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The Role of Mathematics in Quantum Physics for High School Students: a Case Study

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Abstract

Within the research literature in STEM education, documented difficulties concern the epistemological issues arisen by the relationship between the technical aspects of mathematical models and the empirical reality. The transition from classical to quantum physics makes this aspect more problematic because of the need to give up familiar images or space-time descriptions. These problems were addressed by designing and implementing a teaching/learning path whose design principles cohere with the theoretical construct of 'Appropriation'. In this paper, we focus on a case study built on the analysis of an interview in which a female student expressed a problematic position towards mathematical reasoning in physics. The analysis of the student's discourse is based on Habermas's rationality construct. We show both the productiveness and the limits of her forms of epistemic rationality in appropriating quantum physics.

Keywords: Epistemic rationality; Mathematical models; Quantum Physics, High-school students; Appropriation

1. Introduction

The teaching/learning of quantum physics (QP) is an articulated educational issue in which sophisticated mathematical models are intertwined with epistemological obstacles [1][2]. Even more than in classical physics (CP), QP requires to pose and discuss questions like "*What does the model represent? Where and how do the mathematical outputs of the model relate to reality?*". The use of mathematical models in science is often an obstacle because of their abstractness. In the transition from CP to QP this aspect becomes more problematic, because the criticalities associated to the observation and visualization of quantum phenomena. The research group in STEM education of the University of Bologna developed an educational reconstruction that problematizes the epistemological and metacognitive issues that characterize the mathematical models used in QP [1][2]. One of the main goals of the path was to promote a process of 'Appropriation' through which students are encouraged to attribute personal meanings to words and expressions of scientific discourse, loading them with nuances which reflect their epistemological positions [3]. Since Appropriation is strictly related to the formation of identity in a lifelong learning perspective, it is also relevant for adult learning, both for formal and informal contexts. Indeed, by using Appropriation is possible to design activities based on authentic scientific concepts in order to develop not only knowledge but also skills that are relevant for facing the complex problems of this century [4]. The path was experimented with high-school students. The main goal of this paper is to show the emergence of the games between different epistemic rationalities in students' discourses. We focus on a specific case study built up on the analysis of a student's interview, carried out by adapting a model based on Habermas' construct of epistemic rationality, which can be applied to scientific discourses requiring the validation of statements.

2. Framework and research problem

The construct of rationality, introduced by Habermas in 1998 in reference to discursive practice, was adapted to the investigation of mathematical activity in educational contexts [5]. In this paper, we focus

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our attention on the epistemic rationality, “consisting in the conscious validation of statements according to shared premises and legitimate ways of reasoning” [7] in the case of a student facing the transition from CP to QP. Physical reasonings are more complex than the mathematical ones from the epistemic point of view, since they require logical inferences and modelling processes in which observed phenomena are intertwined with empirical data, mathematical models, physical laws and theories [6]. The interweaving of different epistemic rationalities has already been observed in mathematical problem solving. In this case we considered relevant three kinds of epistemic rationalities, defined as follows:

- *Mathematical-inferential* (MI): physical theories are used as formal-inferential systems; the validity of reasonings relies on mathematical inferences on abstract objects. What follows logically from inferences can be considered a valid statement concerning the reality.
- *Empirical-experimental* (EE): reasonings are based on regularities in empirical data or phenomena; general statements are valid with certain degree of uncertainty, depending on how much they fit with data coming from observation of phenomena and experiments; what emerges from the data is real and allows to produce valid statements concerning the reality.
- *Modelling* (MO): aspects of the “real world” are described and interpreted in terms of physical models; the statements connect observations, experiments and models and are considered valid if theoretical outputs and empirical data are consistent. The ways of reasoning are mainly circular, from the observations and data to models and theory and back; the validity of reasoning does not imply our possibility to grasp the essence of reality but concern just the effectiveness of people description and interpretation of phenomena of the real world.

Given the problematic relationships between observations-experiments-models in QP, we considered interesting to investigate the role played by those different epistemic forms. We hypothesized that students might have personal criteria to accept and validate the new reasoning, giving idiosyncratically attention and importance to the different epistemic rationalities, according to their identities [3][5].

3. Context and methods

The path was implemented with high-school students in a 13-grade class (18-19 years-old) of a *Scientific Lyceum*. We choose the case of a female student whose reasoning was particularly characterized by the intertwining between different forms of epistemic rationality and we analysed the following data: i) two initial questionnaires concerning historical debates on QP; ii) final semi-structured interview. The analysis of the student’s discourse aims to show the multiplicity of epistemic rationalities that lie behind her discourse. In the interview transcripts, we looked for sentences in which the students referred to validation criteria, ways of reasoning and premises that could be framed in the classification of epistemic rationalities proposed above. Then we looked for differences between epistemic rationalities used in CP and QP.

4. Data analysis and discussion

In the initial questionnaire, the student stated that the mathematical model had always helped her to connect experiments and theories and, in the first test, she introduced precise mathematical explanations (MI). In the final interview, she focused immediately the attention on physical phenomena in terms of electromagnetic interaction between objects (EE) and not in terms of mathematical models. The analogy between experiments is proposed in terms of same role of the measuring processes; experiments were presented in depth and was very careful in separating the phenomenon and the model used to interpret it, giving priority to the experimental setting and observations and more relevance to *a posteriori* tools. The MO rationality was always weak. In the interview, her attention was focused on three experiments that opened the path towards the discretization, presented by her as the “right” description of reality. The Stern-Gerlach experiment was said to be the most traumatic event, because it destroys the causality principle and the continuity assumption. The prevailing epistemic rationality was EE. Metaphors are considered unsuitable for her comprehension; in particular she disliked a metaphor that completely neglected the senses ("it was like losing the touch, the sight..."). CP was identified with: i) describing phenomena by means of trajectories and differential equations; ii) continuity of "everything", discrete measurements of continuous entities; iii) nature of the objects independent from measuring instruments and processes. Mathematics was crucial in her characterization of CP and was at the basis of her main epistemic rationality in physics before QP. We comment some excerpts, labelling the previous codes the most relevant sentences, according to the criteria listed before. In the first excerpt, it emerges that she was used to "convert" every reasoning into a mathematical reasoning, necessary for her understanding.

> *1. I (Interviewer): What was the useful way to understand quantum indeterminacy and its revolutionary role?*
> *2. S (Student): The hyperbole! I thought Δx=h/Δp, which is exactly a hyperbole. I put y and x, and when one became more precise the other less precise. I understood it in that way. I feel I am a mathematician. Mathematicians are like that, you know? They go always into detail!* **(MI)**
> *3. I: Can you give me an example of how your approach led you to understand?*
> *4. S: In the photoelectric effect. Up to a certain frequency there was nothing, it was constant, and then it increased like a straight line and so h was the angular coefficient between energy and frequency and it determined that they were linearly linked. This made me understand.* **(MI)** *[...] Again, I understood the amplitude of probability with the circle: calculate the angle, the sine and the cosine according to the angle and then the probability as the square of sine and cosine, the first fundamental relationship.* **(MI)**

Even though her interpretation of the indeterminacy principle is currently not acceptable, the example shows her attitude to move to the MI rationality as long as she could.
In the second excerpt, when encouraged to reflect about the differences between CP and QP, she said she felt confused and mentioned a crisis in her prevailing epistemic rationality (MI): mathematics was criticized since it "tried to impose the continuity" and physics reasonings resulted to be identified with the EE rationality, that seems to better grasp the essence of reality.

> 5. *S: A theory based on the continuum could not go on. People started to wonder what their roles were within the reality and the experiment.* **(EE)** *[...] we have been so much accustomed to believe that everything is continuous. We have stopped to ask ourselves the question: is there anything behind? Is there anything discrete behind?*
> 6. I: *What is your point of view now?*
> 7. S: *Actually, now everything is discrete. Mathematics is a bit strange within this discourse, because it tries to show the continuum in every possible way. It tried to impose the continuity! […] The fact that the line is continuous, that it passes through all the points. It seems to me a stretch, after established that the reality is discrete.* **(EE)***.*
> 8. I: *So, how can continuous mathematics and discrete physics be combined? Did you try to do it?*
> 9. S: *Indeed, I don't undertand why they go on distancing themselves more and more. Mathematics seems a "superbeing", while physics is close to reality* **(EE).** *Perhaps mathematics tricks itself… I said that describes reality but it does not seem to me that this is the truth, if it approaches it with the continuum. Now I have doubts that I must solve.*

The student approached physics almost exclusively with a MI rationality (line 2-4), then she reported a change in her prevailing rationality and explains that she moved to the EE one, identifying "physics" with such a way of reasoning (line 5; 7-9). In lines 7 and 9 she presented the main cause: mathematics was "blind", trying to impose the continuity and resulting to be not effective in grasping the true nature of reality. Mathematics had provided her with metaphorical and argumentative tools to represent phenomena and reason on them following her mathematical inferences. Talking about metaphors, she declared that the means she uses to reach the comprehension can't betray the reality itself. She couldn't accept such an ambiguity in mathematical reasonings, since she had been used to trust them to grasp better the reality, to overcome what deceives human beings trying to investigate the reality deeply. The MO rationality was nearly absent in her interview.

5. Conclusions

The first result of this *case study* concerns the effectiveness of the Bologna path, built on Appropriation, in fostering the development of the student's scientific discourse. Indeed, the path made her reflect deeply and in a personal and authentic way on different approaches to the investigation of reality through the lenses of mathematics and physics. The crucial questions about the relationship between mathematical models and reality, abstract concepts and real objects, that we posed in the introduction, were addressed by the student, but the epistemic forms of rationalities she adopted did not allow her to engage properly with such an issue. She passed, seamlessly, from a form of rationality to another without finding a way to entail them as it is necessary to carry out a physical reasoning, in particular in the case of QP. Indeed, a pure MI rationality implies that: abstract objects are mainly used in reasonings; inferences on their properties guarantee the validity of a reasoning; real objects are cleaned up from discourses, focusing on the logical structure. In the case of a pure EE rationality, physics and reality are identified, giving to experimental data the power to determine what is real (e.g. "Now everything *is* discrete"). The MO rationality that could have helped her in managing the relation between mathematics, phenomena and empirical data is not taken in account seriously by the student as the "third way" to solve her internal conflict. The importance of a modelling epistemology is confirmed also by the result of another study [8] in which a refinement in scientific reasoning appears only when math models merged with epistemological reflections about the modelling process. The lens of different epistemic rationalities allowed us to describe the student's ways of reasoning and to identify the main source of difficulties in collocating QP in her background.

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