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Prospective Primary School Teachers' Difficulties when Dealing with Multiplying Fraction Word Problems

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Abstract

The connections between the learning of a concept and its teaching are a source of great concern to our Mathematics Education Department teachers, particularly in the case of fraction word problems. The TPACK framework provides appropriate tools for the study of this topic. We presented to 47 couples of prospective primary school teachers a multiplying fraction word problem in an equal sharing context. The problem was supposed to be solved in two ways: using graphic strategies (with a GeoGebra applet) or arithmetically. Additionally, our trainees were asked about the indications they would give to their future pupils in order to solve similar problems. All the tasks were assigned to one of the TPACK sub domains, and analyzed according to it. The most important findings were related to the predominance of the arithmetic-based methods over the computer-based ones. Moreover, a high percentage of the couples did not check the necessary coincidence of the results when solving the same problem in a different way. When comparing solving results and indications for pupils, we found that many couples with a mathematically correct answer delivered poor indications to their future pupils.

Keywords: *TPACK, fraction teaching difficulties, fraction learning difficulties;*

1. Introduction

Our prospective primary school teachers are used to work with different technological instruments in their everyday life. However, these instruments rarely become an essential part of their learning/teaching processes. Furthermore, they have been taught through traditional methods that included limited interpretations of the rational number. The goals of our work are: i) to analyze if they are willing to include technology in their teaching activities and ii) to study if they are prepared to overcome these interpretations of the rational number.

2. Theoretical framework

Firstly, we briefly introduce the basic concepts of TPACK (technological pedagogical content knowledge) framework. It shows the general domains our prospective primary school teachers' should cover in our course, and the interpretations of the rational number, which is the specific content of our work.

2.2 The TPACK framework

TPACK is the framework that we use to study the prospective primary school teachers' knowledge for technology integration, as explained in Koehler & Mishra [1]. This framework builds on Lee Shulman's construct of pedagogical content knowledge (PCK) to include technology knowledge. The acronyms in Figure 1 mean:

CK: Teacher's knowledge of the mathematical content, including concepts, theories, ideas, organizational frameworks, etc.

PK: Teacher's knowledge of the processes, practices and methods involved in the teaching and learning of mathematics.

PCK: Teacher's knowledge of the possible adaptations of the mathematical content to its teaching.

TK: Teacher's knowledge that permits him/her to do different tasks using IT and to find different ways of solving a given task.

TCK: Teacher's knowledge of the mutual influences and limitations of technology and content.

TPK: Teacher's knowledge of the changes that technology generates in learning and teaching.

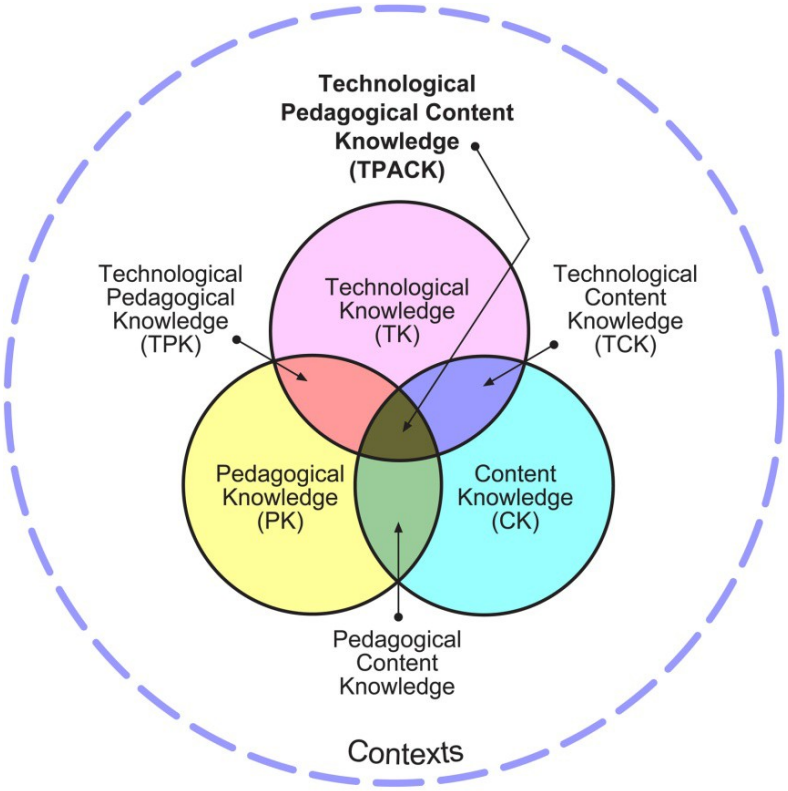

Fig.1. TPACK diagram. Source: http://tpack.org

2.2 The interpretations of the rational number

Five different constructs or interpretations of the rational number are classically accepted by Kieren [2]: part-whole, measure, quotient (division), operator and ratio. Following the explanations in [3]:

- Part-whole interpretation is related to the partition of a continuous quantity in equal-sized subparts.
- Measure interpretation is related to the parts in which the unit is divided (denominator) and the number of these parts considered (numerator) taking into account the fractional representation of the number.
- Quotient interpretation is related with the idea of equal sharing *a* objects or units with *b* persons if we consider the fractional representation of the number.
- Operator interpretation is related with the idea of multiplying the rational number by another number (possibly rational too).
- Ratio interpretation is related with the idea of comparing the sizes of two sets or two measurements.

Although our course covered all five interpretations, the part-whole interpretation is the most frequent in Spanish text-books. Freudenthal [4] explains how restricted this interpretation is, both phenomenologically and mathematically. Moreover, students are forced to believe that "½ times means the same as ½ of"" and learn arithmetical rules that build the concept of rational number. These facts, among others, lead to a mechanical understanding of the algorithms, but no to a true understanding of the concepts.

3 Methods and sample

In January 2017, a four-task questionnaire (a, b, c and d) was presented to 47 couples of prospective primary school teachers who had previously followed up the course "Teaching of the rational number in primary school":

Antonio had pizza for lunch with his friends on Monday and Thursday. On Monday they were 5 friends and shared 3 pizzas. On Thursday, they were 8 friends and shared 5 pizzas. On Monday, he gave one fourth of his food to his sister Sara, eating the rest of his lunch. On Thursday Antonio decided to eat all the food he received, but he dropped one fifth of it on the ground. Which day did Antonio eat the most? (Note: all the pizzas are alike.)

a) Solve the problem without using arithmetic operations, but using the graphic support of the given applet (available at https://www.geogebra.org/m/b3XaeVVV). Justify your answer. (You can use as many screenshots as you want to clarify the resolution.)
b) Considering your previous justifications, what could you say about the graphics used?
c) Solve the problem without using any graphic strategy, just by using arithmetic operations.
d) Imagine that you are preparing a mathematics class for your primary school pupils to teach them how to solve problems about comparing quantities coming from the application of operators. Describe step by step the mathematical instructions you would give to your students to teach them how to solve the given problem.

This activity was designed to connect the quotient, the operator and the measure interpretations. All of these interpretations had been previously studied by our students.
Each task has its corresponding sub-domain in the TPACK framework as follows: a (TCK), b (TCK), c (CK) and d (TPACK).

4. Results

In this work we show only the results corresponding to task 'd' (42 couples answered) due to the limited space that we have.

- 3 couples (out of 14) used the given applet in their instructions, even if they have used previously the applet to solve the problem.
- 2 couples considered in their instructions that the problem can be solved in more than one way.
- None of the couples suggested that the answer of the problem could be checked by solving it by a second method.
- 14 couples based their instructions in the measure interpretation of the rational number.
- 10 couples gave no interpretation to the fractions in their instructions.

In Table 1, we present the references given in the instructions to the main mathematical topics (the operator interpretation and the comparison of quantities) and the contextualization level of the instructions, (classified as T-theoretical, A-abstract, Cn-concrete, Cm-complete). The number of corrects answers in each box is in brackets.

	T	A	Cn	Cm
No reference in the instructions to the operator interpretation nor the meaning of the comparison	1 (0)	1 (1)	1 (1)	
References only to the operator interpretation	2 (2)	1 (0)	9 (6)	
References only to the meaning of the comparison	5 (5)	0 (0)	2 (0)	
References to both of the concepts	7 (3)	2 (0)	5 (3)	6 (3)

Table 1. Mathematical topics and contextualization level

Results about contextualization level and mathematical content in the instructions:

- 20 out of 42 couples considered both mathematical aspects in their instructions. These couples are divided in three almost balanced groups: T, A/Cn and Cm.
- 27 couples considered the meaning of the comparison in their instructions.
- 32 couples considered the operator interpretation in their instructions.
- 68% of the couples did not consider both mathematical aspects have correct mathematical answers.
- 45% of the couples considered both mathematical aspects have correct mathematical answers.

To illustrate our findings, we show in Figure 2 part of the instructions written by a couple with an incorrect mathematical answer but complete instructions.

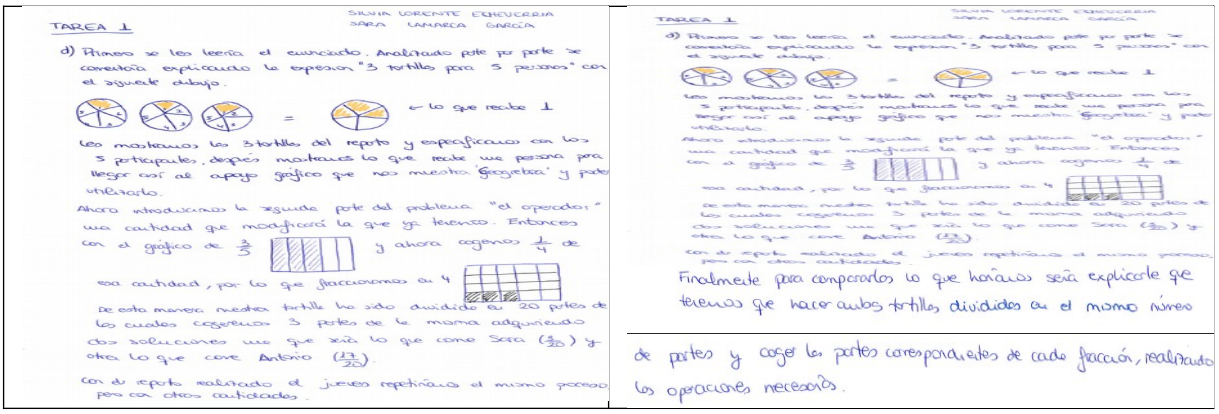

Fig.1. Complete instructions

5. Discussion

Very few couples decided to use technology, which could be due to the traditional instruction received during their school days. In this sense, they did not modify their approach to mathematics, and gave more value to a traditional/numeric answer over a technological/graphic one.

Only two couples considered solving the problem in two different ways, and none proposed to check the answer by solving it in two different ways. These heuristic strategies have been considered very important to learn mathematics in previous studies [5]. Moreover, these facts make a great contrast with the approach we take in our course, where we solve problems with rational numbers by using different interpretations that imply different solving techniques.

Ten of the couples in our sample missed any interpretation of the rational number when writing instructions in task d, and focused them in a formal explanation of the arithmetic operations. This fact could be related with the preponderance of the part-whole interpretation of fraction [6].

The prospective teachers emphasized the most difficult mathematical aspects, mainly the operator interpretation and the meaning of the comparison. From this point of view they may have thought that comparison is easier to be understood by a primary school kid than the operator interpretation. Moreover, instructions about comparison are shown to be more theoretical than the ones about the operator interpretation.

We observed that couples with more complete instructions have had worse mathematical answers than couples with incomplete instructions. We link these facts to the pedagogical difficulties shown by others with a better domain of the mathematical content, probably due to the fact that many preservice primary school teachers think that traditional school mathematics content is not difficult and, hence, it needs few explanations [7, 8].

6. Consequences for the teaching training in mathematics education

Some ideas could be taken into account for the design of future activities in Mathematics Education for prospective primary school teachers:

- To include tasks covering all the TPACK subdomains.
- To combine different interpretations of the rational number.
- To promote the use of one technique by making more difficult the use of the others. It means, for example, we have to to use higher figures in the activities to promote the use of GeoGebra by making more difficult for them the use of other techniques.
- To include actual answers of primary school kids to analyze errors and give tips to correct them by using different techniques.
- To ask for an analysis of the mathematical content before writing the instructions.
- To include role-playing activities with prospective teachers to make them understand better that, when designing instructions, they should focus in pupils' troubles rather than in their own ones

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