Comparison of Mathematical Activities with Preservice Teachers: Manipulatives vs. Paper and Pencil

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Abstract

In the Spanish Primary Education curriculum, basic arithmetic properties are first introduced in the 3rd and 4th grades. Working with arithmetic expressions gives rise to relational thinking, which connects the algebra with the generalization of patterns and relationships. Hence, it is of vital importance to also consider arithmetic properties in the training of preservice teachers encompassing both scientific and pedagogical dimensions (MKT, Mathematical Knowledge for Teaching). This communication aims to compare aspects of mathematical knowledge (in particular, Specialized Content Knowledge and Knowledge of Content and Teaching) when explaining an arithmetic property using pencil and paper versus the use of manipulative materials. The sample consists of the answers of 35 preservice teachers to the following task: "Create a written explanation that demonstrates that the arithmetic property a:(b:c) = (a:b)×c is true (a, b, and c are natural numbers)". The variables used for this new analysis of written productions were: contextualization, variety of representations, choice of numerical values, meaning of intermediate operations, and property verification. Among the results found in both analyses, some similarities can be observed, such as the significance of numbers and the choice of numerical values. However, there are also certain differences, such as the contextualization present in explanations and the meaning attributed to intermediate operations. It is clear that conducting the activity with manipulative materials requires more time on the part of the university professor. Nevertheless, it fosters more engaging experiences for PSTs and, most importantly, closely aligns with their forthcoming teaching practice.

Keywords: *Preservice teachers, arithmetic properties, paper and pencil, manipulatives.*

1. Introduction

The arithmetic properties for addition, subtraction, multiplication, and division of natural numbers are part of the current Spanish primary education curriculum, particularly introduced for the first time in 3rd-4th grade [1]. Working with arithmetic expressions gives rise to relational thinking, connecting the algebraic part with the generalization of patterns and relationships. This enables the examination of expressions globally and utilizing them to solve a problem, make a decision, or continue learning about a concept [2]. Furthermore, considering relational thinking involves examining arithmetic expressions and equations as a whole, i.e., using the properties of operations to relate expressions [3]. Therefore, it is crucial to also consider arithmetic properties in the training of future teachers.

The training in mathematical education that these future teachers receive should encompass both scientific and pedagogical aspects. Both aspects are outlined in the MKT (Mathematical Knowledge for Teaching) theoretical framework [4] based in the seminal work of [5]. This model includes two domains: SMK (subject matter knowledge) and PCK (pedagogical content knowledge). In turn, each of these two domains is divided into three subdomains. SMK includes subdomains such as common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK), while PCK is divided into knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). See Fig 1.

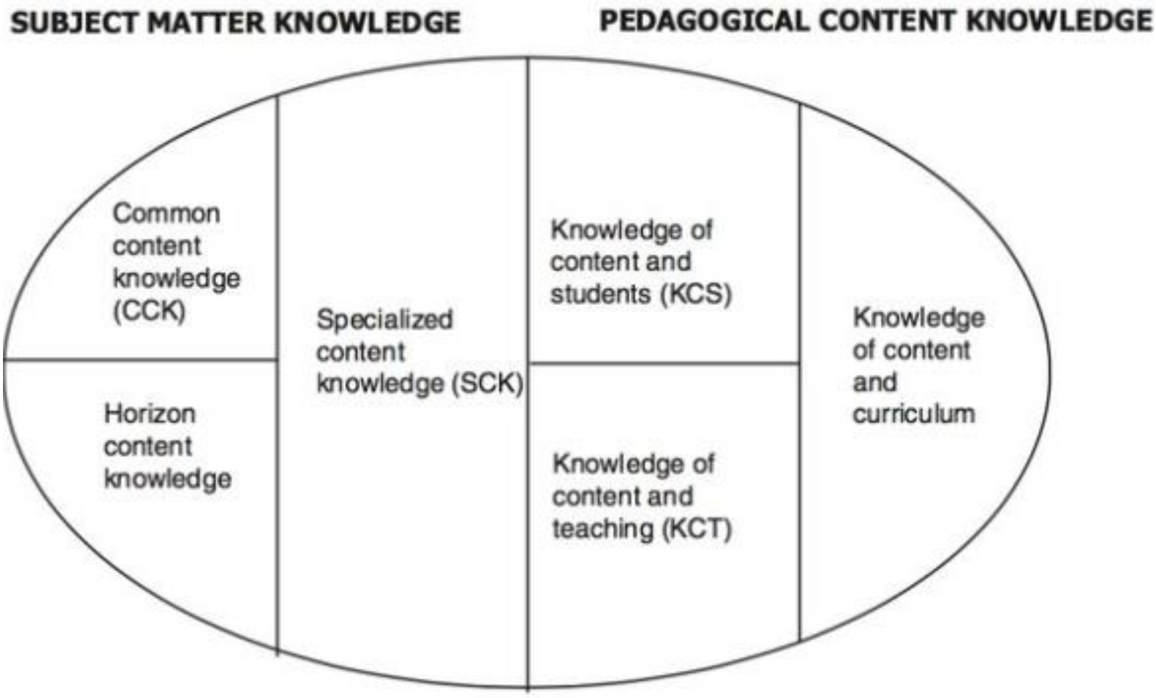

Fig 1. MKT model [4]

In a previous study [6], some aspects of the content knowledge and pedagogical knowledge of future teachers were described when explaining arithmetic properties using manipulative materials. Continuing precisely with this work, this study aims to compare both aspects of specialized content knowledge (SCK) and knowledge of content and teaching (KCT) when explaining an arithmetic property using pencil and paper compared to the use of manipulative materials.
The objective of this contribution is to compare the results obtained in this previous study with manipulative materials and those obtained when the task is presented for solving on paper.

2. Method

The sample consisted of 34 pre-service teachers, all of whom were in their second year of the Primary Education Degree program at a Spanish public university. Specifically, they were enrolled in the course Mathematics and its Didactics I. This course represented the first in their university education where mathematical properties of natural numbers were addressed. Prior to delving into the teaching and learning of natural numbers, the course covered three previous units: 1) legislative framework and curricular design in the Mathematics field, 2) planning and design in the teaching-learning process of Mathematics, and 3) assessment in the teaching-learning process of Mathematics. Each pre-service teacher was individually assigned the following task (Fig 2):

Verify the following property: $a:(b:c) = (a:b)xc$

Fig. 2. Task proposed to pre-service teachers.

Through a qualitative approach, the analysis of the following categories is conducted (Table 1). To facilitate comparison with the previous study [6], values are presented from both the analysis of the current study and the previous study.

Table 1. Variables and categories used in the analysis

Variable (subdomain)	Categories written task	Categories vídeo recorded task [6]
Contextualization (KCT)	Includes explicit context/does not include [7]	
Choice of numerical values (SCK)	No indication/some value is 1/all values are powers/quotient equal to third/different and not powers [8]	
Meaning of intermediate operations (SCK)	Division: Partitive/quotative [9] Multiplication: Repeated addition/meaningless	
Variety of representations (KCC)	No representation (only numerical expressions) / with graphical representation	NA
Variety of materials (KCC)	NA	Single material/different materials
Meaning given to indeterminate in relation to material (SCK)	NA	Representation only/variable/stable
Comprobación de la propiedad (SCK)	NA	Not tested/only with material/only numerically/with material and numerically

3. Results

Out of the 34 PSTs, 32 of them completed the assigned task (all except #29 and #34). We present below the most relevant results regarding the variables of contextualization, choice of numerical values, and meaning of intermediate operations. We chose these two variables as they are observable in both the written task and the videos analyzed in [6]).

3.1 Choice of numerical values

Regarding how the 32 PSTs choose the numerical values for a, b, and c at the beginning of their work, it is noteworthy that 30 of them approached the task with a single example, while the other 2 used two sets of different values, resulting in 34 different elaborations exemplifying the studied property. Referring to these examples, 4 of them present 1 as the third numerical value, in 10 sets, all values are powers of the smallest number, 6 sets show numerical values where the division of the first two results in the third value, and 14 sets exhibit numerical values that do not have any of the aforementioned characteristics.

Table 2. Variables and categories used in the analysis

	N (written task)	% written task	% video recorded task
c is equal to 1	4	11,76%	14,8%
Two of the numbers are powers of the smallest	10	29,41%	40,80%
Quotient equal to the third number	6	17,65%	25,90%
Different choices	14	41,18%	14,80%

To compare these data with those obtained in the previous study where students created a video, we present in Table 2 a summary with the percentages of each category in both studies. The most striking aspect is that the three categories in which the data are somewhat simplistically taken, meaning that the numbers have a strong relationship among them, have much higher percentages in the video study. This accounts for a total of 81.5% of the productions as opposed to 58.9% in the written responses. Another notable element in this comparison is that when intermediate quotients of the proposed numbers are calculated, in the case of written responses, there are 7 that yield non-integer results; whereas, in the video responses, no intermediate quotient is non-integer.

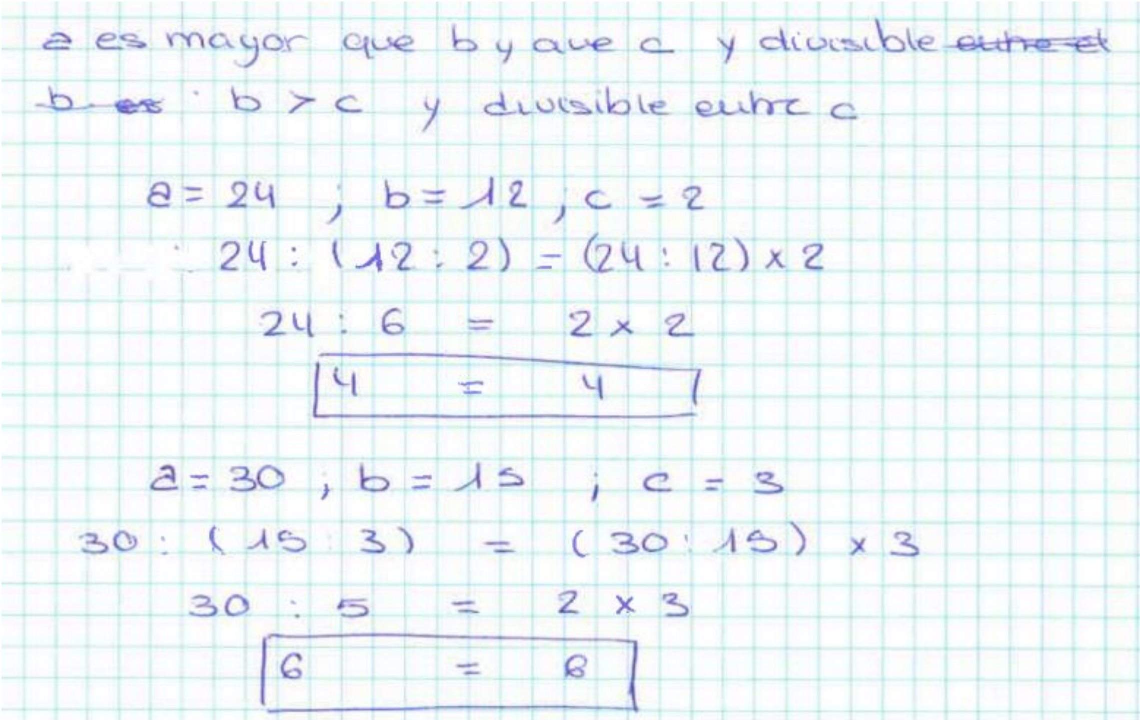

Fig 3. PST #2 who selects quotient equal to the third number

The above image (Fig 3) is presented as an example of a student who, although chooses a set (a=24, b=12, c=2) that appears to have intermediate structuring (c is not the unit, and the numbers are not powers of the third), states in their initial comment that the numbers must satisfy "a is greater than b and c, b > c, and divisible by c," which highlights a lack of understanding of the arithmetic rule as something that holds true for all natural numbers.

3.2 Meaning of intermediate operations

Most of the PSTs do not attribute any kind of meaning to the chosen numerical values or the operations they perform among them. Only 9 PSTs assign some meaning to the operations carried out, with distribution occurring in 8 out of the 9 cases. We differentiate those who do not attribute any meaning and also do not make any representation beyond numeric notation, which includes 16 PSTs, from those who graphically represent the cardinality of the selected numbers, amounting to 15 PSTs. See Table 3.

Table 3. Meaning of intermediate operations

Intermediate operation	written task			video recorded task		
	Distribution	Grouping	Total	Distribution	Grouping	Total
$b: c$	4 (12,5%)	1 (3,1%)	15,6 %	51.9%	25.9%	77,8 %
$a: (\quad)$	7 (21,9%)	0	21,9 %	55.6%	18.5%	74,1 %
$a: b$	5 (15,6%)	0	15,6 %	55.6%	18.5%	75 %
	Repeated addition			Repeated addition		
$(\,)xc$	2 (6.3%)		6,3 %	59,3 %		59,3 %

With respect to the meanings presented, it is noteworthy that in the video works, a certain balance was found between those who represented divisions as distribution (17 out of 77, 22.1%) and those who did it as grouping (44 out of 77, 57.1%). In written works, out of the 17 divisions performed with some meaning attached to it, only one had the meaning of grouping, as opposed to 16 with the meaning of distribution.

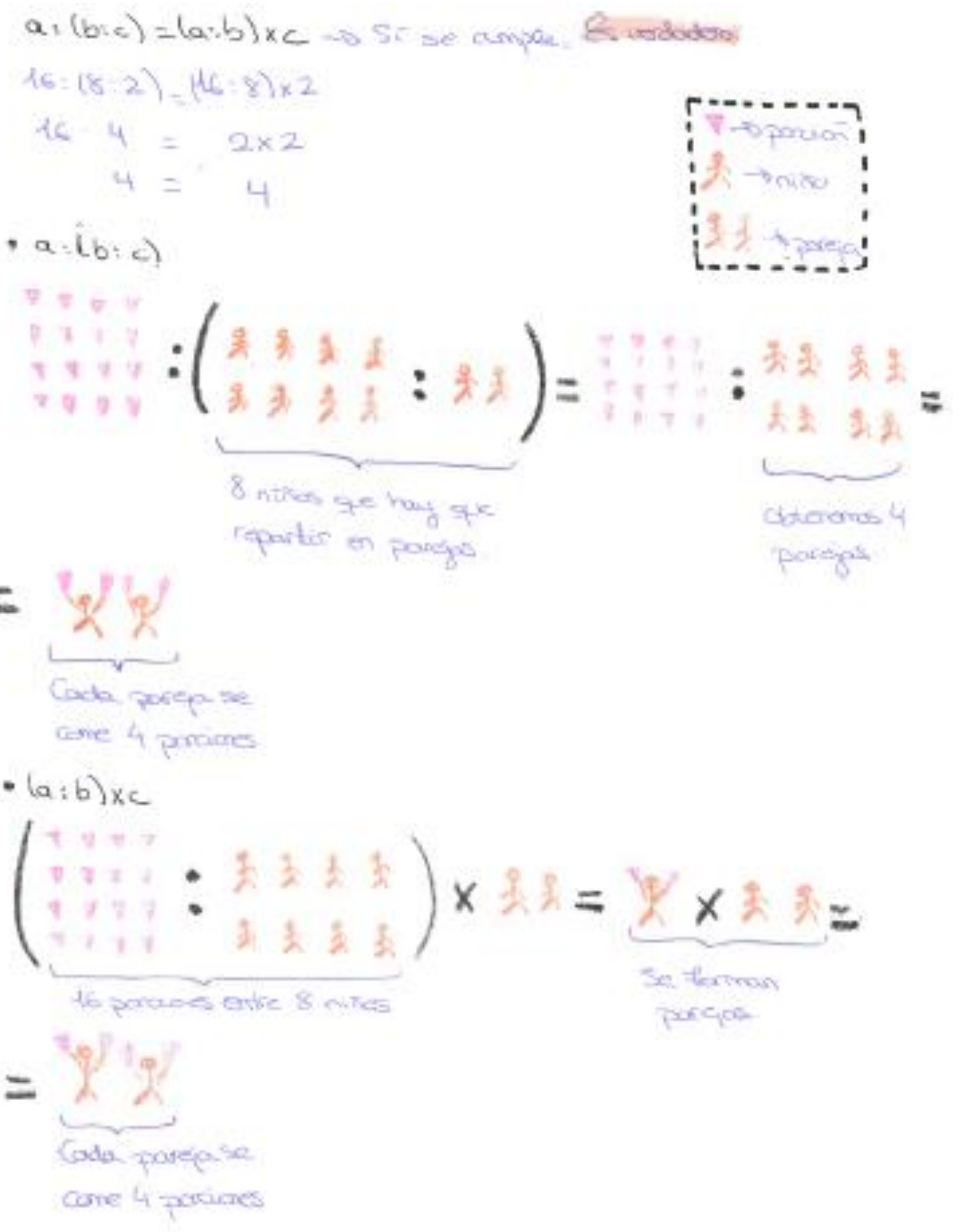

Fig 4. PST # 27 who combines the meanings of grouping and distribution

We present in Figure 4 the answer of PST #27 who combines the meanings of grouping (by pairing with the 8 children in the operation 8:2) and distribution by distributing the portions represented by a pink triangle, giving 4 portions to each pair. It is noteworthy that this student is the only one who has combined both meanings in the task.

3.3 Contextualization

None of the PSTs carried out any written contextualized task in the sense of presenting a word problem to solve, however student #27 used implicitly a context without presenting a word problem. In the task recorded on video, 18.5% of the students proposed a problem to solve in order to explain the task, with the distribution of candies among children being the most common context (Fig 5).

Fig 5. PST who proposed a context in the previous study (Arnal-Bailera & Arnal-Palacián, 2023)

4. Conclusions

Three out of the seven analysis categories that appeared in the previous study with manipulative materials [6] were deemed suitable for analysis when the task is presented in written form. These were contextualization, contributing to understanding in the KCT subdomain of the MKT; the choice of numerical values (SCK); and the meaning of intermediate operations (SCK).
Regarding contextualization, it has been observed that merely asking for the verification of a property is not sufficient to prompt students to create a context in which that property becomes visible. In the case of the choice of numerical values, students made a richer selection in the written task, generating examples with more internal relationships among the data compared to the task performed in video. Finally, concerning the meaning of intermediate operations, the percentages of operations to which some meaning beyond merely formal was attributed were much higher when manipulative materials were used.
In summary, in order to contribute to the development of the KCT subdomain with regard to the understanding of the property in concrete terms, the video along with the use of manipulative materials proved to be a suitable tool. However, if the aim is to attend to the development of SCK, the richness of the chosen sets was greater in the written examples, prompting us to consider that both ways of presenting the task have positive aspects.
As a future perspective that gives continuity to the present study, it is considered appropriate to repeat this same didactic proposal, incorporating as an additional task a reflection on the context, the choice of numerical values and the meaning of the operations involved, as we consider that these three aspects are the ones that favour a richness in the proposal made by preservice teachers.

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