction

Introduction to Computer Science is a foundation course in undergraduate programs in computer and computational sciences. It provides not only with the comprehensive overview of the field [1], but also contributes to the development of general problem solving skill and prepares students for subsequent programming disciplines. Wide spectrum of topics in hardware and software systems includes binary arithmetic, data representation as bit patterns, logic gates and circuits. These are central topics in the computing curriculum. We prioritize careful selection of exercises in the teaching of Introduction to Computer Science to freshmen students in their first semester, and develop a module on logic gates based on puzzles taken from [2]. As pointed by the author, these puzzles are tightly related to the fundamental theory and are of interest to logicians, mathematicians and computer scientists.

Different approaches have been developed for teaching similar subjects. Difficulties in transition from theoretical knowledge to practical experience in digital electronics education are addressed in [3]. An interactive tutorial framework for teaching combinatorial and sequential circuits is developed in [4]. The method covers all core stages: Boolean algebra, truth-tables, reduction techniques and verification. A survey on open-source simulators of computer systems is conducted in [5], where the pedagogical value of simulations is also discussed. An embedded hardware platform is designed for teaching binary numbers in [6], and efficiency of serious toy-based method is demonstrated by the implemented curriculum.

2. Ladies and tigers

The second chapter of [2] is structured as a set of fairy tail episodes – a field hardly associated with binary logic. In each puzzle a prisoner must enter one of the prepared rooms knowing that either a lady or a tiger can be found there. To help the process, the king puts signs on the room doors and explains the rules. We represent rooms by binary digits – the input bits 〉–, and assign 0 to the state with a tiger and 1 – with a lady. The signs are modelled by logic gates, and the output bit –〉 represents the truthfulness of the sign or the entire problem. We start with a single door and a simple sign on it – "This room contains a lady". The truth-table connecting the input and output bits appears as:

Room ⟩–	Sign –⟩
0 (tiger)	0 (false)
1 (lady)	1 (true)

Table 1. Truth-table for the sign “This room contains a lady”

This is the identity operation, which is graphically represented by a plain wire that directly connects the input and output bits. Let us also consider the opposite sign – “This room contains a tiger”. The equivalent truth-table appears as:

Room ⟩–	Sign –⟩
0 (tiger)	1 (true)
1 (lady)	0 (false)

Table 2. Truth-table for the sign “This room contains a lady”

This is the NOT-gate. Both gates are depicted in Fig. 1:

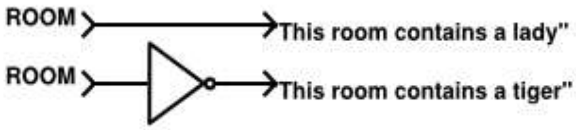

Figure 1. Identity and NOT gates

2.1 The trials of the first day

The puzzles are called “Trials” and grouped by days. Trials from the same day share certain similarities. The trial signs are reproduced from the second chapter [2]. We construct the truth-table and draw the corresponding circuit using standard gate definitions (see, for example, pages 21-26 in [1]). Trial 2 reads:

- Sign I: At least one f the two rooms contains a lady
- Sign II: A tiger is in other room
- Rule: Either both signs true or both false

Room I	Rom II	Sign I	Sign II	Sign I	Sign II	Rule
0 (tiger)	0 (tiger)	0 (false)	1 (true)	0	0	1 (satisfied)
0 (tiger)	1 (lady)	1 (true)	1 (true)	0	1	0 (unsatisfied)
1 (lady)	0 (tiger)	1 (true)	0 (false)	1	0	0 (unsatisfied)
1 (lady)	1 (lady)	1 (true)	0 (false)	1	1	1 (satisfied)

Table 3. Truth-tables for the signs and rule of Trial 2

Sign I is the output of OR-gate applied to both wires, while Sing II – NOT-gate applied to Room I. The rule connects both outputs with XNOR-gate. The circuit is shown in the left panel of Fig. 2. Students can verify that only the input (0, 1) – a tiger in Room I and a lady in Room II, satisfies the rule and, thus, results in the overall output 1, and compare it with the explained solution in [2]. Trial 3 reads:

- Sign I: Either a tiger is in this room or a lady is in the other room
- Sign II: A lady is in other room
- Rule: The same as in Trial 2

Room I	Rom II	Sign I	Sign II
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	1

Table 4. Truth-tables for the signs of Trial 3

The corresponding circuit in the right panel of Fig. 2 is obtained from the one of Trial 2 by swapping the branches of Room I wire. Such change is not commutative, and the solution is (1, 1) – there are ladies in both rooms.

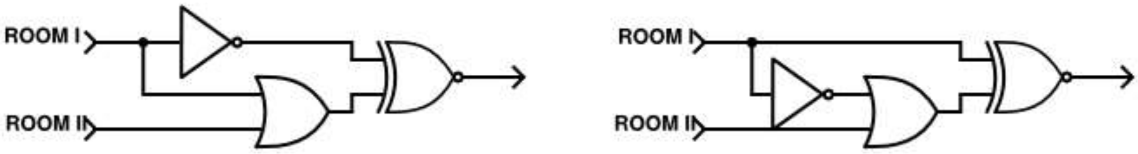

Figure 2. Circuits of Trial 2 (left) and Trial 3 (right)

Finally, Trial 1 reads:

- Sign I: In this room there is a lady, and in the other room there is a tiger

- Sign II: In one of these rooms there is a lady, and in one of these rooms there is a tiger
- Rule: One sign is true, but the other one is false

Literally, Sign I is the output of AND-gate applied to plain Room I wire (lady) and negated Room II wire (tiger), while Sing II – XOR-gate. The circuit is shown in Fig. 3.

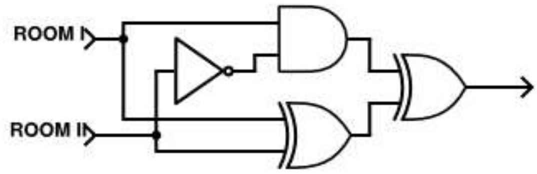

Figure 3. Circuit of Trial 1

2.2 The trials of the second day

All five trials share the same rule – Sign I is true, if there is a lady in Room I, and false, if there is a tiger there; and Sign II is true, if there is a tiger in Room II and false, if there is a lady there. Formally, Room I and Sign I wires are combined by XNOR-gate, Room II and Sign II wires – by XOR-gate, and the outputs – by AND-gate (left panel in Fig. 4).

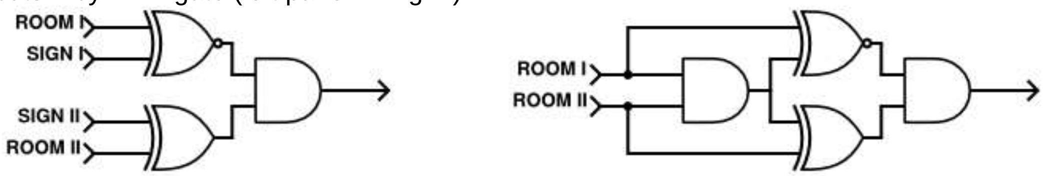

Figure 4. Circuits of the second day rule (left) and Trial 4 (right)

Building of circuits for subsequent trials, then, is reduced to the implementation of the signs and plugging them in the rule circuit. For example, both signs in Trial 4 state that both rooms contain ladies. Thus, the same AND-gate is branching for the signs, as shown in the right panel of Fig. 4. The next three trials have similar structure (Fig. 5):

- Sign I of Trial 5: At least one room contains a lady – OR-gate
- Sign I of Trial 7: It does make a difference which room to pick – XOR-gate
- Sign I of Trial 6: It makes no difference which room to pick – XNOR-gate
- Sign II of all three: The other room contains a lady – plain Room I wire

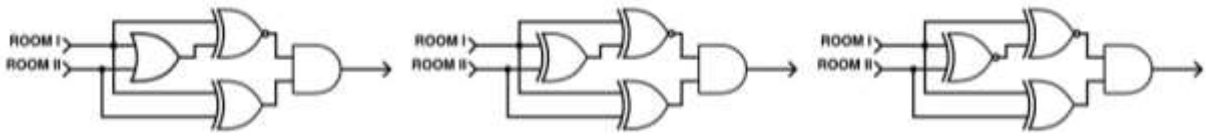

Figure 5. Circuits of Trial 5 (left), Trial 7 (center) and Trial 6 (right)

It is instructive to follow the solutions. All trials have the output of the bottom XOR-gate as an input to the concluding AND-gate. Its output is 1, if and only if both inputs are 1. Meanwhile, the output of XOR-gate is 1, if and only if its inputs are different. Therefore, the cases of two tigers (0, 0) and two ladies (1, 1) are excluded. It is easy to check that the solution to Trial 5 is (1, 0). In Trial 7 the OR-gate is replaced with XOR-gate, which differs only in (1, 1). Such input, however, is already eliminated. Therefore, (1, 0) solves Trial 7 as well. Finally, in Trial 6 the XOR-gate is replaced with the opposite XNOR-gate. Therefore, its solution is (0, 1).

Trial 8 formulates the signs but does not specify which one belongs to which room:

- Upper Sign: This room contains a tiger
- Lower Sign: Both rooms contain tigers

It is easy to check that the upper sign can only be the Sign II, as shown in the left panel of Fig. 6. So, the circuit of Trial 8 is reduced to the lower sign plugged in the upper branch of the rule circuit (right panel of Fig. 6).

Figure 6. Testing of the upper sign (left) and reduced circuit of Trial 8 (right)

2.3 The trials of the third day

The last group of trials involves more than two rooms. Trial 9 reads:

- Sign I: A tiger is in this room – negated Room I wire
- Sign II: A lady is in this room – plain Room II wire
- Sign III: A tiger is in Room II – negated Room II wire
- Rule: one room contains a lady and the other two contain tigers; and at most one sign is true

The rule is equivalent to an arithmetical statement that the carry of the sum of binary representations of all three signs remains 0. So, we start from the 3-input adder (left panel in Fig. 7) and then plug the sign gates in it (right panel in Fig. 7).

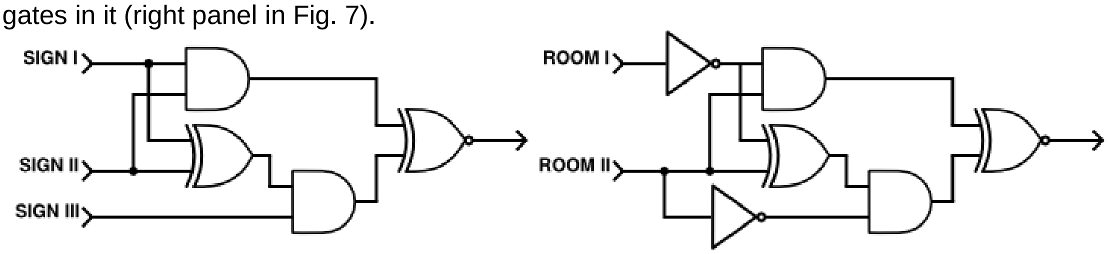

Figure 7. Circuits of 3-input adder (left) and Trial 9 (right)

Trial 10 reads:

- Sign I: A tiger is in Room II – negated Room II wire
- Sign II: A tiger is in this room – negated Room II wire
- Sign III: A tiger is in Room I – negated Room I wire
- Rule: The sign of the room with the lady is true and at least one of the other two signs is false

The rule is symmetric and eliminates cases (0, 0, 0) and (1, 1, 1). It is easy to check that pair-wise connection of signs by XOR-gates, and the outputs – by OR-gate works. The initial version of the solution is obtained after adding the sign gates (top panel in Fig. 8).

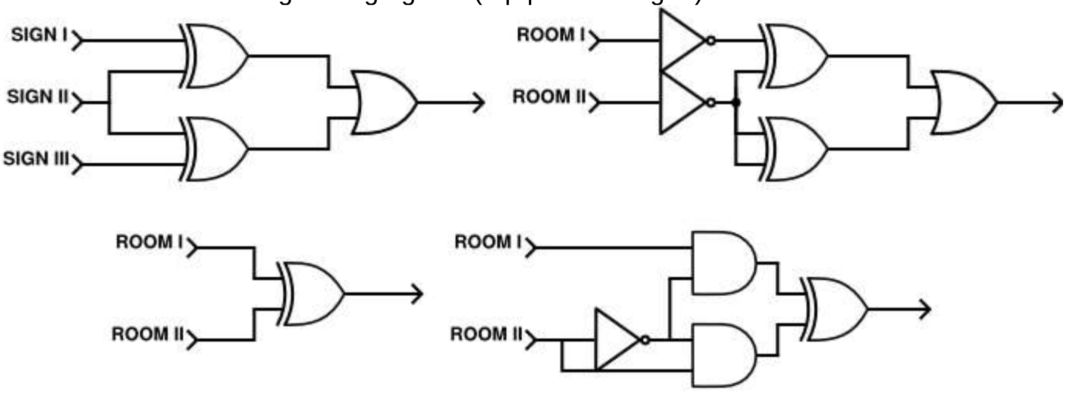

Figure 8. Circuits of Trial 10: rule and initial (top), reduced and final (bottom)

The bottom XOR-gate gets identical inputs and, therefore, outputs 0. So, the result of the entire circuit is determined by the top XOR-gate that checks if Room I and Room II inputs are identical. It is an example of circuit reduction. The solution is finalized by inclusion of the first part of the trial rule.

3. Conclusions

We presented a set of logic puzzles of increasing complexity and demonstrated a gate-based solution method. They serve as consistent exercises for introduction to the main concepts and properties of logic gates and circuits. Below we suggest a sample syllabus for a 4.5-hour block on logic gates within a 45-hour Introduction to Computer Science course:

Trials	In-class	Quiz	Homework	Project
1			+	
2	+			
3		+		
4			+	
5	+			
6		+		
7			+	

8		+		
9	+			
10				+
11				+

Table 5. Sample syllabus

More importantly, the method illustrates the idea that logic gates are not limited to the field of electronic design and binary calculations, but serve as a universal computing model. Finally, the approach allows students concentrating on the problems and developing solving skills, rather than merely memorizing tools the gates are by their nature. They are effectively constructing their own specific computers for each puzzle. In the next paper we will investigate possibilities of hardware implementation of the presented solutions.

References

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